Coordinate Systems- Supplement

See If You Need This Video!

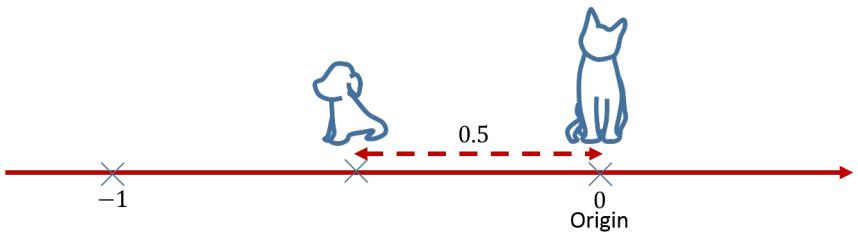


Figure 1: For Q1 and Q2

1. What is the coordinate of the dog?
   A. 2
   B. 0.5
   C. 0
   D. -0.5
   E. -1

2. The system is a...
   A. 1-dimensional system.
   B. 2-dimensional system.
   C. 3-dimensional system.
   D. no dimensional system.
   E. high dimensional system.

3. At least, how many numbers we should use to describe a position of a point in a 6-dimensional system?
   A. 2
   B. 4
   C. 6
   D. 8
   E. 10

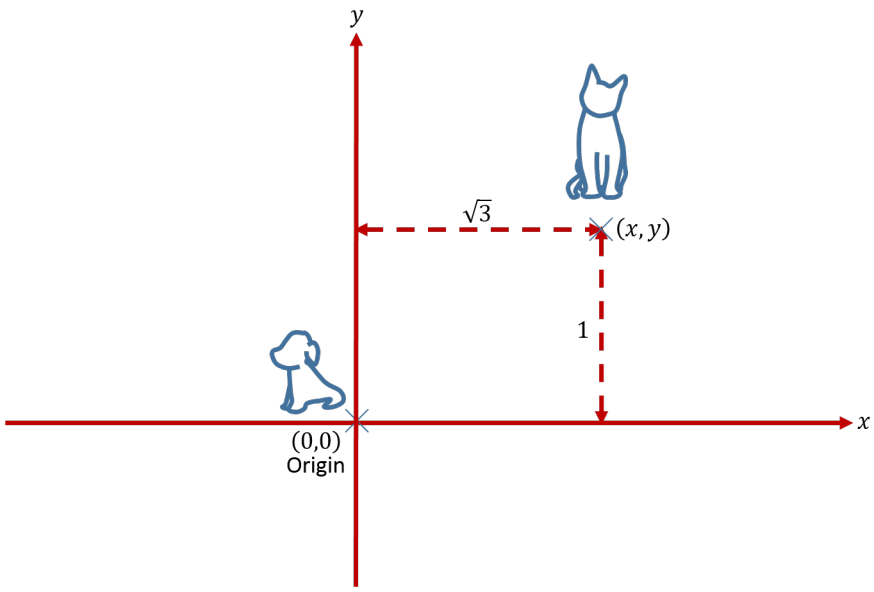


Figure 2: For Q4 to Q6

4. In reference frame of the dog, what is the coordinates (x, y) of the cat?
   A. 1
   B. $\sqrt{3}$
   C. $\left(1, \sqrt{3}\right)$
   D. $\left(\sqrt{3}, 1\right)$
   E. $(0, 0)$

5. In reference frame of the dog, find the polar coordinates (ρ, ϕ) of the cat.
   A. $(1, \sqrt{3})$
   B. $(\sqrt{3}, 1)$
   C. $(2, \pi/6)$
   D. $(2, \pi/3)$
   E. 2

6. What is the Cartesian coordinate of the dog, in cat's reference frame?
   A. $(0, 0)$
   B. $(\sqrt{3}, 1)$
   C. $(-\sqrt{3}, 1)$
   D. $(\sqrt{3}, -1)$
   E. $(-\sqrt{3}, -1)$

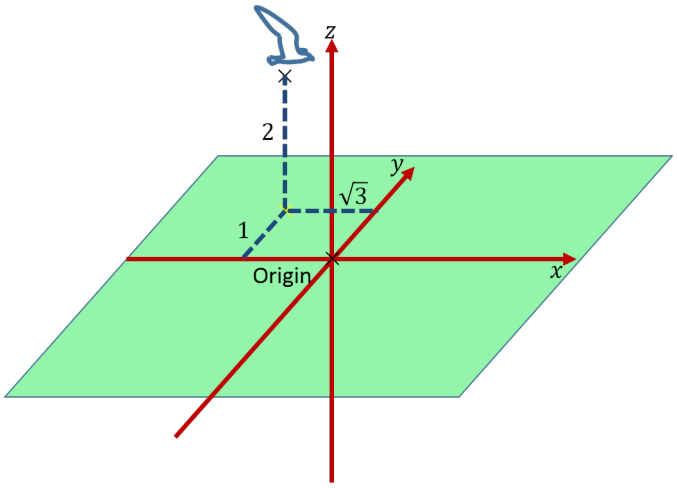


Figure 3: For Q7 to Q9

7. What is the Cartesian coordinates (x, y, z) of the bird?
   A. $(\sqrt{3}, 1, 2)$
   B. $(-\sqrt{3}, 1, 2)$
   C. $(-\sqrt{3}, -1, 2)$
   D. $(\sqrt{3}, 1, -2)$
   E. $(0, 0, 0)$

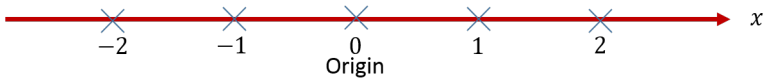
8. What is the cylindrical coordinates (ρ, ϕ, z) of the bird?
   A. $\left(\sqrt{3}, 1, 2\right)$
   B. $\left(-\sqrt{3}, 1, 2\right)$
   C. $(2, \pi/3, 2)$
   D. $(2, 5\pi/6, 2)$
   E. $(0, 0, 0)$

9. What is the spherical coordinates (r, θ, ϕ) of the bird?
   A. $\left(-\sqrt{3}, 1, 2\right)$
   B. $(2, 5\pi/6, 2)$
   C. $\left(2\sqrt{2}, \pi/4, 5\pi/6\right)$
   D. $\left(2\sqrt{2}, \pi/4, \pi/3\right)$
   E. $(0, 0, 0)$

10. Convert the spherical coordinates $(2, \pi/2, \pi)$ into Cartesian coordinates (x, y, z).
    A. $(-2, 0, 0)$
    B. $(2, 0, 0)$
    C. $(0, 2, 0)$
    D. $(0, -2, 0)$
    E. $(0, 0, 2)$

Learn More

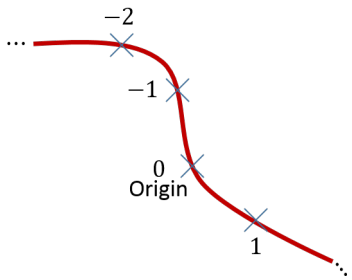
Before we start, consider the following one-dimensional system includes all points $-\infty < x < \infty$.



Please think carefully, why we say this is a one-dimensional system. Because we have only one axis? Formally, it is because all the points $-\infty < x < \infty$ are labelled with one number.

Consider all the points on the following line,

Of course we may describe them with 2-dimensional Cartesian coordinates. However, if we ONLY include the points on the line in the system, it is still a one-dimensional system, a curved one like this,



When you consider the dimension of a system, please think carefully how many parameters you need to describe all the points included.

That is why we can draw a map of the Earth on a piece of paper (2D) even though the Earth is a 3D object. The surface of the Earth is a curved 2D surface.

And Drill Deeper

We have a few challenges for you. If you really have no idea, take a look at the "Guide" and learn the way of thinking. The way to think may help you in solving problems even in real life. And no solution will be given for this part, just enjoy yourself. :)

Challenge 1.
In the video we mentioned the conversion from spherical to Cartesian coordinates,

$$\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned}$$

Please find carefully the formula for conversion from Cartesian to spherical coordinates.

Challenge 2.

A snail and a cat move in the same direction with different speeds as shown above.
What is the speed of the cat in the reference frame of the snail?
What is the speed of the snail in the reference frame of the cat?

You need the idea of vector in the last part.
Now, the snail moves in velocity $\overrightarrow{v_s}$, the cat moves in velocity $\overrightarrow{v_c}$. What is the velocity of the cat in the snail's reference frame? What is the velocity of the snail in the cat's reference frame?

"Guide"

You can always get other ways to solve the problem. This "guide" is just a helping hand, not the law.

Challenge 1.
You may obviously foresee that there will be arc-sine, arc-cosine or arc-tan, please think carefully how many points that these functions give you. Make sure your functions are one-to-one.

In physics, there are many cases that you have to transform the coordinates into other forms, so you have to make sure the transformation method is consistent.

Check your formulas with a few test points like Cartesian coordinates $(1, 2, 3)$, make sure your formulas will not give an other Cartesian coordinates when you have Cartesian$\rightarrow$spherical$\rightarrow$Cartesian transformation.

Challenge 2.
The first two questions are not difficult, just imagine if you were the snail or the cat, what would you see.

Separate the velocities of the snail and the cat in components, and repeat what you have done in the last two questions.