Limit- Supplement

See If You Need This Video!

1. Solve the following limit.

$$\lim_{x\to\infty} \frac{1}{x}$$

   A. 0
   B. 0.01
   C. 0.0001
   D. 1
   E. Limit does not exist, because ∞ is not a fixed number.

2. Find $\lim_{x\to 0} f(x)$

$$f(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } x \neq 0 \end{cases}$$

   A. 0
   B. 0.5
   C. 1
   D. Not enough information is given
   E. Limit does not exist because it is not continuous.

3. Solve the following limit.

$$\lim_{x \to \infty} \cos x$$

A. 0
B. 1
C. -1
D. Limit does not exist, because ∞ is not a fixed number.
E. Limit does not exist, because $\cos x$ is oscillating at infinity.

4. Solve the following limit.

$$\lim_{x \to 0} [f(x) + g(x)]$$

Where

$$f(x) = 3x \quad , \quad g(x) = x^2$$

A. $3x$
B. x^2
C. $3x + x^2$
D. 0
E. Limit does not exist.

5. Solve the following limit.

$$\lim_{x \to 0} [f(x)\, g(x)]$$

Where

$$f(x) = 10^x \quad , \quad g(x) = x^{10}$$

A. 0
B. 1
C. 10
D. 10^x
E. Limit does not exist.

6. Solve the following limit.

$$\lim_{x\to\infty} \frac{2x^2 + sinx}{1 + x^2}$$

A. 0
B. 1
C. 2
D. Limit does not exist, because $\lim_{x\to\infty} \sin x$ does not exist.
E. Limit does not exist, because $(2x^2 + \sin x) \ll (1 + x^2)$ when $x \to \infty$.

The following 4 questions may needs differentiation. You can watch the first half of this video if you have not learnt it yet. But for the second part, please check the respective chapter first.

7. Which of the following(s) satisfies $\lim_{x\to\infty} f(x) = 0$?

   1. $f(x) = \cos x$
   2. $f(x) = \cos x/x$
   3. $f(x) = 1/x$
   4. $f(x) = \sin x/x$

A. 1 only
B. 3 only
C. 1, 2 only
D. 2, 3 only
E. 2, 3, 4 only

8. Solve the following limit.

$$\lim_{x \to 0} \frac{x^2}{\sin x}$$

A. 0
B. 1
C. 2
D. $2x$
E. Limit does not exist, because 0/0 is undefined.

9. Which of the following equals to

$$\lim_{x \to 0} \frac{x^3 + x^2}{1 - \cos x}$$

A. $\lim_{x \to 0} (0/0)$
B. $\lim_{x \to 0} [(6x+2) / \sin x]$
C. $\lim_{x \to 0} [(3x^2+2x) / \cos x]$
D. $\lim_{x \to 0} [(6x+2) / \cos x]$
E. 1

10. Solve the following limit.

$$\lim_{x \to 0} \frac{\sin x}{x - \sin x}$$

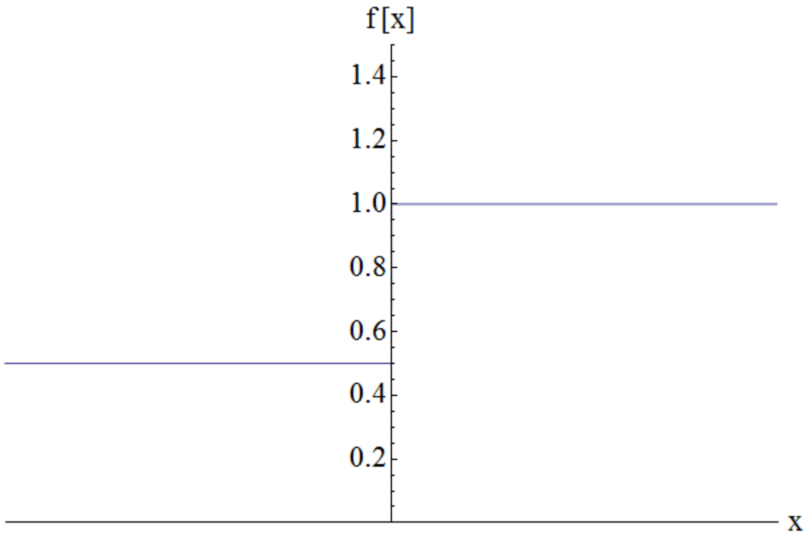
A. 0
B. 1
C. $\cos x$
D. $-\sin x$
E. Limit does not exist.

Learn More

Take a look at the following function:

$$f(x) = \begin{cases} 0.5 & \text{if } x \leq 0 \\ 1 & \text{if } x > 0 \end{cases}$$

If we plot it out, it looks like this.



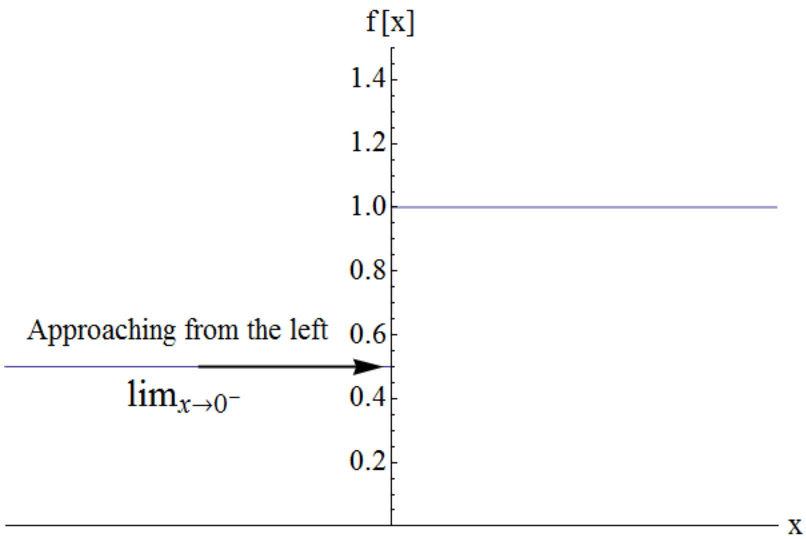
Here comes a question, what is the following limit?

$$\lim_{x \to 0} f(x)$$

Of course, the limit does not exist obviously. But there are terms like "left-hand" and "right hand" limit.

Left-hand Limit	$\lim_{x \to 0^-} f(x)$
Right-Hand Limit	$\lim_{x \to 0^+} f(x)$

We use 0^+ for right hand and 0^- for left hand limit. And from their names, you can immediately figure out what they mean. Left hand limit means approaching a point from left hand side(from smaller value). Like this



Then the left hand limit is thus

$$\lim_{x\to 0^-} f(x) = 0.5$$

With the same idea for the right hand limit. We have

$$\lim_{x\to 0^+} f(x) = 1$$

Try to figure out how to get this result by yourself. Back to our question, the reason for the limit $\lim_{x\to 0} f(x)$ does not existing is not due to oscillating nor blowing up to infinity but the non-equalling left and right hand limits.

$$\begin{array}{ll} \text{If} & \lim\limits_{x\to C^-} \neq \lim\limits_{x\to C^+} \\ \text{Then} & \lim\limits_{x\to C} \text{ does not exist} \end{array}$$

And Drill Deeper

We have a few challenges for you. If you really have no idea, take a look at the "Guide" and learn the way of thinking. The way to think may help you in solving problems even in real life. And no solution will be given for this part, just enjoy yourself. :)

Challenge 1.
Now open your mind to higher dimension, if a function gets 2 arguments like this

$$g\left(x,y\right)=\frac{xy^2+y}{x^2+y}$$

Try to show why

$$\lim_{x\to 0,y\to 0} g\left(x,y\right) \quad \text{doe not exist.}$$

Challenge 2.
Sometimes we may have to deal with functions with unknown exact form like this

$$0\leq g\left(x\right)\leq x^{-2}$$

What is the following limit?

$$\lim_{x\to\infty} g\left(x\right)$$

”Guide”

You can always get other ways to solve the problem. This ”guide” is just a helping hand, not the law.

Challenge 1.
When you learn something new, try to understand the reason at the back of what you learnt. You may extend your knowledge based on it though it is not easy sometimes.

1. My first suggestion is try to understand the arguments in the last section about left and right hand limits. What is the reason for the limit not existing?

2. Similar to the left and right hand limits, how many path can we approach $(x, y) \to (0, 0)$?

3. Can you choose a pair of them that helps you to show the limit does not exist?

Taking limit at higher dimension may need other coordinates system (not necessary for this challenge).

Challenge 2.
This challenge is not that challenging. You have learnt that $\lim_{x\to\infty} \sin x/x = 0$.
Do you remember why?

There is a famous theorem called ”Sandwich Theorem”, check on web and learn this useful theorem by yourself .