Partial Specialization and Heterogeneous Task Assignment

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Abstract

US inequality has increased dramatically during recent decades and much of the rise occurred among workers within the same occupation. This paper proposes a new mechanism for how demand has driven the rising within-occupation inequality: workers perform multiple and different tasks within-occupation. I develop a model that features worker-level partial specialization and heterogeneous task assignment, consistent with the data. I structurally estimate the joint skill distribution using the distributions of occupational wages and task assignments. The estimated model provides a close approximation to the observed changes in wages and employment, and accounts for most of the observed rise in inequality within-occupation.

JEL Codes: J22; J23.

Key Words: partial specialization, multi-dimensional skills, within-occupation task assignment, between and within occupation inequality

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"Fundamentally, assigning task measures to occupations overlooks all heterogeneity in task among individuals within an occupation, and only captures the demand feature of occupation-level tasks. It's self-evident that individual worker skills and actual job tasks differ among workers within an occupation, and it seems likely that these within-occupation skill-task assignments are an important component of the overall equilibrium relationship between skills and tasks. Thus, at best, occupation level task measures provide a rough approximation to the microeconomic assignment process." (Autor, 2013)

1 Introduction

Rising US inequality is one of the most important topics in economics (Katz and Murphy, 1992). Technological changes have been viewed as the main driver of US inequality (Autor et al., 2003, 2008, Goos and Manning, 2007, Autor and Dorn, 2013, among others). Crucially, most US inequality growth occurred within-occupation.[1] While this suggests understanding mechanisms driving within-occupation inequality is at the heart of studying the overall inequality, to date much evidence examines between-occupation outcomes rather than within.

This paper proposes a new mechanism for how demand has driven rising inequality within-occupation: workers perform multiple and different tasks within-occupation. I begin using a module of the Princeton Data Improvement Initiative (PDII) to provide supporting evidence on the mechanism. Within narrowly defined occupations, (1) workers perform multiple tasks, and the frequency of performing each task differs across workers; (2) workers sort positively into tasks—highly educated workers more frequently perform tasks that have higher returns (cognitive or social tasks);[2] and (3) the heterogeneity in task performance (and sorting) accounts for a significant portion of the earning variation within occupations.

I bring these important aspects of the data into an assignment model that features partial specialization and heterogeneous task assignment within-occupation.[3] The model explicitly distinguishes occupation, tasks, and skills. I consider four-dimensional skills (cognitive, social, routine, and manual) that are used in all occupations. Each type of skill produces a specific task (e.g., cognitive skills for cognitive tasks). Tasks are bundled to produce occupational outputs; however, occupations differ in task demand. Demand changes take place in two distinct forms: occupation-specific productivity changes and relative task demand changes within occupation. Each worker chooses an occupation. Once entering the occupation, she chooses *one* task that maximizes utility at each time but performs *multiple* tasks over time. I structurally estimate the general equilibrium model using micro-data from the Current Population Survey (May/ORG CPS) and the PDII.

[1] See Appendix A for a variance decomposition. Similar decomposition results also appear in Burstein, Morales and Vogel (2019). See Edmond and Mongey (2022) for similar findings in residual inequality.

[2] This fact has been documented in Autor and Handel (2013).

[3] Throughout the paper, I use task assignment interchangeably with time allocation.

I use the general equilibrium model to address two questions. First, wage has evolved deferentially across US occupations: the inequality increases substantially at cognitive-intensive occupations but changed modestly at manual or routine-intensive occupations (see section 5.2). I ask whether and to what extent the demand changes can account for these patterns, and how does the inequality impact depend on the new mechanism proposed.[4] Second, Atalay, Phongthiengtham, Sotelo and Tannenbaum (2020) recently document that the changes in task content changes within-occupation are at least as pronounced as the task changes resulting from changes in occupational employment. I quantify how the task demand changes within-occupation have contributed to the overall and within-occupation inequality, relative to occupation-specific productivity changes.

In the model, workers earn from multiple tasks. Comparative advantage determines the earning shares of each task and shapes the within-occupation inequality response. For example, workers who have a cognitive comparative advantage have high earning shares in cognitive tasks for two reasons. First, their relative efficiency units supplied in cognitive tasks is high per unit of time. Second, they allocate more time to cognitive tasks—their cognitive-task supply is even higher. An increasing cognitive task demand benefits high cognitive-comparative-advantage workers more, and would increase within-occupation inequality if they are concentrated among the top wage earners.

Taking the model to the data requires information on the skill distribution. I estimate the joint skill distribution by exploring the distributions of occupational wages and task assignments under parametric assumptions. Following Ohnsorge and Trefler (2007) and Adão (2015), I transform the equilibrium systems in terms of comparative and absolute advantage schedule, under which, I estimate of distribution of comparative advantage schedules, separately from the absolute advantage. Because task assignments are only determined by comparative advantages, I first explore the joint distribution of task assignments to identify the variance-covariance matrix of log comparative advantages, up to scale. Intuitively, more dispersed cognitive task assignments (relative to a baseline task) within-occupation suggests a high variance for cognitive comparative advantage; and if workers performing more cognitive tasks are also inclined to social tasks, this reveals a strong positive correlation in comparative advantages between the two.

Second, I estimate the remaining skill parameters to target three sets of micro-level moments (employment, the mean and variance of log wage) by groups and occupations. These skill parameters are estimated specific to 16 demographic groups (4 education categories, 2 age groups, and gender). Since employment and wages depend on equilibrium task prices,

[4] In the data, there has been a pronounced increase in inequality within cognitively- and socially-intensive occupations, yet inequality increased modestly or even compressed within manual/routine-intensive occupations.

the estimation is carried out while fully solving the general equilibrium model. I implement a two-step Simulated Method of Moments (SMM) procedure and explore cross-sectional variation in the year 2000: as relative task demand differs across occupations, variation in the group-occupation log wage distribution disciplines the location of log comparative advantages and the correlation between log comparative and absolute advantage; the distribution of absolute advantage is disciplined by the aggregate group-level wage distribution, as absolute advantage shifts wages up or down by the same amount for all occupations.

The estimated model reveals two main findings. First, the prices of cognitive, social and manual tasks, relative to routine, have all increased, for all occupations. Second, within each cognitively or socially intensive occupation, comparative advantage in cognitive and social relative to routine skills is positively associated with earnings; among manual or routine occupations, however, comparative advantage in manual relative to routine skills varies modestly. These patterns of comparative advantage indicate that, as the relative prices change, inequality grows significantly for cognitive/social occupations but modestly for manual/routine occupations. I show the general equilibrium model fits reasonably well with the untargeted changes in the occupational wage distribution for cognitive, social, and manual-intensive occupations.[5] I also perform extensive validation exercises to show the model can capture other untargeted micro-moments.

Having validated the model, I quantify the inequality implications of demand changes involving two forms: occupation-specific productivity and within-occupation relative task demand. Using measures of log wage variance and the percentile wage gap, I find the impacts of relative demand changes within-occupation are at least twice as large as the impact of occupation-specific productivity. I also find that the within-occupation relative task demand changes are the primary contributor to the rising within-occupation inequality and inequality at the top end. These findings are similar using alternative occupational classifications.

The key mechanisms that generate unequal within-occupation wage response are partial specialization and heterogeneity in task assignments. I analyze the quantitative role of each feature in two alternative models, where I re-estimate each model to target the same sets of moments. First, I analyze a Roy model where all skills are paid at the same price. In this model, partial specialization and task assignments are both absent. The model predicts limited within-occupation inequality responses and loads much of the inequality effects into the between-occupation component. Second, I evaluate the prediction of a model that has partial specialization but common task assignments across workers. In this model, wages are linear in comparative advantages and the estimated model requires the skills to be more

[5]The model fits routine-intensive occupations less well.

dispersed to match the cross-sectional wage distribution in 2000. The model predicts nearly linear wage changes within-occupation that cannot match the observed smooth occupational wage changes at the bottom end of the distribution.

The task-based approach has been the workhorse tool to analyze the inequality impact of technological changes (Autor, Levy and Murnane, 2003, Acemoglu and Autor, 2011) and trade (Grossman and Rossi-Hansberg, 2008). Lindenlaub (2017) develops a novel theory of sorting and inequality in the context of multi-dimensional skill. An extensive and growing literature built upon the Fréchet-Roy model (Acemoglu and Autor, 2011, Lagakos and Waugh, 2013, Hsieh, Hurst, Jones and Klenow, 2019) to perform model-based counterfactuals, and much of existing work examines variation across occupations (Burstein, Morales and Vogel, 2019, Atalay, Phongthiengtham, Sotelo and Tannenbaum, 2018) or industries (Adão, 2015, Galle et al., 2020) but is silent on within variation. One related exception is Helpman, Itskhoki, Muendler and Redding (2017), who use Brazilian employer-employee data to study within-occupation inequality. In their work, the source of within-occupation inequality arises from differential firm exposure to trade shocks. My contribution is to introduce partial specialization and task assignment into a Roy model of occupational choice. This is an important contribution for two reasons. First, conceptually, my model generates one-to-many task assignments that are heterogeneous across workers, inspired by the Ricardian model (Eaton and Kortum, 2002)—these are the mechanisms generating unequal responses within-occupation. Second, I show the general equilibrium model can be structurally estimated using micro-data, at a more granular level with multi-dimensional skills, occupations, and demographic groups, without relying on external measures of skills.

Growing empirical evidence has suggested the returns to skill are multi-dimensional (Heckman and Kautz, 2012) and that the returns to skill have increased more for social skills (Deming, 2017). A few recent studies using data from job postings document a dramatic shift in the tasks demanded within occupations (Atalay, Phongthiengtham, Sotelo and Tannenbaum, 2020, Hershbein and Kahn, 2018). Since workers tend to perform different tasks within occupations (Autor and Handel, 2013), in theory, they would be unequally affected by demand shifts (Costinot and Vogel, 2010, 2015). Complementing these studies, I develop and estimate a general equilibrium model that incorporates these salient features. My analysis shows that demand changes, specifically, the dramatic changes in the task content within US jobs, played a major role in driving the overall inequality increases and were the major contributor to the rising inequality within-occupation.

The paper is organized as follows. Section 2 presents motivating facts. Section 3 describes the model. Section 4 structurally estimates the model and skill distribution. Section 5 examines my model fitness to the data. Section 6 quantifies the inequality implication of demand changes, and Section 7 concludes.

2 Motivating Facts

The focus of this paper is to explain the rise of within–occupation inequality. To highlight its importance, I decompose the total variance of log wages into within- and between-occupation components. Using different levels of occupational classification with or without composition adjustments, I find the within component always accounts for the majority of the changes in the log wage variance between 1980 and 2000. See Appendix A for details. In what follows, I use the PDII data to present facts that motivate my quantitative model.

2.1 The Princeton Data Improvement Initiative (PDII)

The PDII collects representative samples of US workers and has been analyzed in a few reduced-form studies, e.g., Autor and Handel (2013). Of the 2513 US adults interviewed, 1333 provided information on wages, demographic characteristics, occupations, and, importantly, how often they perform different types of tasks. These task variables are categorically coded in two ways. One asks the frequency of performing certain tasks (e.g., using advanced math such as algebra or geometry). The variable is coded into five categories: (1) never; (2) less than a month; (3) monthly; (4) weekly; and (5) daily. Another asks the proportion of the workday used to perform tasks (e.g., managing or supervising workers). This variable is coded into four categories: (1) almost none; (2) less than half; (3) more than half; and (4) almost all.

2.2 Facts

Fact 1: Workers perform multiple tasks. In the data, among workers who use advanced math on a daily or weekly basis, 38% spend more than half of their time supervising other workers, 42% spend more than half of their time performing repetitive tasks, and 59% spend more than half of their time performing physical tasks involving standing, operating machinery or vehicles, or making or fixing things by hand.

To summarize, I define a binary variable to represent whether a worker often performs a given task, which equals one if they do so more frequently than on a weekly basis or for more than half the workday, and zero otherwise. See Appendix B.1 for details. It appears that 78% of workers often perform at least two tasks, among which 41% often perform two tasks, 29% often perform three, and 8% workers often perform all four.

Fact 2: Task sorting on observable characteristics within-occupation. I estimate the following regression

$$T_k^i = \sum_j \beta_j X_j^i + \alpha_o + v^i, \quad k \in \{\text{Cognitive, Social, Routine, Manual}\}. \tag{1}$$

where T_k^i is worker i's type-k task intensity, X_j^i is observable characteristics j, α_o is occupational fixed effects, and v^i is a residual. β_j captures the differences in task sorting across observable characteristics within-occupation.

Table 1: Reduced-Form Evidence Using PDII Data

	A. Estimated Coefficients for Equation (1)					
	Cognitive			**Manual**		
	(1)	(2)	(3)	(4)	(5)	(6)
HS graduate	0.0152	0.0492	0.0330	-0.00653	-0.0215	-0.0157
	(0.0262)	(0.0261)	(0.0279)	(0.0208)	(0.0207)	(0.0212)
Some college	0.0869	0.0907	0.0523	-0.0216	-0.0137	0.0123
	(0.0278)	(0.0274)	(0.0291)	(0.0220)	(0.0218)	(0.0221)
College and above	0.144	0.123	0.0791	-0.135	-0.124	-0.102
	(0.0289)	(0.0284)	(0.0310)	(0.0230)	(0.0226)	(0.0235)
Age	0.00996	0.00431	0.00180	-0.00876	-0.00597	-0.00581
	(0.00373)	(0.00359)	(0.00385)	(0.00296)	(0.00285)	(0.00292)
Age^2	-0.000135	-0.0000746	-0.0000479	0.0000817	0.0000533	0.0000554
	(0.0000465)	(0.0000446)	(0.0000476)	(0.0000369)	(0.0000354)	(0.0000361)
Male	0.0492	0.0185	0.0235	-0.00557	-0.00171	-0.0121
	(0.0159)	(0.0158)	(0.0174)	(0.0126)	(0.0126)	(0.0132)
Black	-0.0149	0.000228	0.0196	0.0384	0.0336	0.0269
	(0.0225)	(0.0221)	(0.0234)	(0.0179)	(0.0175)	(0.0178)
2-digit Occup.	✓			✓		
3-digit Occup.		✓			✓	
5-digit Occup.			✓			✓
	B. Estimated Coefficients for Equation (2)					
Cognitive	0.358	0.255	0.215	0.282	0.240	0.208
	(0.0673)	(0.0679)	(0.0704)	(0.0646)	(0.0662)	(0.0691)
Social	0.0771	0.0283	0.0523	0.0964	0.0476	0.0754
	(0.0530)	(0.0535)	(0.0558)	(0.0505)	(0.0519)	(0.0546)
Routine	-0.269	-0.207	-0.211	-0.160	-0.124	-0.153
	(0.0536)	(0.0527)	(0.0549)	(0.0514)	(0.0514)	(0.0538)
Manual	-0.585	-0.560	-0.538	-0.383	-0.405	-0.395
	(0.0697)	(0.0723)	(0.0771)	(0.0682)	(0.0717)	(0.0768)
2-digit Occup.	✓			✓		
3-digit Occup.		✓			✓	
5-digit Occup.			✓			✓
Demographic controls				✓	✓	✓

Notes: All reduced-form equations are estimated using PDII data. The omitted group is high school dropout females who are 41-60 years old. $N = 1333$ for all models. The number of occupational fixed effects is 21 for 2-digit, 76 for 3-digit, and 193 for 5-digit. Standard errors are reported in the parenthesis.

Following Autor and Handel (2013), I measure T_k^i as the first component of a principal components analysis, then convert to percentile rankings. I use three variables for cognitive tasks. The first component accounts for 59% of the variation. The social task is measured using one variable. Routine task intensity is calculated as the first component of four variables. The first component accounts for 55% of the variation. Manual tasks are measured using one variable. See Appendix B.1 for details.

To conserve space, Table 1.A reports the estimates for cognitive and manual tasks only, for which task sorting is prominent. I control for occupational fixed effects using 2, 3, and 5-digit OCCSOC codes at different columns. For all models, college graduates tend to perform more cognitive tasks but fewer manual tasks. These coefficients become smaller but remain statistically significant with disaggregated occupational fixed effects. Notably, within 5-digit occupations, college graduates on average perform 0.38 of a standard deviation fewer cognitive tasks and 0.64 of a standard deviation fewer manual tasks, compared to high school dropouts (the omitted group).[6] Estimates for social and routine tasks appear in Appendix Table E.4.

Fact 3: Task heterogeneity and sorting explain over 30% of the observed within-occupation earning variation. I estimate the following regression:

$$\ln w^i = \sum_j \beta_j X_j^i + \sum_k \gamma_k T_k^i + \alpha_o + u^i, \tag{2}$$

where the dependent variable is log hourly wage and u_i is the unobserved idiosyncratic heterogeneity that affects earnings.

The first three columns of Table 1.B report the estimates using different occupational fixed effects, without demographic characteristics. The coefficients for worker-level tasks are sizable and statistically significant for cognitive, routine, and manual in all columns, and the magnitude generally becomes smaller as more disaggregated occupational fixed effects are included. Adding demographic controls, the coefficients for cognitive, routine, and manual tasks fall but remain statistically significant (see the last three columns). Worker-level tasks, therefore, capture distinct sources of wage variation unexplained by demographic characteristics. Based on the estimates in Column (6), a standard deviation increase in cognitive tasks is associated with a 0.208 $\times$ 0.289 = 0.06 increase in log wage (0.14 of a standard deviation in log wage within-occupation), and a standard deviation increase in manual tasks is associated with a 0.395 $\times$ 0.256 = 0.10 decrease in log wage (0.24 of a standard deviation).[7]

[6] The standard deviation of $T_k^i - \widehat{\alpha}_o$ is 0.206 for cognitive and 0.155 for manual, where $\widehat{\alpha}_o$ uses 5-digit occupational fixed effects. Using the estimated coefficients for the college-educated dummy, this implies a $\frac{0.079}{0.206} = 0.38$ (of a standard deviation) increase for cognitive and a $\frac{-0.108}{0.155} = -0.64$ decrease for manual.

[7] The numbers inside parentheses are calculated as the ratio between the changes in log wage and 0.41, which is the standard deviation of the log wage residual within 5-digit occupations.

I use the regression estimates to account for the contribution of worker tasks to within-occupation inequality as follows:

$$\mathrm{Var}(\ln w^i - \widehat{\alpha}_o) = \underbrace{\mathrm{Var}(\sum_j \widehat{\beta}_j X^i_j)}_{15.3\%} + \underbrace{\mathrm{Var}(\sum_k \widehat{\gamma}_k T^i_k)}_{15.1\%} + \underbrace{2\mathrm{Cov}(\sum_j \widehat{\beta}_j X^i_j, \sum_k \widehat{\gamma}_k T^i_k)}_{16.0\%} + \underbrace{\mathrm{Residual}}_{53.6\%}.^{8} \quad (3)$$

The left-hand side is the within-occupation log wage variance, which decomposes into four terms. The first is the dispersion of worker-level tasks, which account for 15.3% of within-occupation wage variation. Worker-level task intensity operates through two channels, as shown in the second and third terms. The first is the heterogeneity of worker tasks, which accounts for 15.1% of within-occupation wage variation. The second is the covariance term, which measures the contribution due to task sorting based on observable characteristics and accounts for 16% of the within inequality. The term is positive because workers with higher earning potential (e.g., high level of education) tend to perform more tasks that have higher returns (cognitive tasks) within-occupation. Task heterogeneity and sorting jointly explain over 31% of all within-occupation inequality. The residual absorb the variance of idiosyncratic ability and also task sorting on unobserved ability.

3 Model

I present an assignment model featuring partial specialization and heterogeneous task assignment within-occupation. The production unit in each occupation bundles all types of tasks to produce output; however, occupations vary in their relative demand for tasks, denoted as $\lambda_{o,k}$. Every worker supplies one unit of time $[0, 1]$, chooses an occupation, and then allocates time to different tasks. Workers are grouped based on observable characteristics (education, age, and gender), with mass N^G for group G. The unobserved skills differ among workers within each G-group. In particular, each worker has a K-dimensional vector of skills, denoted as $\nu^G = \left(\nu^G_1, ..., \nu^G_K\right)$ drawn from a joint distribution F^G_ν.

3.1 The Final and Occupational Goods Producers

The economy has a final good producer who combines outputs from multiple occupations using a CES technology with elasticity of substitution ρ:

$$Q = \Big[\sum_o A_o Q_o^{\frac{\rho-1}{\rho}}\Big]^{\frac{\rho}{\rho-1}}, \quad (4)$$

[8] The residual absorbs the variance of idiosyncratic ability and also task sorting on unobserved ability.

where A_o is the occupational factor productivity. Occupational outputs are produced by perfectly competitive production units using a constant returns to scale Cobb-Douglas technology:

$$Q_o = \prod_{k=1}^{K} \Big[L_{o,k}\Big]^{\lambda_{o,k}}, \qquad \sum_k \lambda_{o,k} = 1 \tag{5}$$

where $L_{o,k}$ is the aggregate efficiency units of task k in occupation o. $\lambda_{o,k}$ is the relative demand for task k that varies across occupations—for example, STEM occupations may intensively use cognitive tasks, whereas routine tasks are essential for machine operators. Demand changes take place in two forms: A_o and $\lambda_{o,k}$.

Firm profit maximization implies the price per unit of occupational output is:

$$P_o = \prod_{k=1}^{K} \left[\frac{p_{o,k}}{\lambda_{o,k}}\right]^{\lambda_{o,k}}, \tag{6}$$

which bundles the equilibrium occupational task price, $p_{o,k}$.

3.2 Workers' Problem

The timing is as follows. Before entering an occupation, workers observe their skill vector ν^G and task prices $p_{o,k}$. Workers know they will work for one unit of time and are paid according to the total value of task outputs produced. However, they do not know the specific task they will perform at t. They choose an occupation based on their realized occupational preference ε^i_o and wages. After entering an occupation, at each time t, they realize an idiosyncratic task shock $\varepsilon^i_{o,k,t}$. $\varepsilon^i_{o,k,t}$ captures, for example, that some workers might prefer performing cognitive tasks on Monday morning but routine tasks on Friday afternoon. At each t, workers perform one task that delivers the highest utility. The model is solved through backward induction.

3.2.1 Partial Specialization and Heterogeneous Task Assignments

Conditional on choosing an occupation o, at each time t, each worker performs one task to maximize:

$$p_{o,k} \times \nu^G_k \times \varepsilon^i_{o,k,t}. \tag{7}$$

Here, $p_{o,k} \times \nu^G_k$ is the value of k-task output produced per unit of time. For tractability, I assume $\varepsilon^i_{o,k,t}$ follows independent and identical Fréchet distributions across k and t, with shape parameter $\theta > 1$. The probability of performing task k at time t is:

$$\Pi_{k|o,t}\big(\nu^G\big) = \frac{\big(p_{o,k} \times \nu^G_k\big)^\theta}{\sum_k \big(p_{o,k} \times \nu^G_k\big)^\theta}.$$

Note that, in equation (7), because ν_k^G is idiosyncratic, task assignment is also idiosyncratic. In a similar fashion as Eaton and Kortum (2002), the combination of i.i.d. Fréchet across t and the Law of Large Numbers implies the aggregate fraction of time allocated to task k is:

$$\Pi_{k|o}\left(\nu^G\right) = \int_0^1 \mathbb{1}_{k|t}\left(\nu^G\right)dt = \frac{\left(p_{o,k} \times \nu_k^G\right)^\theta}{\sum_k \left(p_{o,k} \times \nu_k^G\right)^\theta}, \tag{8}$$

where $\mathbb{1}_{k|t}(\nu^G)$ is an indicator function that equals 1 if task k is chosen at t. Note that, individuals perfectly foresee their aggregate time allocation before entering an occupation, although they do not know the tasks they would perform at each t.

3.2.2 Wages

Workers earn the value of task outputs produced, which is:

$$W_o(\nu^G) = \sum_k p_{o,k} \times \nu_k^G \times \Pi_{k|o}(\nu^G).^{9} \tag{9}$$

$\nu_k^G \times \Pi_{k|o}(\nu^G)$ measures individuals' efficiency units of k-task supply, which is endogenous. Notably, equation (9) differs from the wage equation estimated in Firpo, Fortin and Lemieux (2014), where workers simultaneously supply all of their skills or spend the same amount of time on each task, $\Pi_{k|o}(\nu^G) = \frac{1}{4}$. I argue in Section 6.3 that heterogeneous time allocation is important in replicating the smooth occupational-level wage changes.

3.2.3 Occupational Choice

Given time allocations and earnings, workers choose an occupation that maximizes utility:

$$U_o^i\left(\nu^G\right) = W_o(\nu^G) \times \Gamma_o^G \times \varepsilon_o^i, \tag{10}$$

where Γ_o^G captures non-wage factors that affect occupational choices, such as occupational barriers (Hsieh, Hurst, Jones and Klenow, 2019). ε_o^i is i.i.d. Fréchet with shape parameter $\vartheta > 1$, which implies the share of workers choosing occupation o is:

$$\Pi_o\left(\nu^G\right) = \frac{\left[W_o(\nu^G)\Gamma_o^G\right]^\vartheta}{\sum_o \left[W_o(\nu^G)\Gamma_o^G\right]^\vartheta}. \tag{11}$$

[9] Note that $W_o(\nu^G)$ is the expected earnings and equals the actual earnings because $\varepsilon^i_{o,k,t}$ is i.i.d.

Note that $W_o(\nu^G)$ potentially generates arbitrary correlations across occupations through similarity in task prices, $p_{o,k}$—for example, high cognitive-skill workers have high $W_o(\nu^G)$ in all cognitive-intensive occupations. This feature delivers a realistic prediction that is consistent with existing empirical evidence (Gathmann and Schönberg, 2010, Traiberman, 2019): when workers switch occupations, they are more likely to move to jobs that have similar returns to their skills as their previous jobs.

3.3 General Equilibrium

Aggregate task supply. The aggregate efficiency units of task supply k in occupation o are:

$$L_{o,k}^{\text{supply}} = \sum_G \int N^G \cdot \nu_k^G \cdot \Pi_o(\nu^G) \cdot \Pi_{k|o}(\nu^G)\, dF_\nu^G. \tag{12}$$

And aggregate occupational employment is:

$$L_o = \sum_G N^G \int \Pi_o(\nu^G) dF_\nu^G. \tag{13}$$

Aggregate task demand. The Cobb-Douglas technology implies that task demand is:

$$L_{o,k}^{\text{demand}} = \frac{1}{p_{o,k}} \lambda_{o,k} Y_o. \tag{14}$$

The CES production function of final goods implies:

$$Y_o = P_o^{1-\rho} \cdot A_o^\rho \cdot P^{\rho-1} \cdot Y, \tag{15}$$

where $Y_o = P_o Q_o$ and $Y = \sum_o Y_o$ are the value of occupational and total outputs, respectively. Total output then equals:

$$Y = \sum_o \sum_G \int N^G \cdot \Pi_o(\nu^G) \cdot W_o(\nu^G)\, dF_\nu^G. \tag{16}$$

Equilibrium. Given the labor stock N^G, labor demand parameters, $\{A_o, \lambda_{o,k}, \rho\}$, supply parameters, $\{\vartheta, \theta, \Gamma_o^G\}$, and the distribution of skills, F_ν^G, the following conditions hold:

1. Taking $p_{o,k}$ as given, workers' occupational choice probability follows equation (11).
2. Workers allocate time according to equation (8) and earn wages according to (9).
3. $p_{o,k}$ takes values such that the labor supply in equation (12) equals the demand in (14).
4. Occupational output is defined in equation (15), and total output is given in (16).

Setting the aggregate price index, P, as numeraire, there is a unique vector, $\{p_{o,k}\}$, that sets $L_{o,k}^{\text{supply}} = L_{o,k}^{\text{demand}}$. See Appendix C.4.

Two remarks regarding my model are in order. First, the Cobb-Douglas production function in equation (5) implies a unit elasticity of substitution across tasks within-occupation. Appendix D presents quantitative results of an alternative model that aggregates tasks using a CES aggregator within-occupation. This setting allows task substitution within-occupation to be more responsive to relative prices. Second, the empirical I/O literature traditionally expresses market shares as functions of unobservables (Berry et al., 1995, Nevo, 2011). This approach was pioneered by Adao, Costinot and Donaldson (2017) in the international trade literature, see also Redding and Weinstein (2020), Lind and Ramondo (2018). In my model, the equilibrium allocations and wages are all functions of observable G and unobserved idiosyncratic skills ν^G. Demand changes in terms of A_o or $\lambda_{o,k}$, therefore, not only unequally affect workers of distinct observable characteristics (between-group inequality) but also unequally impact workers with the same observable characteristics (within-group inequality).

3.4 Intuition

Let $z^G = \{z_1^G, z_2^G, z_3^G\}$ denote the comparative advantage schedules, and $z_k^G = \nu_k^G/\nu_4^G$. ν_4^G for absolute advantage. I index $k = 1$ for cognitive skill, $k = 2$ for social, $k = 3$ for routine, and $k = 4$ for manual. Following Ohnsorge and Trefler (2007) and Adão (2015), I transform the equilibrium task assignment and wages in terms of z^G and ν_4^G as:

$$\Pi_{k|o}(z^G) = \frac{\left(p_{o,k} \times z_k^G\right)^\theta}{\sum_\ell \left(p_{o,\ell} \times z_\ell^G\right)^\theta}, \tag{17}$$

$$W_o\left(z^G\right) = \left(\sum_{k=1}^{4} p_{o,k} \times z_k^G \times \Pi_{k|o}\left(z^G\right)\right)\nu_4^G. \tag{18}$$

The transformed system serves two purposes: (1) it simplifies the structural estimation carried out in Section 4, and (2) it eases the comparative statics on how task demand affects inequality within-occupation, illustrated below. To a first-order approximation, the changes in log wages to changes in task prices are:

$$\Delta \ln W_o\left(z^G\right) = \sum_k \Delta \ln p_{o,k} \times B_{k|o}\left(z^G\right), \tag{19}$$

where

$$B_{k|o}\left(z^G\right) = \frac{p_{o,k} \times z_k^G \times \Pi_{k|o}\left(z^G\right)}{\sum_k p_{o,k} \times z_k^G \times \Pi_{k|o}\left(z^G\right)} \tag{20}$$

is workers' earning share for type-k tasks. $B_{k|o}(z^G)$ governs the unequal responses to task price changes within occupation.[10] For example, an increase in the cognitive task price within management occupations is more beneficial to managers who have cognitive comparative advantages, and, hence, high earnings shares in cognitive tasks. $B_{k|o}(z^G)$ is high for two reasons. First, per unit of time, managers who have cognitive comparative advantages supply relatively more cognitive tasks.[11] Second, cognitive-comparative-advantage workers spend more time on cognitive tasks, making their earnings share from cognitive tasks even higher. The inequality implication depends on the correlation between wage profiles and cognitive comparative advantage, conditional on selection. If managers who have cognitive comparative advantages are among the top earners, an increasing cognitive task price raises overall inequality.

4 Estimation

This section structurally estimates the model. It is important to point out that the estimation is carried out using cross-sectional variation, using the May/ORG CPS in year 2000 and the PDII. Changes in the wage structure are not targeted in the estimation and will be used for validation. I use the May/ORG CPS because it directly reports point-in-time measures of usual hourly or weekly earnings, whereas other data might bias the residual inequalities (Lemieux, 2006). My baseline estimation and quantitative exercise consider 16 demographic groups by gender: 4 education categories (high school dropouts, high school graduates, some college, and college graduates and above) and 2 age groups (21-40 and 41-60). I use 20 broad aggregate occupations, which yields a sufficient number of observations in each of the $16 \times 20 = 320$ group-occupation cells.

I assume the following multivariate normal distribution for group G:

$$\begin{bmatrix} \ln z^G \\ \ln \nu_4^G \end{bmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mu_z^G \\ \mu_\nu^G \end{pmatrix}, \begin{pmatrix} \underbrace{\Sigma_z^G}_{3\times 3} & \underbrace{\Sigma_{z\nu}^G}_{3\times 1} \\ \Sigma_{z\nu}^{G\,\prime} & \Sigma_\nu^G \end{pmatrix} \right], \tag{21}$$

[10] Differentiating equation (18) gives the following exact relationship:

$$\Delta \ln W_o(z^G) = \sum_k B_{k|o}(z^G) \times \Big(\Delta \ln p_{o,k} + \Delta \ln \Pi_{k|o}(z^G) \Big).$$

Note that $\Delta \ln \Pi_{k|o}(z^G) \approx \Delta \ln p_{o,k} - \sum_k \left(\Pi_{k|o}(z^G) \times \Delta \ln p_{o,k} \right)$, which is close to zero as $\Delta \ln p_{o,k}$ is similar across all tasks.

[11] In this case, $B_{k|o}(z^G)$ becomes:

$$B_{k|o}(z^G) = \frac{p_{o,k} \times z_k^G}{\sum_k p_{o,k} \times z_k^G}.$$

One can show $B_{k|o}(z^G)$ is increasing in z_k^G or $\frac{\partial B_{k|o}(z^G)}{\partial z_k^G} > 0$.

where μ_z^G and μ_ν^G are the means for the log comparative and absolute advantages, respectively. Σ_z^G is the covariance matrix for the log comparative advantage schedules, $\Sigma_{z\nu}^G$ is the correlation vector between log comparative and absolute advantages, and Σ_ν^G is the variance of log absolute advantage.

The estimation uses equations (17) and (18). Because absolute advantage does not affect task assignments, the transformed systems allows me to estimate Σ_z^G using only the joint distribution of task assignments. The parameters μ_z^G, μ_ν^G, and Σ_ν^G will be estimated from the occupational wage distribution by groups. Next, I discuss the estimation in steps.

4.1 Parameters Estimated Without Solving the Model

Parameters obtained from the literature. ρ measures the elasticity of substitution across occupations and ϑ measures the occupational labor supply elasticity to wages. Since both parameters have been estimated in the literature, I set $\rho = 2.1$ and $\vartheta = 1.5$ as estimated in (Burstein, Morales and Vogel, 2019).

Occupational barriers. I parameterize Γ_o^G to match group-occupation employment (Berry, 1994) as:

$$\Gamma_o^G = \ln \Pi_o^G - \ln \Pi_{o*}^G - \vartheta(\ln W_o^G - \ln W_{o*}^G), \tag{22}$$

where Π_o^G is the share of occupational employment in overall employment for group G, W_o^G is the observed average group-occupation wages. o^* is the choice of normalization, $\Gamma_{o*}^G = 0$.

Within-occupation task demand in 2000. I use the O*NET database to measure $\lambda_{o,k}$ for the year 2000 in terms of four conventionally used tasks: cognitive, social, routine, and manual tasks. I adopt Deming (2017)'s measurements for cognitive and social tasks and follow Acemoglu and Autor (2011) to measure routine and manual tasks. Appendix B.2 details the O*NET variables used. I follow Autor, Levy and Murnane (2003) to convert task variables into percentile rankings, denoted as $\text{ptl}_{o,k}$, and calculate the occupation-level relative task demand as:

$$\lambda_{o,k}^{2000} = \frac{\text{ptl}_{o,k}}{\sum_k \text{ptl}_{o,k}}.^{12} \tag{23}$$

Within-occupation task demand in 1980. O*NET is known for offering a static view of occupational task content without indicating how tasks are changing over time. I apply Atalay, Phongthiengtham, Sotelo and Tannenbaum (2020)'s data to measure within-occupation task changes over time. The data is collected based on job advertisements using text analysis

[12] $\text{ptl}_{o,k}$ is obtained by first computing the percentile rankings among the detailed SOC occupations. I then calculate the average for 20 aggregate occupations, weighted by hours.

(Spitz-Oener, 2006). Following Atalay et al. (2018), I measure the changes in task demand using the changes in the frequency of task-related words, and compute task shares in 1980 as:

$$\lambda_{o,k}^{1980} = \frac{\lambda_{o,k}^{2000} \times \left(F_{o,k}^{2000} \big/ F_{o,k}^{1980}\right)}{\sum_k \lambda_{o,k}^{2000} \times \left(F_{o,k}^{2000} \big/ F_{o,k}^{1980}\right)}, \tag{24}$$

where $F_{o,k}^{2000}$ and $F_{o,k}^{1980}$ are measures of word frequency related to task k in occupation o in 2000 and 1980.[13] Task-related words that are mentioned more frequently in job advertisements indicate an increase in demand. Appendix Table E.1 reports the estimated relative task shares in 1980 and 2000. As documented in Atalay et al. (2020), US jobs became more cognitively and socially intensive, but less routine.

The variance-covariance of log comparative advantage schedules. I estimate Σ_z^G from the joint distribution of task assignments. Since the variance-covariance of relative task assignment varies modestly across education, gender, and age (see Appendix Table E.5), I assume Σ_z^G is common across groups (to increase precision), denoted as Σ_z.

Equation (17) implies a log-linear relationship between comparative advantage and relative task assignments

$$\begin{bmatrix} \ln z_1^G \\ \ln z_2^G \\ \ln z_3^G \end{bmatrix} = \frac{1}{\theta} \begin{bmatrix} \ln \Pi_{1|o}(z^G) - \ln \Pi_{4|o}(z^G) \\ \ln \Pi_{2|o}(z^G) - \ln \Pi_{4|o}(z^G) \\ \ln \Pi_{3|o}(z^G) - \ln \Pi_{4|o}(z^G) \end{bmatrix} - \begin{bmatrix} \ln p_{o,1} - \ln p_{o,4} \\ \ln p_{o,2} - \ln p_{o,4} \\ \ln p_{o,3} - \ln p_{o,4} \end{bmatrix} \tag{25}$$

I measure worker-level task share as the ratio of PDII percentile rankings:

$$\ln \Pi_{k|o}^i = \ln \frac{\mathrm{T}_k^i}{\sum_\ell \mathrm{T}_\ell^i}, \tag{26}$$

Note that relative task prices also affect assignment. To isolate the price effect, I regress $\ln \Pi_{k|o}^i$ on 5-digit occupational fixed effects to obtain the residual, denoted as $\ln \widetilde{\Pi_{k|o}^i}$. Another common concern when estimating comparative advantages is selection. Here, the occupational fixed effects could largely address the selection effect if the conditional distribution in $\ln z_k^G$ is mainly a rightward or leftward shift from the unconditional occupations. Appendix C.1 provides suggestive evidence that the occupational fixed-effects filter out selection in cognitive and social comparative advantage to a reasonable degree.

I then estimate the joint distribution of $\ln z_k^G$ based on the residuals of task assignment

[13] The IPUM occupational codes are available in Atalay, Phongthiengtham, Sotelo and Tannenbaum (2020)'s dataset. I aggregate the data to the same 20 occupations and compute changes in the frequency of words related to each task.

following:

$$\begin{bmatrix} \ln z_1^G \\ \ln z_2^G \\ \ln z_3^G \end{bmatrix} = \frac{1}{\theta} \begin{bmatrix} \ln \widetilde{\Pi^i_{1|o}} - \ln \widetilde{\Pi^i_{4|o}} \\ \ln \widetilde{\Pi^i_{2|o}} - \ln \widetilde{\Pi^i_{4|o}} \\ \ln \widetilde{\Pi^i_{3|o}} - \ln \widetilde{\Pi^i_{4|o}} \end{bmatrix}. \tag{27}$$

Calculating the variance-covariance matrix for both sides of (27), Σ_z is estimated (up to scale) as:

$$\Sigma_z = \frac{1}{\theta^2}\Xi, \tag{28}$$

where Ξ is the 3×3 covariance matrix for the residual task assignments.[14]

4.2 Parameters Estimated While Solving the Model

Denoting $\Upsilon = \{\rho, \lambda_{o,k}, \vartheta, \Xi\}$ as the parameters previously estimated. I estimate the rest skill parameters $\Theta = \{\mu_z^G, \mu_\nu^G, \Sigma_{z\nu}^G, \Sigma_\nu^G, \theta\}$ by targeting group-occupation employment and group-occupation log wage distribution (the first and the second moments). Because wages and employments are all functions of equilibrium task prices, $p_{o,k}$, the estimation procedure is implemented while fully solving for the equilibrium.

The structural estimation can be understood in two steps.[15] Given an initial guess Θ_0, the first step (inner problem) solves task price at each occupation that clear the markets, and then calibrates the A_o that the predicted average occupational wages match the data. I construct the sample following Lemieux (2010) and Acemoglu and Autor (2011), and target five sets of moments using the May/ORG 2000 CPS. Appendix B.3 details the sample.

The second step (outer problem) searches for the Θ that minimizes the distance of the targeted moments between the model and the data. The three sets of moments are: (1) the average group-occupation log wage, $\overline{W}_{G,o}^{\text{model}} = \overline{W}_{G,o}^{\text{data}}$; (2) the group-occupation log wage variance, $(\sigma^2_{G,o})^{\text{model}} = (\sigma^2_{G,o})^{\text{data}}$; and (3) group-occupation employment, $\Pi_{G,o}^{\text{model}} = \Pi_{G,o}^{\text{data}}$.[16] I outline the procedure below and provide the full technical details in Appendix C.2.

Solving for General Equilibrium (Inner Problem). Denote ϖ as the collection of data and

[14] Here, I only use the residual task assignments to estimate the covariance matrix. The means, μ_z^G, are pinned down by matching the wage distribution.

[15] While the exercise essentially estimates Θ, $p_{o,k}$, and A_o to target five sets of moments, it is computationally burdensome to estimate all parameters jointly using gradient-based methods because of the large dimensions. The 2-step procedure greatly eases the computation burden, similar to Berry, Levinsohn and Pakes (1995), by realizing $p_{o,k}$ being uniquely determined as a function of Θ, Υ, and A_o and can be solved quickly using contraction mapping algorithm (Berry, 1994, Alvarez and Lucas, 2007).

[16] Because wages and employments are all functions of equilibrium task prices, $p_{o,k}$, the SMM procedure is implemented while fully solving for the equilibrium.

parameters used to solve for the general equilibrium:

$$\varpi = \Big\{ N^G, \Gamma_o^G, \overline{W}_{G,o}^{\text{data}}, (\sigma_{G,o}^2)^{\text{data}}, \Pi_{G,o}^{\text{data}}, \Upsilon \Big\}. \tag{29}$$

Given an initial guess of Θ_0, and an initial guess of occupational productivity A_0, I draw $(\ln z^G, \ln \nu_4^G)$ from the distribution (21) for each G-group with a mass of N^G. I compute aggregate task demand and supply for occupation and applies the contraction mapping algorithm (Alvarez and Lucas, 2007) to solve the task price $p_{o,k}(\Theta_0, A_0, \varpi)$ where

$$L_{o,k}^{\text{supply}} = L_{o,k}^{\text{demand}}. \tag{30}$$

Given $p_{o,k}(\Theta_0, A_0, \varpi)$, I then update $A_0(\Theta_0)$ that the predicted average occupational wages match the data. The inner problem, therefore, obtains task price as a function of Θ, $A(\Theta)$, and ϖ, denoted as $p_{o,k}(\Theta, A(\Theta), \varpi)$. I denote $\mathcal{F}(\Theta) = \{p_{o,k}(\Theta, A(\Theta), \varpi), A(\Theta), \varpi\}$.

GMM Estimation (Outer Problem). The second step then computes $\overline{W}_{G,o}^{\text{model}}$, $(\sigma_{G,o}^2)^{\text{model}}$, and $\Pi_{G,o}^{\text{model}}$, and uses gradient-based methods to minimize the following micro-level moments:

$$\widehat{\Theta} = \underset{\Theta}{\operatorname{argmin}} \;\; \Psi_o^G\Big(\Theta; \mathcal{F}(\Theta)\Big) \times \Omega \times \Psi_o^G\Big(\Theta; \mathcal{F}(\Theta)\Big)', \tag{31}$$

where

$$\Psi_o^G\Big(\Theta; \mathcal{F}(\Theta)\Big) = \Big[\overline{W}_{G,o}^{\text{model}} - \overline{W}_{G,o}^{\text{data}}, (\sigma_{G,o}^2)^{\text{model}} - (\sigma_{G,o}^2)^{\text{data}}, \Pi_{G,o}^{\text{model}} - \Pi_{G,o}^{\text{data}}\Big] \tag{32}$$

is the vector of targeted moments that stack 3 sets of moments over 16 groups and 20 occupations. Ω is the weighting matrix, chosen as a diagonal matrix with each element containing group-occupation employment. Under this weighting matrix, the minimizer of the objective function also targets group-level wages and employment.[17] The variance for the SMM estimator is constructed in Appendix C.3.

The skill parameters are estimated for each of the 16 demographic groups and requires normalization. I set $\mu_z^G = \mu_\nu^G = 0$ for females who are high school dropouts and 41-60 years old.[18] For each group, $20 \times 3 = 60$ moments are used to estimate 8 skill parameters in $\{\mu_z^G, \mu_\nu^G, \Sigma_{z\nu}^G, \Sigma_\nu^G\}$ and θ (common across groups).

[17] I confine attention to the consistent one-step procedure by setting Ω as group-occupation employment. The estimator is consistent but might not be the one with the smallest asymptotic variance.

[18] Here, I normalize the mean of a normal distribution $\mu_z^G = \mu_\nu^G = 0$ without restriction on the variance. Alternatively, one can normalize based on the mean of a log-normal distribution, which is $\exp(\mu_z^G + \frac{\sigma_z^2}{2})$. The normalization choice would be absorbed by $p_{o,k}$ and have no impact on my results.

4.3 The Sources of Variation

I now discuss the sources of variation used in the SMM estimation.

Relative task demand within occupations. The equilibrium task prices are strongly disciplined by occupational relative demands $\lambda_{o,k}$. Appendix Figure E.1 shows a strong positive correlation between the two. Θ is then estimated by exploring variation in the group-level log wage distributions across occupations as $p_{o,k}$ varies.

Between-occupation inequality. Variation in the average log wages (the between-occupation inequality) and equilibrium task prices $p_{o,k}$ across occupations discipline μ_z^G and θ. For example, consider a small change $\Delta \ln p_{o,k}$ (k-task and occupation o). Equation (19) implies the change in the average log wage in occupation o for group G is:

$$\Delta \mathbb{E}\big(\ln W_o(z^G) \big| o\big) = \mathbb{E}\big(B_{k|o}(z^G) \big| o\big) \times \Delta \ln p_{o,k}.^{19} \tag{33}$$

Equation (33) suggests that, for example, as the cognitive task price varies across occupations, if we observe larger differences in average log wages for group G than for other groups, it means the earnings share in cognitive tasks is high for group G. Because $B_{k|o}(Z^G)$ monotonically increases in comparative advantage, given that z_k^G follows a log normal distribution with a mean of $\exp\left(\mu_z^G + \sigma_z^2/2\right)$, this forces μ_z^G or Σ_z to be high. As Σ_z is estimated from the task assignments (up to scale), a high Σ_z forces θ to be small.

Within-occupation inequality. Variations in the log wage variance (within-occupation inequality) and $p_{o,k}$ across occupations discipline the parameters μ_z^G, $\Sigma_{z\nu}^G$, and θ. Given $\Delta \ln p_{o,k}$, the changes in log wage variances are:

$$\Delta \mathrm{Var}\big(\ln W_o(z^G) \big| o\big) = \mathrm{Var}\big(B_{k|o}(z^G) \big| o\big) \left(\Delta \ln p_{o,k}\right)^2 + 2\mathrm{Cov}\big(B_{k|o}(z^G), \ln W_o(z^G) \big| o\big) \Delta \ln p_{o,k}.^{20} \tag{34}$$

Equation (34) suggests that, as cognitive task prices vary across occupations, if we observe larger differences in within-occupation inequality for group G than for other groups, $B_{k|o}(z^G)$ (the cognitive comparative advantages) are either more dispersed (captured by the first term) or more positively correlated with earnings within group G (captured by the second term).

Specifically, two underlying forces are at work. First, the cognitive comparative advantages need to be more dispersed, again, forcing μ_z^G to be large or θ to be small.[21] Second,

[19] The equation captures the partial equilibrium impact (which is of first-order importance), neglecting the adjustments in occupation choices and time allocation.

[20] Because $\Delta \mathrm{Var}\big(\ln W_o(z^G) \big| o\big) = \mathrm{VAR}\big(\ln W_o^G + \Delta \ln p_{o,k} \times B_{k|o}\big) - \mathrm{VAR}\big(\ln W_o^G\big)$, the equality then follows.

[21] Since z_k^G follows a log normal distribution, its variance is $[\exp(\sigma_z^2) - 1] \exp(2\mu_z + \sigma_z^2)$ which increases in μ_z and σ_z^2.

for $\text{Cov}(B_{k|o}(z^G), \ln W_o(z^G))$ to be more positive (or less negative), the monotonicity between $B^G_{k|o}(z)$ and z^G_k suggests the sorting along each single dimension of skill is more positive (or less negative)—meaning high k-skill workers have large k-comparative advantages and thus spend more time in k-tasks. The positive sorting for manual tasks means $\text{Cov}(\ln \frac{\nu^G_4}{\nu^G_k}, \nu^G_4)$ is more positive. This forces $\Sigma^G_{z\nu} = -\text{Cov}(\ln \frac{\nu^G_4}{\nu^G_k}, \nu^G_4)$ to be more negative.

The average log wage across groups. Because absolute advantage shifts wages up or down by the same amount for all occupations, it does not affect task assignments or wage differences across occupations. The parameters for absolute advantage, μ^G_ν and Σ^G_ν, are disciplined by the group-level log wage distribution. Generally, μ^G_ν and Σ^G_ν would be large for groups that are high in average log wage and log wage variance.

4.4 The Estimated Parameters

The estimates of Ξ. I estimate Ξ using the residual task assignment from the PDII. Table 2.A (left) reports the estimates. The implied variances of log comparative advantage are:

$$\text{var}(\ln z^G_1) = \frac{1.71}{\theta^2}, \qquad \text{var}(\ln z^G_2) = \frac{2.52}{\theta^2}, \qquad \text{var}(\ln z^G_3) = \frac{1.63}{\theta^2}.$$

Mirroring the distribution of residual task assignments, social comparative advantage appears to be widely dispersed across workers, while the comparative advantages in cognitive and routine skills are less dispersed.

Table 2 (right) reports the correlation coefficients, which are also equal to the correlation coefficients of the log comparative advantage schedules. In the data, because workers who perform more cognitive tasks also tend to perform more social tasks, this reveals a strong positive correlation between cognitive and social comparative advantages (0.48). The cognitive-to-routine and social-to-routine correlations are also both positive.

Table 2: The Estimated Parameters

A. The Covariance Matrix of $\ln \widetilde{\Pi^i_{k\|o}} - \ln \widetilde{\Pi^i_{4\|o}}$						
	Covariance, Ξ			**Correlation Coefficient**		
	Cognitive	*Social*	*Routine*	*Cognitive*	*Social*	*Routine*
Cognitive	1.71			1		
Social	1.00	2.52		0.48	1	
Routine	0.69	0.66	1.63	0.41	0.32	1

B. Parameter Estimated from the SMM				
	$\mu^G_{z_1}$	$\mu^G_{z_2}$	$\mu^G_{z_3}$	μ^G_{ν}
HS dropouts	-.42 (.003)	-.41 (.020)	.111 (.078)	.301 (.209)
HS graduates	-.19 (.002)	-.66 (.004)	.398 (.076)	.375 (.129)
Some college	.389 (.008)	-.58 (.001)	.303 (.005)	.501 (.001)
College graduates	.738 (.030)	.666 (.056)	-.06 (.000)	.315 (.092)
	Cov$(\ln z^G_1, \ln \nu^G_4)$	**Cov**$(\ln z^G_2, \ln \nu^G_4)$	**Cov**$(\ln z^G_3, \ln \nu^G_4)$	**Var**$(\ln \nu^G_4)$
HS dropouts	-.06 (.004)	-.04 (.021)	-.06 (.002)	.110 (.010)
HS graduates	-.02 (.010)	.017 (.036)	-.08 (.009)	.142 (.047)
Some college	-.06 (.002)	.042 (.013)	-.13 (.003)	.198 (.013)
College and above	-.22 (.009)	-.23 (.006)	-.17 (.012)	.416 (.038)

Notes: Standard errors are reported in the parenthesis.

The SMM estimates of the other skill parameters. To conserve space, Table 2.B reports the estimates of $\{\mu^G_z, \mu^G_\nu, \Sigma^G_{z\nu}, \Sigma^G_\nu\}$ for 41-60 year old males by four education groups. The full set of parameter estimates for all 16 groups is reported in Appendix Table E.6. I also estimate a value for θ of 2.7 (and a standard error of 0.4).

I highlight a few results. First, the mean of log cognitive comparative advantage, $\mu^G_{z_1}$, monotonically increases in education, driven by the fact that both between- and within- occupation inequality are larger for more educated groups. I also find that the mean of log social comparative advantage, $\mu^G_{z_2}$, is high for college graduates but low for other groups, and the mean of log routine comparative advantage, $\mu^G_{z_3}$, is high for medium-education groups. Second, because more educated groups are high in average log wage and log wage variance, the mean and variance parameters for absolute advantage, μ^G_ν and Σ^G_ν, generally increase in education.[22] Third, because there are larger differences in log wage variance across occupations for college graduates than for other groups, following the same intuition discussed earlier, comparative advantages are more negatively correlated with absolute advantages for college graduates than for other groups.

[22]The only two exceptions are college-educated males, young and old. For these groups, since their μ^G_z is high—due to their large observed gap between $\mathbb{E}(\ln W_o(z^G)|o)$ and $\text{Var}(\ln W_o(z^G)|o)$ —μ^G_ν needs to be low to match the observed group-level wage.

4.5 Additional Results

My quantitative exercise focuses on rising US inequality between 1980 and 2000.[23] As pointed out by Burstein, Morales and Vogel (2019), one cannot separately measure the changes in A_o from the changes in skills. In my analysis, I assume the skill distribution is unchanged over time (Chay and Lee, 2000). Given the estimated skill distribution, I then solve the model for 1980 and present additional results below.

Changes in task prices. Appendix Table E.2 reports the changes in log task prices between 1980 and 2000 by occupation. Cognitive and social task prices increased dramatically for all occupations, and by more among occupations that are cognitively and socially intensive (e.g., Executive Management, STEM). Notably, these estimated task prices are the general equilibrium notion of returns to skills, and are in line with recent reduced-form estimates regarding the increasing return to social skills (Deming, 2017).[24] Manual task prices increased for most occupations, but the magnitudes of increase are modest. Routine task prices declined substantially for all occupations.

The comparative advantage schedules. As discussed in Section 3.4, within-occupation inequality depends on the correlation between comparative advantage and the log wage profile, conditional on selection. Because prices in cognitive, social, and manual tasks have all increased relative to routine tasks, I plot comparative advantages relative to routine skills to clarify the illustration. Figure 1 (upper panel) plots the log comparative advantages in cognitive and social tasks against the log wage profile in four occupations: management, STEM, education, and sales. In these occupations, cognitive or social tasks are the main source of earnings, and I find the cognitive and social comparative advantages both increase in wages. The lower panel plots comparative advantages in manual skills for construction and mechanics & repair occupations where manual tasks are the main earnings sources, and for administrative and machine operation occupations where routine tasks are the main earnings sources. See Appendix Table E.3 for the average earnings shares. The comparative advantage monotonically increases in wages within administrative occupations, although to a much more modest degree compared to the upper panel. The curves are flat for the other three occupations.

The implication is that for the upper-panel occupations, demand changes benefit the rich more than the poor within-occupation. The magnitude of this inequality-increasing effect also depends on the level of relative price increases. For occupations in the lower panel, the

[23] Because Atalay et al. (2020)'s data measuring within-occupation demand ended in 2000.

[24] The period of my study differs from Deming (2017), who focus on 25-33 year-olds during the period 2004-2012.

relative task price changes have only modest impacts on inequality.

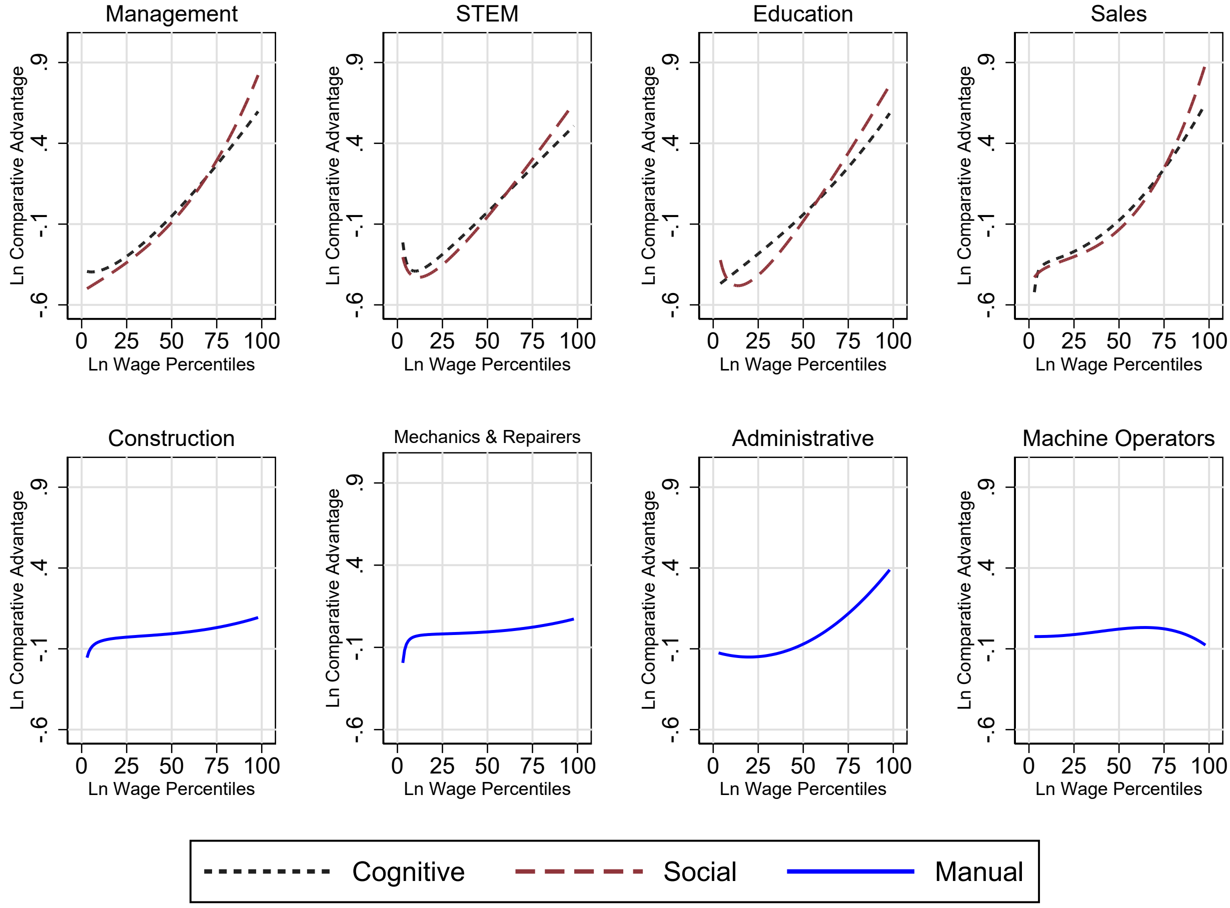

Figure 1: The Log Comparative Advantage Schedules (relative to routine skills) against Log Wages by Occupation. I normalize each curve to have a mean zero.

5 Model Fit

This section performs extensive exercises to show that the estimated model closely replicates the micro-moments of both the cross-section and the changes in wages and employment shares—all of which are essential to perform quantitative exercises.

5.1 The Targeted Moments

The wage distribution in 2000. Table 3 compares the predicted and the observed average log wage by four education groups as displayed in Columns (1) and (2), and the log wage variance as reported in Columns (3) and (4). The model fits the data closely for most cases. Since the skill distribution is estimated by targeting the conditional log wage distribution, this model fitness is not surprising. Appendix Figures E.2 and E.3 show the model closely replicates both the overall log wage distribution and those of disaggregated education and age groups in 2000.

Table 3: The Average and Variance of Log Wages in 2000: Model and Data

	Average Log Wage		**Variance Log Wage**	
	Data	Model	Data	Model
	(1)	(2)	(3)	(4)
HS dropouts	2.365	2.432	0.132	0.139
HS graduates	2.634	2.665	0.181	0.166
Some college	2.763	2.798	0.205	0.201
College and above	3.138	3.191	0.275	0.239

Notes: The wages are in real terms, for which I deflate the hourly wages by the PCE price deflator.

Group-level occupational employment. I now compare the model fit in terms of group-level occupational employment.[25] Figure 2 plots the observed group-level occupational shares on the x-axis against the predicted shares on the y-axis for 1980 and 2000, respectively. In equation (22), since Γ_o^G targets group-occupation employment, the variation in occupational choice across groups is well targeted by the model. For each year, the coefficient slope and the R-square are all close to 1.

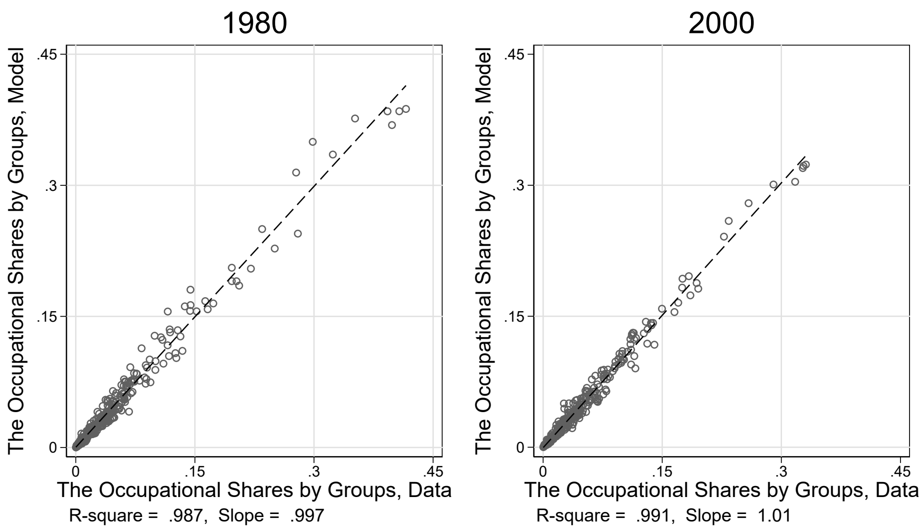

Figure 2: The Fit of Occupational Shares, Data, and the Model, by education-age-gender-occupation groups (each circle presents one of the $4 \times 2 \times 2 \times 20 = 320$ cells)

[25]The predicted group-level occupational shares are computed as

$$\Pi_o^G = \int \Pi_o(\nu^G) dF_\nu^G = \frac{1}{R} \sum_{r=1}^{R} \Pi_o(\nu^{G,r}).$$

where the superscript r refers to the rth pseudo-individual, and $\Pi_o(\cdot)$ is the function of probability choice defined in equation (11).

5.2 The Non-Targeted Moments

I also examine model fit in terms of log wage changes over time, which are the non-targeted moments in the estimation.

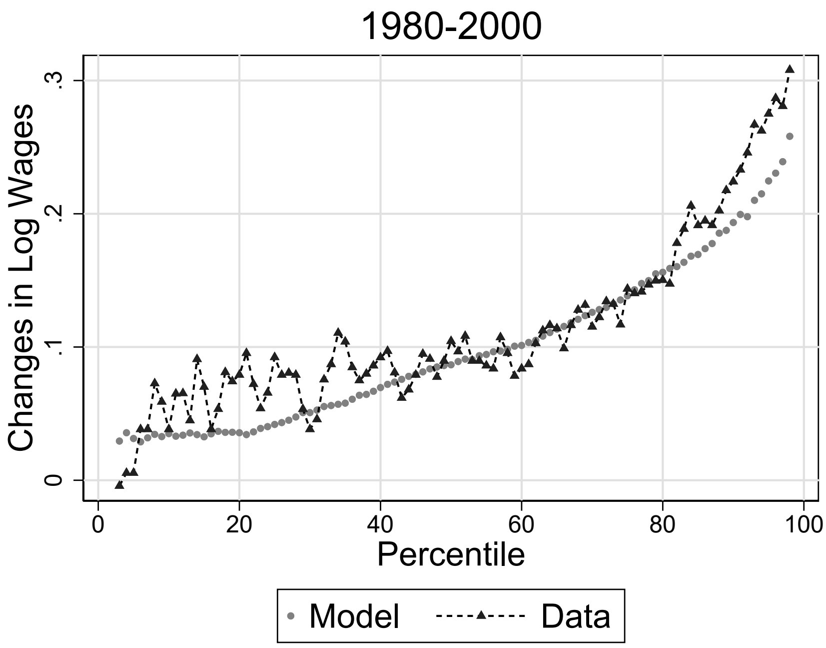

Figure 3: Changes in Log Wage By Percentiles: Model and Data

Notes: This figure plots the percentiles on the horizontal axis and the changes in the log wage on the vertical axis. For each year, the wages are measured in real terms by deflating the CPS hourly wages by the PCE price deflator following (Autor et al., 2008).

Changes in the overall wage distribution. Between 1980 and 2000, real wage inequality grew everywhere along the distribution, and this growth was more pronounced at the upper tail but relatively flat from the middle to the lower tail (Autor, Katz and Kearney, 2008). Figure 3 shows the predicted log wage changes generally match the overall wage distribution.

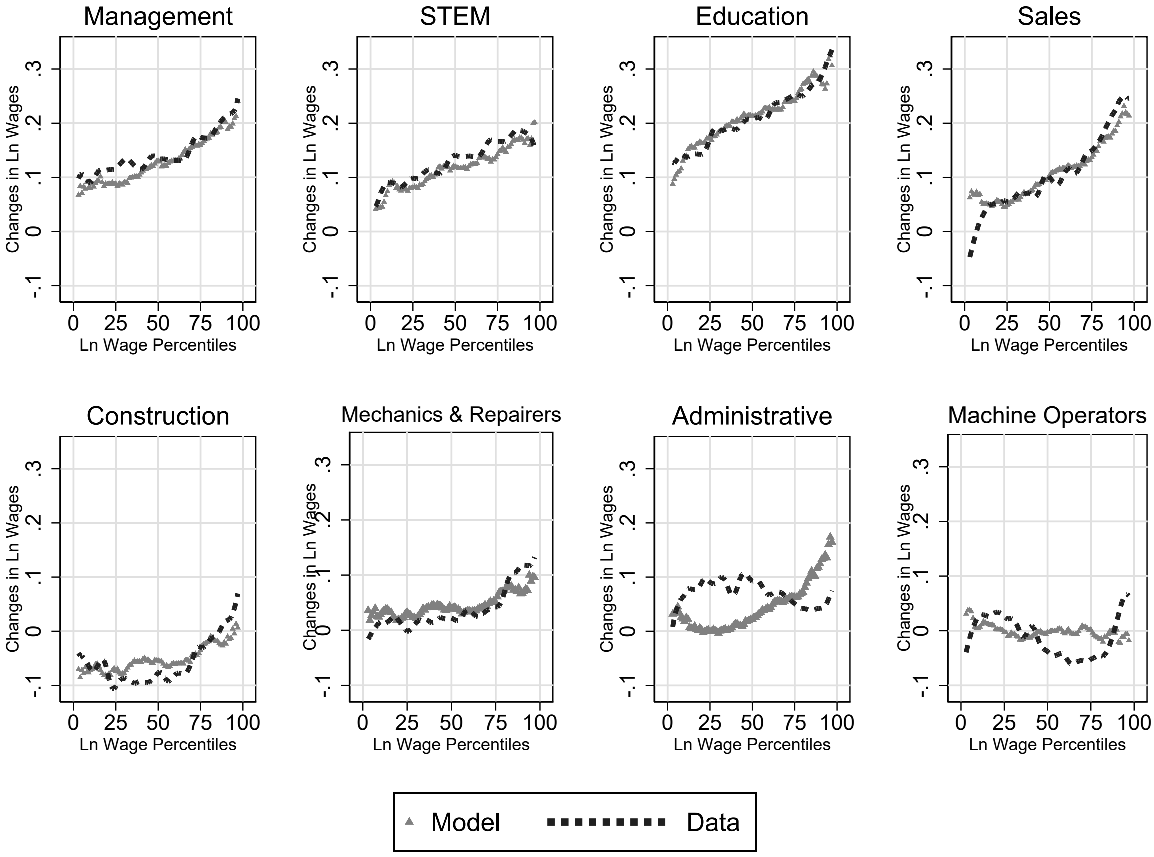

Figure 4: Changes in Occupational Log Wage Percentiles, 1980-2000: Model and Data

Notes: This figure plots the percentile on the horizontal axis and changes in the log wage on the vertical axis. The changes are in real terms, for which I deflate the hourly wages by the PCE price deflator.

Changes in occupation-level wage distributions. Figure 4 plots the predicted and observed log wage changes between 1980 and 2000 for each of the largest eight US occupations. Most of these occupations mimic the overall pattern of wage changes, as shown in Figure 3, where the wage growth is more pronounced at the top end but flat at the middle-to-bottom of the distribution. This is true for cognitively-intensive occupations (Management, STEM) and socially-intensive occupations (education, sales). This is also true for manually-intensive occupations (construction, mechanics & repairers), although the magnitude of the increase in inequality is small compared to the plots in the upper panel. The model can replicate much of the observed within-occupation wage changes. In contrast, in routine-intensive occupations (administrative, machine operators), the wage changes are relatively flat and growth is faster at the bottom of the distribution. The model matches these occupations relatively poorly, especially for administrative occupations. Overall, I take this as evidence that the model can capture much of changes across occupations in the data. In Section 6.3, I show the model fit deteriorates substantially when either partial specialization or heterogeneity in task assignment is abstracted from the model.

The within-occupation task assignment. While I estimate the covariance of comparative advantage schedules by targeting the distribution of the residual relative task assignments, the levels of task assignments are not targeted. I now investigate model fit in terms of task assignments.

I calculate worker-level task shares based on equation (26), and report the mean and variance of task shares by 1-digit broad occupations in Table 4.[26] In the data, task shares vary substantially between and within occupations. In terms of the average task shares, unsurprisingly, management and professional specialist occupations are high in cognitive and social tasks; clerical occupations have a high routine task share; production workers, transportation, and service occupations are mostly routine and manual tasks. The model predicted shares replicate much of the variation in the average task shares across occupations. The variances of task shares are reported in parenthesis. The predicted dispersion in task assignments fit the data reasonably well for most cases.

Table 4: The Mean and Variance of Task Shares by Broad Occupations: Data and Model

	Management		**Professional Specialist**		**Sales**		**Clerical**	
Task	Data	Model	Data	Model	Data	Model	Data	Model
Cognitive	.369 (.035)	.347 (.031)	.411 (.050)	.326 (.027)	.268 (.052)	.294 (.026)	.235 (.033)	.256 (.026)
Social	.387 (.039)	.292 (.042)	.218 (.034)	.271 (.040)	.215 (.026)	.247 (.037)	.131 (.031)	.230 (.029)
Routine	.129 (.015)	.204 (.028)	.195 (.036)	.210 (.025)	.267 (.028)	.250 (.029)	.422 (.045)	.319 (.035)
Manual	.112 (.021)	.156 (.011)	.174 (.034)	.191 (.015)	.249 (.030)	.207 (.015)	.210 (.030)	.193 (.012)
N	195		462		162		201	
	Construction/Repair		**Production workers**		**Transportation**		**Service Occupations**	
Task	Data	Model	Data	Model	Data	Model	Data	Model
Cognitive	.292 (.017)	.211 (.018)	.186 (.025)	.185 (.015)	.191 (.020)	.140 (.010)	.170 (.025)	.159 (.012)
Social	.183 (.025)	.157 (.022)	.138 (.028)	.143 (.017)	.110 (.023)	.186 (.024)	.150 (.027)	.194 (.026)
Routine	.216 (.020)	.272 (.023)	.354 (.026)	.343 (.028)	.358 (.026)	.304 (.025)	.333 (.026)	.298 (.025)
Manual	.307 (.013)	.359 (.025)	.320 (.016)	.327 (.022)	.339 (.017)	.368 (.024)	.346 (.023)	.346 (.023)
N	105		80		80		216	

Notes: In each column, the first figure denotes the mean of task shares, and the figure in parentheses denotes the variance of task shares within 1-digit occupations.

6 Quantitative Results

This section quantifies the extent to which has the changes in A_o and $\lambda_{o,k}$ contributed to increasing US inequality, using multiple inequality measures. I present results based on 20 occupations below. Appendix D reports results using 30 or 40 occupational categories.

[26]I report 1-digit occupation because of the small sample size of the PDII.

6.1 Log Wage Variance

First, I use the variance of log wages to measure inequality. When doing so, I construct composition-adjusted measures using a constant weight. Specifically, I first compute the mean and variance of the log wage for 16 demographic groups within the 20 occupations, and aggregate to the occupation-level using constant weights averaging over 1980 and 2000.

Table 5: Decomposing Changes in Inequality Between 1980 and 2000, Baseline Model

	Data	Occupation Demand, A_o	Within occupation, $\lambda_{o,k}$	Residual
	(1)	(2)	(3)	(4)
A. Composition-adjusted Between and Within Occupation Inequality				
Between-occupation	0.006	0.022	0.023	-0.039
Within-occupation	0.034	0.001	0.026	0.007
Total	0.040	0.023	0.049	-0.032
B. Composition-adjusted Between and Within Group Inequality				
Between-group	0.021	0.010	0.034	-0.023
Within-group	0.025	0.015	0.012	-0.003
Total	0.046	0.025	0.047	-0.025
C. Percentile log wage gap				
90-10 Gap	0.159	0.065	0.149	-0.055
50-10 Gap	0.030	0.029	0.035	-0.034
90-50 Gap	0.129	0.036	0.113	-0.020

Notes: The impacts in Columns (2) and (3) are obtained by evaluating what would happen in 1980 if each of the four shocks is set its level from 2000. I use equations (8), (9), and (11) to compute counterfactual task assignments, wages, and occupational employment. Each inequality measure is constructed using a constant weight over time to average over the log wage changes of the disaggregated groups.

Table 5.A decomposes total inequality into between- and within-occupation components. The majority of the observed increases in log wage variance occurred within occupation, see Column (1). Columns (2) and (3) display the inequality impact if only the changes in A_o or $\lambda_{o,k}$ are taken into consideration. For the overall variance, the impact of $\lambda_{o,k}$ is twice as large as that of A_o. These two forms of demand shocks operate through different channels. A_o impacts inequality only through between-occupation variance. $\lambda_{o,k}$, in contrast, shapes inequality both between- and within-occupation, and explains most of the observed rising inequality within-occupation.

Column (4) reports the residual impacts, which equal the differences between the data and the aggregate effects of the two types of demand shocks. The residual captures the inequality effect of other factors such as the changes in labor composition (Card and Lemieux, 2001), the reduction in occupational barriers (Hsieh, Hurst, Jones and Klenow, 2019), and changes in minimum wages and labor unionization (DiNardo, Fortin and Lemieux, 1996). These factors tend to affect inequality negatively, compensating the differences.

Did the demand changes have a larger impact among workers of distinct observable characteristics (between-group inequality) or among workers with the same observable characteristics (within-group inequality)? While previous studies primarily focus on between-group inequality measures, my approach also speaks to the consequences for within-group inequality. The unequal responses arise because task assignments (equation 8) and occupational choice (equation 11) are both functions of unobserved skills.

Table 5.B decomposes the log wage variance into between and within group inequality. Again, the changes in $\lambda_{o,k}$ had a much larger overall impact than the changes in A_o; the changes in $\lambda_{o,k}$ operate mainly by increasing between-group inequality. This is because with 16 disaggregated groups, the estimated $B_{k|o}(z^G)$ within the same occupation varies more between than within groups. In contrast, the changes in $A_{o,k}$ have a similar impact on between and within-group inequality. The residual factors, again, compensating the differences.

6.2 Along the Wage Distribution

While the log wage variance can be decomposed into between and within components, it is not informative about what happened at different points of the wage distribution. To this end, Table 5.C reports the commonly used 90-50 gap—the difference between the 90th percentile and the median of log wages—as well as the 50-10 gap and the 90-10 gap, which are defined analogously.

Between 1980 and 2000, inequality rose more at the top than the bottom end. The 90-50 gap increased by 0.159 log wage points, compared to a 0.030 increase in the 50-10 gap. Table 5.C shows the changes in $\lambda_{o,k}$, which again had a larger impact on all three percentile measures of wage inequality, and is the primary contributor to the rise at the top end. In contrast, A_o had a relatively small impact in rising the overall inequality (See Appendix Figure E.4).

So far, the baseline results assume the within-occupation responses of relative task demand to prices are less elastic than the responses between occupations. Appendix D.2 estimates a model with CES aggregation within occupations. I find that, when the within-occupation demand becomes more elastic, changes in $\lambda_{o,k}$ lead to a larger price response, generating a larger inequality impact.

6.3 The Mechanisms

The unequal wage response within-occupation arises from partial specialization and heterogeneous task assignments. This section highlights these mechanisms through two alternative models.

Model # 1: The Roy Model. I assume the production function follows equation (4). In contrast to the baseline model, all skills are paid at the same price, $p_{o,k} = P_o, \forall k$. Workers choose one occupation that maximizes their utility given in equation (10), where ε_o^i is the idiosyncratic preference for occupation o following i.i.d. Fréchet with shape parameter ϑ. Γ_o^G captures non-wage factors that affect occupational choices and is, again, parameterized in equation (22). Workers earn $W(\nu^G) = p_o \times \nu^G$, and the probability of choosing occupation o follows equation (11).

Every worker draws a single-dimensional skill, where the log skill, $\ln \nu^G$, follows a normal distribution $\mathcal{N}(\mu^G, \sigma^G)$.[27] While the literature commonly uses Roy models with Fréchet skills (Lagakos and Waugh, 2013, Hsieh et al., 2019), I assume normally-distributed skill to draw direct comparisons with the baseline model. Irrespective of the skill distribution, these models all imply that demand shocks equally affect workers within-occupation in partial equilibrium.

I adopt the two-step SMM procedure to estimate μ^G and σ^G. The estimator is defined in equations (31) and (32), where I target the same sets of moments using the May/ORG CPS 2000: group-occupation employment shares and the mean and variance of log wages. Note that demand changes only involve the changes in A_o. I calibrate A_o to match average occupational wages in each year. I assign the same values to ρ and ϑ as in the baseline model.

Model # 2: Partial specialization with common task assignments. To highlight the importance of heterogeneous task assignments, I estimate a model where every worker has multi-dimensional skills but simultaneously supplies all of their skills. The production functions follow equations (4) and (5). Every worker draws multi-dimensional normally-distributed skills, where the log absolute and comparative advantage schedules follow a joint normal distribution defined in (21). Workers choose one occupation that maximizes utility, where ε_o^i and Γ_o^G are the same as before. The share of workers choosing occupation o is, again, given in equation (11). After the occupational choice is made, workers simultaneously supply all of their skills, or equivalently all workers spend the same amount of time on each task, $\Pi_{k|o}(\nu^G) = \frac{1}{4}$. Since the common time allocation is absorbed by task prices, the equilibrium wage is:

$$W_o(\nu^G) = \sum_k p_{o,k} \times \nu_k^G, \tag{35}$$

similarly to the reduced-form wage equation studied in Firpo, Fortin and Lemieux (2014). I

[27] Although I assume the idiosyncratic skill is invariant across occupations, it is equivalent to a Roy model of occupation-specific draws, where $\ln \nu_o^G$ follows a normal distribution $\mathcal{N}(\mu_o \mu^G, \mu_o^2 \sigma^G)$. These two setups are equivalent as μ_o is absorbed in A_o.

assign the same values to ρ, ϑ, and Σ_z as in the baseline model. Demand changes involve changes in A_o and $\lambda_{o,k}$, where A_o matches average occupational wages and $\lambda_{o,k}$ is obtained using the O*NET and Atalay et al. (2020)'s datasets. Again, I adopt the two-step SMM procedure to estimate $\mu_z^G, \mu_\nu^G, \Sigma_{z\nu}^G, \Sigma_\nu^G$, and I target the same sets of moments using the 2000 CPS. The estimator is defined in equation (31) and (32). Appendix Table E.7 reports the SMM estimates for this alternative model.

Predictions. Using the estimated parameters, I also solve each model in the year 1980. Figure 5 compares the model fit in terms of occupational wage changes across these alternative models. In the Roy model, demand changes equally affect all workers within an occupation. Unsurprisingly, the model thus predicts limited within-occupation inequality responses (see the flat curve in red dashes).

In model # 2, wages are linear in comparative advantages. Here, one needs to load more dispersion into skills in order to fit the observed occupational wage distribution. As a result, the estimated parameters μ_ν^G and Σ_ν^G are generally larger than the baseline estimates.[28] See Appendix Table E.7 for the complete parameter estimates.

Figure 5 shows that the model predicts linear changes for all occupations. The model fits the wage changes well at education, mechanics & repairs occupation. However, for occupations such as management, STEM, education, the predicted linear wage changes only replicate the top-end but fail to match the smooth wage changes at the middle-to-bottom end of the distribution (in blue dash). Again, the model predicts less well for routine-intensive occupations. On the aggregates, both the model fit less well for the middle-to-bottom end of the overall wage changes. See Appendix Figure E.5.

Finally, Table 6 reports these model predictions in terms of the aggregate inequality measures. Using the log wage variance and decompose it into between and within occupations, the Roy model can predict closely the total inequality changes but load the impact into between-occupation inequality. Model # 2 tends to overpredict the within occupation inequality. I also decompose the log wage variance into between and within demographic groups. Similarly, columns (4)-(6) in Table 6 show that the Roy generates inequality responses mostly through between-group, whereas model # 2 overpredicts both between and within group inequality.

[28] One can see this as μ_k^G or Var$(\ln \nu_4^G)$ are generally larger in the alternative model #2.

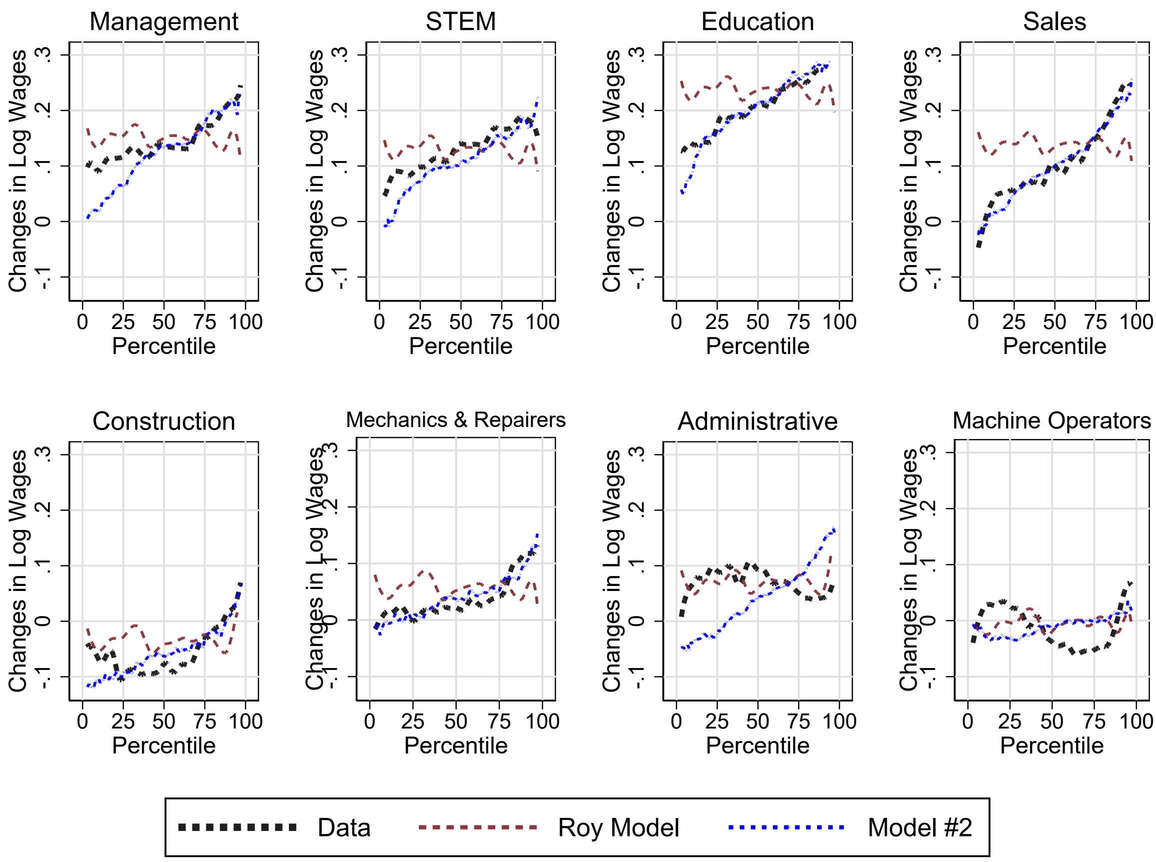

Figure 5: Changes in Occupational Log Wage Percentiles, 1980-2000: Alternative Models and Data

Notes: This figure plots the percentile on the horizontal axis and the changes in the log wage on the vertical axis. The changes are in real terms, for which I deflate the hourly wages by the PCE price deflator.

These exercises underscore that (1) modelling partial specialization is the important in generating unequal wage response; and (2) modelling wage convexity and heterogeneous task assignments is important in matching the pronounced wage changes at the upper tail and the smooth and flat changes at the middle-to-bottom end for many occupations.

Table 6: Changes in Inequality Using Alternative Models

	Between and Within Occupation			**Between and Within Group**		
	(1)	(2)	(3)	(4)	(5)	(6)
	Data	Roy Model	Model # 2	Data	Roy Model	Model # 2
Between	0.011	0.035	0.021	0.021	0.051	0.042
Within	0.030	0.002	0.057	0.025	-0.015	0.034
Total	0.041	0.038	0.078	0.046	0.036	0.077

Notes: Each inequality measure is constructed using a constant weight over time to average over the log wage changes for each disaggregated group.

7 Conclusion

This paper analyzes the impacts of changes in task demand on within-occupation inequality and proposes a new mechanism: workers in the same occupation perform multiple and different tasks. In the data, workers perform multiple tasks and differ substantially in task assignments within the same occupation. This heterogeneity in task assignments strongly predicts earning variation within occupation. Motivated by empirical facts, I build and estimate an assignment model in which every worker has multi-dimensional skills and partially specializes in certain tasks within the occupation. Within occupation, workers' time allocations to tasks vary and are shaped by their comparative advantages. The general equilibrium model can be structurally estimated at a more granular level without relying on test scores as measures of skills. The estimated model can explain much of the differential wage changes across US occupations.

Using the estimated model to quantify the inequality implication of demand changes in two forms: the relative task demand within occupations and occupation-specific productivity. I find the within-occupation relative task demand changes are the primary contributor to the rising within-occupation inequality and the inequality at the top end. The new mechanism proposed in the model are crucial in generating these predictions.

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Online Appendix

A Variance Decomposition

I show that within–occupation component accounts for the majority of both the level and the changes in the log wage variance between 1980 and 2000, using the May/ORG CPS.

Decomposition Results. Table A.1.a reports the contribution of within component as a share of the overall log wage variance, both in level and in changes. The between component equals one minus the value reported. Using 20 aggregate occupations, I find the within component accounts for 82% and 77% of the overall log wage variance in 1980 and 2000, respectively. Over the period, 56% of the rise in log wage variance is driven by the within-component. The last three columns of Table A.1 show the within-component remains large in levels and is more important in driving the changes. Since within-component is more important under composition-adjusted measures, it appears that labor composition had a more pronounced impact driving between than within-occupation inequality.

Table A.1: The Contribution of Within-Occupation Log Wage Variance

	Non-adjusted			**Composition-adjusted**		
	1980	**2000**	**Changes**	**1980**	**2000**	**Changes**
a. Aggregated Occupations Based on OCC1990 Code						
20 Occupations	82.0%	76.7%	56.1%	88.0%	87.1%	81.1%
30 Occupations	80.0%	73.2%	46.8%	81.9%	80.1%	69.6%
40 Occupations	79.0%	73.0%	49.3%	84.2%	82.3%	70.5%
383 Occupations	71.4%	68.1%	54.9%	78.7%	78.5%	78.1%
b. Aggregated Occupations Based on OCCSOC Code						
2-digit	83.2%	77.0%	53.2%	86.2%	84.5%	75.4%
3-digit	78.0%	73.2%	53.9%	85.2%	82.4%	68.7%
4-digit	77.7%	73.0%	54.2%	85.6%	82.9%	68.9%

Notes: the analysis is based on the May/ORG CPS 1980 and 2000. Each number presents the contribution of the within-component to the total log wage variance inequality, in levels or changes. The changes are computed as the changes in within-component divided by the changes in the overall log wage variance. In Panel A, the 20 occupations are based on the aggregation used in quantitative exercise (see Appendix Table E.1). The 30 and 40 occupations are based on less broad aggregation using the OCC1990 code, and the 383 occupations are based on David Dorn's OCC1990dd code. In Panel B, the results are based on the 2, 3, and 4-digit OCCSOC occupation code.

The decomposition results may depend on the occupation categories used. To this end, I report the results using 30 and 40 broad occupations based on the OCC1990 code, and using 383 detailed occupations based on David Dorn's OCC1990dd code, respectively. The next three rows show, although the within-occupation component generally falls with more disaggregate occupations, the within component are substantial in all cases.

Table A.1.b shows the results are similar when I use OCCSOC code at the different levels of aggregation. Using the 2, 3, and 4-digit OCCSOC occupation codes, I find the within-component are substantial in explaining both the levels and the changes. Note that because the OCCSOC code is only available after the year 2000, I assign the OCCSOC code to the CPS data in 1980 and 1990 (using crosswalk of OCCSOC to OCC1990).

Below, I detail how I construct the non-composition and composition adjusted decomposition.

Non-composition adjusted decomposition. I denote the total log wage variance as the sum of between and within occupation components

$$T_t^u = B_t^u + W_t^u, \tag{A.1}$$

where

$$T_t^u = \frac{1}{N_t}\sum_i (w_{it} - \bar{w}_t)^2, \quad B_t^u = \frac{1}{N_t}\sum_o N_{ot}(\bar{w}_{ot} - \bar{w}_t)^2, \quad W_t^u = \frac{1}{N_t}\sum_o \sum_{i \in o}(w_{it} - \bar{w}_{ot})^2$$

where o denotes occupation cell, N_t denotes the overall employment, and N_{ot} denotes the employment in occupation o. w_{it} is the log wage for worker i. $\bar{w}_t$ is the gross average of log wage, and $\bar{w}_{ot}$ is the average log wage at occupation o. The linear relation also holds in changes as

$$\Delta T_t^u = \Delta B_t^u + \Delta W_t^u, \tag{A.2}$$

Composition-adjusted decomposition. Composition changes might mechanically bias the results (Lemieux, 2006). To address this concern, I compute the average and variance of log wage at each occupation and demographic group, then aggregate over groups using constant weights averaging over 1980 and 2000. Specifically, I denote the total composition-adjusted log wage variance as the sum of between and within occupation components

$$B_t^a = \frac{1}{\bar{N}}\sum_o \bar{N}_o(\bar{w}_{ot}^a - \bar{w}_t^a)^2, \quad W_t^a = \frac{1}{\bar{N}}\sum_o \bar{N}_{ot} Var_{ot}$$

where $\bar{N}$ and $\bar{N}_o$ are the average in overall employment and occupational employment between 1980 and 2000. $\bar{w}_{ot}^a$ and $\bar{w}_t^a$ are the composition-adjusted average occupation and gross log wage, respectively, which are calculated using the average group-occupation employment between 1980 and 2000 as the weights. $Var_{o,t}$ denotes the composition-adjusted log wage variance in occupation o and is calculated as

$$Var_{ot} = B_{ot} + W_{ot}, \tag{A.3}$$

$$B_{ot} = \sum_G \frac{\bar{N}_o^G}{\bar{N}_o}(\bar{w}_{o,t}^G - \bar{w}_{ot}^a)^2, \qquad W_{ot} = \sum_G \frac{\bar{N}_o^G}{\bar{N}_o}\sum_{i \in G}(w_{i,t} - \bar{w}_{ot}^G)^2 \tag{A.4}$$

where $\bar{w}_{ot}^G$ is the average group-occupation level wage. Again, the changes can be written as

$$\Delta T_t^a = \Delta B_t^a + \Delta W_t^a, \tag{A.5}$$

B Data Appendix

B.1 PDII Variables

I define a worker *often* cognitive tasks if at least one of the following is true (1) taking 30 minutes to solve problems at least once a week; (2) applying advanced math algebra, geometry, trigonometry, probability/statistics, or calculus) to solve problems at least once a week; (3) often have to read documents that are more than 6 pages. A worker *often* performs social tasks if he/she spends more than half of workdays managing or supervising other workers. A worker *often* performs routine tasks if he/she spends more than half of workdays on repetitive tasks that complete absence from face-to-face interactions. A worker *often* performs manual tasks if he/she spends more than half of workdays on standing, operating machinery or vehicles, making or fixing things by hand.

I use the following variables to construct worker-level task intensity using the first component of principal components analysis, then transfer to percentile rankings to obtain T_k^i.

- Cognitive task intensity: (1) the frequency of using advanced mathematics tasks; (2) the frequency of problem-solving tasks requiring at least 30 minutes to find a good solution; and (3) the length of the longest document typically read as part of the job.
- Social task intensity: the proportion of workday managing or supervising other workers.
- Routine task intensity: (1) proportion of the workday spent performing short, repetitive tasks and complete absence of face-to-face interactions with (2) customers or clients, (3) suppliers or contractors, or (4) students or trainees.
- Manual task intensity: the proportion of the workday spent performing physical tasks (standing, operating machinery or vehicles, making or fixing things by hand).

B.2 O*NET Variables

I follow Deming (2017) to measure cognitive (math) and social task intensities. I follow Acemoglu and Autor (2011) to measure routine and manual task intensities. Below provides the details

1. Cognitive (math) task intensity is the average of three variables: mathematical reasoning ability (the ability to understand and organize a problem and then to select a mathematical method or formula to solve the problem), mathematics knowledge (knowledge of numbers, their operations, and interrelationships including arithmetic, algebra, geometry, calculus, statistics, and their applications), and mathematics skill (using mathematics to solve problems).
2. Social task intensity as the average of four variables: social perceptiveness (being aware of others' reactions and understanding why they react the way they do), coordination (adjusting actions in relation to others' actions), persuasion (persuading others to approach things differently), and negotiation (bringing others together and trying to reconcile differences).
3. Routine task intensity as the average of six variables: the importance of repeating the same tasks (How important is repeating the same physical activities or mental activities over and over, without stopping, to performing this job?), the importance of being exact or accurate (how important is being very exact or highly accurate in performing this job?), structured verse unstructured work (to what extent is this

job structured for the worker, rather than allowing the worker to determine tasks, priorities, and goals?), pace determined by the speed of equipment (how important is it to this job that the pace is determined by the speed of equipment or machinery?), controlling machines and processes (using either control mechanisms or direct physical activity to operate machines or processes), and making repetitive motions.

4. Manual task as the average of four variables: operating vehicles, mechanized devices, or equipment (running, maneuvering, navigating, or driving vehicles or mechanized equipment, such as forklifts, passenger vehicles, aircraft, or watercraft), using hands to handle, control or feel objects, tools or controls, manual dexterity (the ability to quickly make coordinated movements of one hand, a hand together with its arm, or two hands to grasp, manipulate, or assemble objects), and spatial orientation (the ability to know one's location in relation to the environment, or to know where other objects are in relation to one's self).

B.3 May/ORG CPS

I draw data from the Current Population Survey May and Outgoing Rotation Group samples (May/ORG CPS), which directly report point-in-time measures of usual hourly or weekly earnings. I construct the sample following Lemieux (2010) and Acemoglu and Autor (2011), and restrict workers to those who are between 21 and 60 years old, not in the military, and not self-employed. For individuals who are paid hourly, their hourly earnings are reported in May/ORG CPS. For other workers who report weekly earnings, I compute their hourly earnings as the ratio between the usual weekly earnings and the hours worked in the previous week. I multiple the top-coded earnings observations by 1.5. Following Autor, Katz and Kearney (2008), all earnings are measured in log real terms, deflated by the personal consumption expenditure (PCE) deflator.[29] This leaves about 110,000-130,000 observations for each year. I apply the occupation concordance developed in Autor and Dorn (2013) to create time-consistent occupation codes.

I start with the 30 broad occupational categories given in OCC1990 (also see Burstein et al., 2019). I obtain the 20 occupations by merge occupations with small employment size. For 40 occupations, I break down occupations with large employment size into separate groups. Below, I list the occupations that include in each classification.

20 Occupations. 1 "Executive Management" 2 "Management Related" 3 "STEM" 4 "Social Service, Lawyers" 5 "Education, Training, Library, legal support" 6 "Health Occupations" 7 "Technicians and Related Support" 8 "Financial Sales and Related Occupations" 9 "Retail Sales" 10 "Administrative Support" 11 "Housekeeping, Cleaning, Laundry" 12 "All Protective Service" 13 "Food Preparation and Service" 14 "Farm operators" 15 "Mechanics and Repairers" 16 "Construction" 17 "Precision production" 18 "Machine Operators, Assemblers, and Inspectors" 19 "Transportation and Material Moving" 20 "Handlers, Equipment Cleaners, and Helpers".

30 Occupations. 1 "Executive Management" 2 "Management Related" 3 "Architect" 4 "Engineer" 5 "Computer and Mathematics" 6 "Life, Physical, and Social Science" 7 "Community and Social Services" 8 "Lawyers" 9 "Education, Training, Library, legal support" 10 "Arts, Design, Entertainment, Sports, Media" 11 "Health Diagnosing Occupations" 12 "Health Assessment and Treating" 13 "Technicians and Related Support" 14 "Financial Sales and Related Occupations" 15 "Retail Sales" 16 "Administrative Support" 17 "Housekeeping, Cleaning, Laundry" 18 "All Protective Service" 19 "Food Preparation and Service" 20 "Health Service" 21 "Building, Grounds Cleaning and Maintenance" 22 "Personal Appearance" 23 "Child Care Workers" 24 "Farm operators" 25 "Me-

[29] The data is available from David Autor's website.

chanics and Repairers" 26 "Construction" 27 "Precision production" 28 "Machine Operators, Assemblers, and Inspectors" 29 "Transportation and Material Moving" 30 "Handlers, Equipment Cleaners, and Helpers".

40 Occupations. 1 "Executive Management" 2 "Management Related" 3 "Architect" 4 "Engineer" 5 "Computer and Mathematics" 6 "Life, Physical, and Social Science" 7 "Health diagnosing occupations" 8 "Health assessment and treating, Therapists" 9 "Teacher postsecondary" 10 "Teacher except postsecondary" 11 "Librarians, Archivists, and Curators" 12 "Social Scientists and Urban Planners" 13 "Social, Recreation, and Religious Workers" 14 "Lawyers" 15 "Writers, Artists, Entertainers, and Athletes" 16 "Health Technologists and Technicians" 17 "Engineering and Related Technologists and Technicians" 18 "Sales Representatives, Finance and Business Services" 19 "Sales Representatives" 20 "Administrative Support" 21 "Information Clerks" 22 "Records Processing Occupations" 23 "Financial Records Processing Occupations" 24 "Duplicating, Mail, and Other Office Machine Operators" 25 "Material Recording, Scheduling, and Distributing Clerk" 26 "Adjusters and Investigators" 27 "Housekeeping, Cleaning, Laundry" 28 "All Protective Service" 29 "Food Preparation and Service" 30 "Health Service" 31 "Building, Grounds Cleaning and Maintenance" 32 "Personal Appearance" 33 "Child Care Workers" 34 "Farm operators" 35 "Mechanics and Repairers" 36 "Construction" 37 "Precision production" 38 "Machine Operators, Assemblers, and Inspectors" 39 "Transportation and Material Moving" 40 "Handlers, Equipment Cleaners, and Helpers".

C Technical Appendix

C.1 Kolmogorov-Smirnov Tests

The occupational fixed effects isolate the price effect and could address the selection effect to a large extent if the conditional distribution in $\ln z_k^G$ is mainly the rightward or leftward shift from the unconditional occupations. In this case, one would expect the conditional distribution of the residual task assignments to be similar across all occupations. To this end, I perform a series of two-sample Kolmogorov-Smirnov tests for the null hypothesis that the marginal distribution of residual relative task assignments in a specific occupation is the same as the residual distribution for the rest of the sample.[30] The null hypothesis is that the marginal distribution of $\ln \widetilde{\frac{\Pi_{k|o}(Z^G)}{\Pi_{4|o}(Z^G)}}$ for a given occupation is the same as the rest of the sample. Table C.1 reports the p-value, where I test each task and occupation separately. Using broad occupations, I reject the null for 2 out of 8 for cognitive tasks, and 3 out of 8 for social tasks. I take these as suggestive evidence that the occupational fixed-effects filter out selection in cognitive and social comparative advantage to a reasonable degree. Occupational fixed-effects filter out selection less well for routine tasks.

Table C.1: Two-Sample Kolmogorov–Smirnov Equality of Distributions Test.

	Management	Professional Occupations	Sales	Clerical	Construction Repair	Production workers	Transportation	Service Occupations
				A. Cognitive tasks				
P-value	0	0.247	0.710	0.382	0.146	0.303	0.737	0.040
				B. Social tasks				
P-value	0	0.069	0.920	0.643	0.540	0.155	0.001	0.003
				C. Routine tasks				
P-value	0.005	0	0.022	0.004	0.415	0.008	0	0

C.2 The Simulated Methods of Moment Estimator

I obtain the SMM estimator using the following steps.

1. Solving $p_{o,k}(\Theta)$. First, given Υ (the set of parameters prior obtained), and a guess of Θ and of A_o, I apply the contraction mapping algorithm (Alvarez and Lucas, 2007) to solve $p_{o,k}(\Theta)$ numerically as follows

1. I draw $R = 300$ pseudo individuals $(\ln z_{r,1}^G, \ln z_{r,2}^G, \ln z_{r,3}^G, \ln \nu_{r,4}^G)$ from multivariate normal distribution given in (21) for each group G. G group has a mass of N^G. r refers to the rth pseudo individual.
2. Initial guess: $\{p_{o,k}^t\} = [1, ..., 1]$, and t index for the number of iteration.
3. For each pseudo individual, compute the occupational probability using equation (11), where P_o is given by (6); compute the time allocation to each task k using equation (8); and compute the wage profile using equation (9).
4. Compute the aggregate efficiency units of labor supply according to equation (12).
5. Compute the labor demand according to equation (14), where the occupational output and the total outputs are computed using equations (15) and (16), respectively.
6. Compute the excess labor demand as:

$$Z_{o,k}\left(\{p_{o,k}^t\}\right) = L_{o,k}^{\text{demand}} - L_{o,k}^{\text{supply}}. \tag{C.1}$$

[30]Given the sample size of PDII, I perform Kolmogorov-Smirnov tests based on 1-digit broad occupation.

Stop the procedure if $\max_{o,k} Z_{o,k}\big(\{p^t_{o,k}\}\big) < 10^{-5}$. Otherwise, update:

$$p^{t+1}_{o,k} = p^t_{o,k} + \alpha_1 \cdot \frac{\sqrt{Z_{o,k}(\{p^t_{o,k}\})^2}}{\max\left\{L^{\text{demand}}_{o,k}, L^{\text{supply}}_{o,k}\right\}}, \quad \alpha_1 \in (0,1). \tag{C.2}$$

2. Solving $A_o(\Theta)$. Given the solved $p_{o,k}(\Theta)$, the outer loop uses contraction mapping to solve $A_o(\Theta)$ that target the average occupational wages as follows

1. Using the solved $p_{o,k}(\Theta)$ and the wages defined in equation (9), I compute the average occupation-level wages as

$$\overline{W}^{\text{Model}}_o = \frac{\sum_G \int N^G \cdot \Pi_o(\nu^G) \cdot W_o(\nu^G)\, dF^G_\nu}{\sum_G \int N^G \cdot \Pi_o(\nu^G)\, dF^G_\nu} \tag{C.3}$$

2. Compute the sum of occupation-level wage gap $\sqrt{\sum_o (\overline{W}^{\text{Model}}_o - \overline{W}^{\text{Data}}_o)^2}$, and stop if the gap smaller than 10^{-5}. Otherwise, update:

$$A^{t+1}_o = A^t_o + \alpha_2 \frac{\sqrt{(\overline{W}^{\text{Model}}_o - \overline{W}^{\text{Data}}_o)^2}}{\max\left\{\overline{W}^{\text{Model}}_o, \overline{W}^{\text{Data}}_o\right\}}, \quad \alpha_2 \in (0,1). \tag{C.4}$$

3. Searching Θ. I then compute $\overline{W}^{\text{model}}_{G,o}$, $(\sigma^2_{G,o})^{\text{model}}$, and $\Pi^{\text{model}}_{G,o}$. I use gradient-based methods that search for the SMM estimator

$$\hat{\Theta} = \underset{\Theta}{\text{argmin}} \left[\Psi^G_o\big(\Theta; p_{o,k}(\Theta), A_o(\Theta)\big)\right]\Omega\left[\Psi^G_o\big(\Theta; p_{o,k}(\Theta), A_o(\Theta)\big)\right]', \tag{C.5}$$

where $\Psi^G_o\big(\Theta; p_{o,k}(\Theta), A_o(\Theta)\big) = \left[\overline{W}^{\text{model}}_{G,o} - \overline{W}^{\text{data}}_{G,o}, (S^2_{G,o})^{\text{model}} - (S^2_{G,o})^{\text{data}}, \Pi^{\text{model}}_{G,o} - \Pi^{\text{data}}_{G,o}\right]$ is the vector of targeted moments. Ω is the weighting matrix, with diagonal element being the size of group-occupation employment. I compute the model predicted average and variance of wages, and the occupational employment as

$$\overline{W}^{\text{model}}_{G,o} = \frac{1}{R}\sum_{r=1}^{R} W_o\Big(\nu^G_r, \Theta; p_{o,k}(\Theta), A_o(\Theta)\Big)\Pi_o\Big[\nu^G_r, \Theta; p_{o,k}(\Theta), A_o(\Theta)\Big], \tag{C.6}$$

$$(S^2_{G,o})^{\text{model}} = \frac{1}{R-1}\sum_{r=1}^{R}\Big(W_o\big(\nu^G_r, \Theta; p_{o,k}(\Theta), A_o(\Theta)\big) - \overline{W}^{\text{model}}_{G,o}\Big)^2 \Pi_o\Big[\nu^G_r, \Theta; p_{o,k}(\Theta), A_o(\Theta)\Big], \tag{C.7}$$

$$\Pi^{\text{model}}_o = \frac{1}{R}\sum_{r=1}^{R}\Pi_o\big(\nu^r, \Theta, \Gamma\big), \qquad \Pi_o\big(\nu^r, \Theta; \Gamma\big) = \frac{\Big[W_o\big(\nu^G_r, \Theta; p_{o,k}(\Theta), A_o(\Theta)\big)\Gamma^G_o\Big]^{\vartheta}}{\sum_o \Big[W_o\big(\nu^G_r, \Theta; p_{o,k}(\Theta), A_o(\Theta)\big)\Gamma^G_o\Big]^{\vartheta}} \tag{C.8}$$

The standard gradient-based method (using Fmincon in Matlab) is used to search for parameters Θ.

C.3 The Variance-Covariance of SMM Estimator

According to Wooldridge (2010), the asymptotic variance-covariance matrix can be obtained as

$$\Big[\Phi'\Omega\Phi\Big]^{-1}\Big[\Phi'\Omega H\Omega\Phi\Big]\Big[\Phi'\Omega\Phi\Big]^{-1} \tag{C.9}$$

where Φ is the gradient matrix of $\Psi_o^G(\Theta; p_{o,k}(\Theta), A_o(\Theta))$ with respect to parameters Θ defined as

$$\Phi = \begin{bmatrix} \frac{\partial(\overline{W}_{G,o}^{\text{model}} - \overline{W}_{G,o}^{\text{data}})}{\partial\Theta} \\ \frac{\partial(S_{G,o}^2)^{\text{model}} - (S_{G,o}^2)^{\text{data}}}{\partial\Theta} \\ \frac{\partial\Pi_{G,o}^{\text{model}} - \Pi_{G,o}^{\text{data}}}{\partial\Theta} \end{bmatrix}_{\Theta_0}.$$

The gradient matrix is evaluated at parameter values Θ_0, which are the SMM estimator. Ω, again, is the diagonal matrix with each element being the size of group-occupation employment. H is the variance-covariance of the moment condition (evaluated at the truth).

$$H = \text{Var}\Big[(\Psi_o^G)' \times \Psi_o^G\Big]_{\Theta_0}, \tag{C.10}$$

where Ψ_o^G is the shorthand for $\Psi_o^G(\Theta; p_{o,k}(\Theta), A_o(\Theta))$. The asymptotic variance of the SMM estimators are then the diagonal elements.

C.4 The Existence and Uniqueness of Equilibrium

Below I show the existence and uniqueness, in which the proof relies heavily on Alvarez and Lucas (2007) and Allen and Arkolakis (2015). Denote $\mathbb{P}$ as the vector of task prices and define the excess labor demand function as

$$D_{o,k}(\mathbb{P}) = L_{o,k}^{\text{demand}} - L_{o,k}^{\text{supply}},$$

where the demand is defined in equations (14) and (15) and the labor supply is defined in equation (12). Following Alvarez and Lucas (2007), I verify the following six conditions hold, which ensures the existence and the uniqueness of a vector $\mathbb{P}$ such that $D_{o,k}(\mathbb{P}) = 0$:

1. $D_{o,k}(\mathbb{P})$ is continuous in $\mathbb{P}$, which holds immediately from the functional form of labor supply and demand.
2. $D_{o,k}(\mathbb{P})$ is homogeneous of degree zero. For any $\alpha > 0$,

$$\begin{aligned} D_{o,k}(\alpha\mathbb{P}) &= L_{o,k}^{\text{demand}}(\alpha\mathbb{P}) - L_{o,k}^{\text{supply}}(\alpha\mathbb{P}) \\ &= \frac{\lambda_{o,k} A_o(\alpha\mathbb{P})^\rho}{\alpha p_{o,k}} \big[P_o(\alpha\mathbb{P})\big]^{1-\rho} \big[P(\alpha\mathbb{P})\big]^{\rho-1} Y(\alpha\mathbb{P}) - \sum_G \int N^G \nu_k^G \Pi_o(\alpha\mathbb{P}, \nu^G) \Pi_{k|o}(\alpha\mathbb{P}, \nu^G) dF_\nu^G \\ &= \frac{\lambda_{o,k} A_o(\alpha\mathbb{P})^\rho}{\alpha p_{o,k}} \alpha^{1-\rho} \big[P_o(\mathbb{P})\big]^{1-\rho} \alpha^{\rho-1} \big[P(\alpha\mathbb{P})\big]^{\rho-1} \alpha Y(\mathbb{P}) - \sum_G \int N^G \nu_k^G \Pi_o(\mathbb{P}, \nu^G) \Pi_{k|o}(\mathbb{P}, \nu^G) dF_\nu^G \\ &= L_{o,k}^{\text{demand}}(\mathbb{P}) - L_{o,k}^{\text{supply}}(\mathbb{P}) \\ &= D_{o,k}(\mathbb{P}). \end{aligned}$$

The third equality holds because $\Pi_{o,k}(\mathbb{P}, \nu^G)$, $\Pi_{k|o}(\mathbb{P}, \nu^G)$, and $A_o(\alpha\mathbb{P})$ are homogeneous of degree zero and $Y(\mathbb{P})$ and $P_o(\mathbb{P})$ are homogeneous of degree one in $\mathbb{P}$.

3. For all $\mathbb{P} > 0$, it is true that

$$\begin{aligned}\sum_{o,k} p_{o,k} D_{o,k}(\mathbb{P}) &= \sum_{o,k} p_{o,k} L_{o,k}^{\text{demand}}(\mathbb{P}) - \sum_{o,k} p_{o,k} L_{o,k}^{\text{supply}}(\mathbb{P}) \\ &= \sum_{o,k} \lambda_{o,k} P_o^{1-\rho} A_o^{\rho} P^{\rho-1} Y - \sum_{o,k} p_{o,k} \sum_G \int N^G \nu_k^G \Pi_{k|o}(\mathbb{P}, \nu^G) \Pi_o(\mathbb{P}, \nu^G)\, dF_\nu^G \\ &= \sum_o \lambda_{o,k} Y_o - \sum_o Y_{o,k} = 0 \\ &= \sum_o \lambda_{o,k} Y_o - \sum_o \lambda_{o,k} Y_o = 0.\end{aligned}$$

The second equality holds by the definition of aggregate labor supply and demand. The third equality holds because of perfect competition, i.e., the total output is the sum of the value added of all tasks by workers. The fourth equality holds because $\lambda_{o,k}$ corresponds to expenditure share under Cobb-Douglas production function.

4. For all $\mathbb{P}$, there is a uniform lower bound. For a specific pair (o, k),

$$\begin{aligned}D_{o,k}(\mathbb{P}) &\geqslant -\sum_G \int N^G \nu_k^G \Pi_{k|o}(\mathbb{P}, \nu^G) \Pi_o(\mathbb{P}, \nu^G) dF_\nu^G \geqslant -\sum_G N^G \int \nu_k^G dF_\nu^G \\ &= -\sum_G N^G \exp(\mu_k^G + \frac{\sigma_k^2}{2}).\end{aligned}$$

The second inequality holds because $\Pi_o(\mathbb{P}, \nu^G) \leqslant 1$ and $\Pi_{k|o}(\mathbb{P}, \nu^G) \leqslant 1$. The equality holds because $\int \nu_k^G dF_\nu^G = \exp(\mu_k^G + \frac{\sigma_k^2}{2})$, which is the expected value of log normal distribution. The uniform lower bound can be set as the $-\sum_G N^G \exp(\mu_k^G + \frac{\sigma_k^2}{2}) < 0$.

5. The following limit holds for any pair (o, k):

$$\lim_{p_{o,k} \to 0} D_{o,k}(\mathbb{P}) = \infty > 0.$$

Following Alvarez and Lucas (2007), there exists at least an equilibrium if conditions 1-5 hold. To ensure the equilibrium is unique, we need the gross substitution property below.

6. Pick an occupation o' and task k' (either $o' \neq o$ or $k' \neq k$, or both are different), it is straightforward to show that $\frac{\partial L_{o,k}^{\text{demand}}}{\partial p_{o',k'}} > 0$ and $-\frac{\partial L_{o,k}^{\text{supply}}}{\partial p_{o',k'}} > 0$. Then

$$\frac{\partial D_{o,k}(\mathbb{P})}{\partial p_{o',k'}} > 0.$$

Since $D_{o,k}(\mathbb{P})$ is homogeneous of degree zero in $\mathbb{P}$, this implies

$$\nabla D_{o,k}(\mathbb{P}) \cdot \mathbb{P} = 0.$$

Combining these two results, it must be the case that

$$\frac{\partial D_{o,k}(\mathbb{P})}{\partial p_{o,k}} < 0.$$

Therefore, the gross substitution property holds.

Conditions 1-6 ensure the equilibrium is unique.

D Quantitative Results Under Alternative Models

This section presents the main quantitative results under alternative model specifications.

D.1 Baseline Model With Alternative Occupation Aggregation

First, I present the quantitative results under 30 or 40 occupation categories, respectively. The 30 broad occupational categories given in OCC1990, used in Burstein, Morales and Vogel (2019). For 40 occupations, I break down occupations with large employment size into separate groups. See Appendix B.3 for the lists of occupations that include in each classification.

Table D.1: Decomposing Changes in Inequality Between 1980 and 2000, Baseline Model

	Data	Occupation Demand, A_o	Within occupation, $\lambda_{o,k}$	Residual
A. Composition-adjusted Between and Within Occupation Inequality				
		30 Occupations		
Between-occupation	0.013	0.024	0.025	-0.036
Within-occupation	0.035	0.001	0.025	0.009
Total	0.048	0.026	0.050	-0.028
		40 Occupations		
Between-occupation	0.011	0.023	0.025	-0.037
Within-occupation	0.026	0.001	0.025	-0.001
Total	0.036	0.024	0.050	-0.038
B. Composition-adjusted Between and Within Group Inequality				
		30 Occupations		
Between-group	0.021	0.009	0.036	-0.024
Within-group	0.025	0.021	0.012	-0.008
Total	0.046	0.029	0.049	-0.032
		40 Occupations		
Between-group	0.021	0.007	0.037	-0.023
Within-group	0.025	0.020	0.012	-0.008
Total	0.046	0.028	0.049	-0.030
C. Percentile log wage gap				
		30 Occupations		
90-10 Gap	0.159	0.075	0.152	-0.068
50-10 Gap	0.030	0.029	0.034	-0.033
90-50 Gap	0.129	0.046	0.118	-0.035
		40 Occupations		
90-10 Gap	0.159	0.067	0.155	-0.063
50-10 Gap	0.030	0.027	0.034	-0.031
90-50 Gap	0.129	0.040	0.121	-0.032

Notes: Each inequality measure is constructed using a constant weight over time to average over the log wage changes of dis-aggregate groups.

In doing so, I re-estimate the model structurally. Because, equation (25) still holds in these alternative models, and Σ_z, the variance-covariance matrix of log comparative advantages is still estimated using the PDII data. Second, I use SMM to estimate $\Theta = \{\mu_z^G, \mu_\nu^G, \Sigma_{z\nu}^G, \Sigma_\nu^G, \theta\}$ that targets the same sets of moments using the CPS 2000. The findings are similar.

D.2 Nested-CES Model

This section assumes the occupational output is produced using CES technology as

$$Q_o = \left[\sum_k \lambda_{o,k} L_{o,k}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{D.1}$$

where $\lambda_{k,o}$ measures the k-task intensity in occupation o. η measures the elasticity of substitution across tasks within-occupation. Firms' profit maximization implies the price per unit of occupational output is

$$P_o = \left[\sum_k \lambda_{o,k}^{\eta} p_{o,k}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{D.2}$$

The supply side remains the same as the baseline model. In equilibrium, the task demand at occupation o becomes

$$L_{o,k}^{\text{Demand}} = \frac{1}{p_{o,k}^{\eta}} \lambda_{o,k}^{\eta} P_o^{\eta-1} Y_o = \frac{1}{p_{o,k}^{\eta}} \lambda_{o,k}^{\eta} P_o^{\eta-\rho} A_o^{\rho} Y. \tag{D.3}$$

where Y_o follows equation (15) and Y follows equation (16), respectively.

To take the model to the data, I need values of $\lambda_{o,k}$ and η. I measure $\lambda_{o,k}$ as the ratio of using the percentile ratio of task intensity following equation (23) for the year 2000, and equation (24) for the year 1980. Under the CES production function, $\lambda_{o,k}$ does not exactly capture the expenditure share but is positively associated with it.[31] I have little information on the elasticity of substitution across tasks within-occupation. Here, I experiment with values of $\eta = 2$ or 3.

I evaluate the model fit and the effects of demand on within-inequality using this model. In doing so, I proceed with the baseline estimation strategy. First, since equation (25) remains hold, Ξ, the variance-covariance matrix of log comparative advantages can be estimated using the PDII data and takes the same value as the baseline model. Second, I also set ρ and ϑ as in the baseline, and use SMM to estimate $\Theta = \{\mu_z^G, \mu_\nu^G, \Sigma_{z\nu}^G, \Sigma_\nu^G, \theta\}$ that targets the same five sets of moments in the year 2000 as described in Section 4.2.

Table D.2 displays the decomposition results with $\eta = 2$ or 3. I find that, as η becomes larger, $\lambda_{o,k}$ plays a bigger role in driving the overall inequality. The intuition is that, when η is large, the aggregate relative task demand $\frac{L_{o,k}}{L_{o,\ell}}$ are more responsive to the relative demand changes $\frac{\lambda_{o,k}}{\lambda_{o.\ell}}$ (which are given exogenously in the data), and the relative task prices $\frac{p_{o,k}}{p_{o,\ell}}$ needs to respond more to induce the changes in $\frac{L_{o,k}}{L_{o,\ell}}$. This causes more inequality responses.

Figures D.1 and D.2 show the model fits in terms of the predicted occupation-level wage changes for $\eta = 2$ and 3, respectively. These nested-CES models still predicts reasonably well for occupation-level wage changes at the upper panel but less well for the bottom panel, compared to the benchmark model.

[31] For occupation o, the expenditure share on k-task equals $\frac{\lambda_{o,k}^{\eta} p_{o,k}^{1-\eta}}{\sum_\ell \lambda_{o,\ell}^{\eta} p_{o,\ell}^{1-\eta}}$.

Table D.2: Decomposing Changes in Inequality Between 1980 and 2000: Nested-CES Model.

	Data	Occupation Demand, A_o	Within occupation, $\lambda_{o,k}$	Residual
A. Composition-adjusted Between and Within Occupation Inequality				
		$\eta = 2$		
Between-occupation	0.006	0.009	0.035	-0.038
Within-occupation	0.034	0.002	0.022	0.010
Total	0.040	0.011	0.058	-0.028
		$\eta = 3$		
Between-occupation	0.006	0.002	0.048	-0.043
Within-occupation	0.034	0.003	0.022	0.009
Total	0.040	0.005	0.070	-0.035
B. Composition-adjusted Between and Within Group Inequality				
		$\eta = 2$		
Between-group	0.021	0.004	0.043	-0.026
Within-group	0.025	0.008	0.011	0.005
Total	0.046	0.012	0.054	-0.020
		$\eta = 3$		
Between-group	0.021	0.003	0.050	-0.032
Within-group	0.025	0.004	0.016	0.005
Total	0.046	0.006	0.066	-0.027
C. Percentile log wage gap				
		$\eta = 2$		
90-10 Gap	0.159	0.023	0.183	-0.048
50-10 Gap	0.030	0.014	0.036	-0.021
90-50 Gap	0.129	0.009	0.147	-0.027
		$\eta = 3$		
90-10 Gap	0.159	-0.002	0.206	-0.046
50-10 Gap	0.030	0.001	0.036	-0.008
90-50 Gap	0.129	-0.003	0.170	-0.038

Notes: Each inequality measure is constructed using a constant weight over time to average over the log wage changes of dis-aggregate groups.

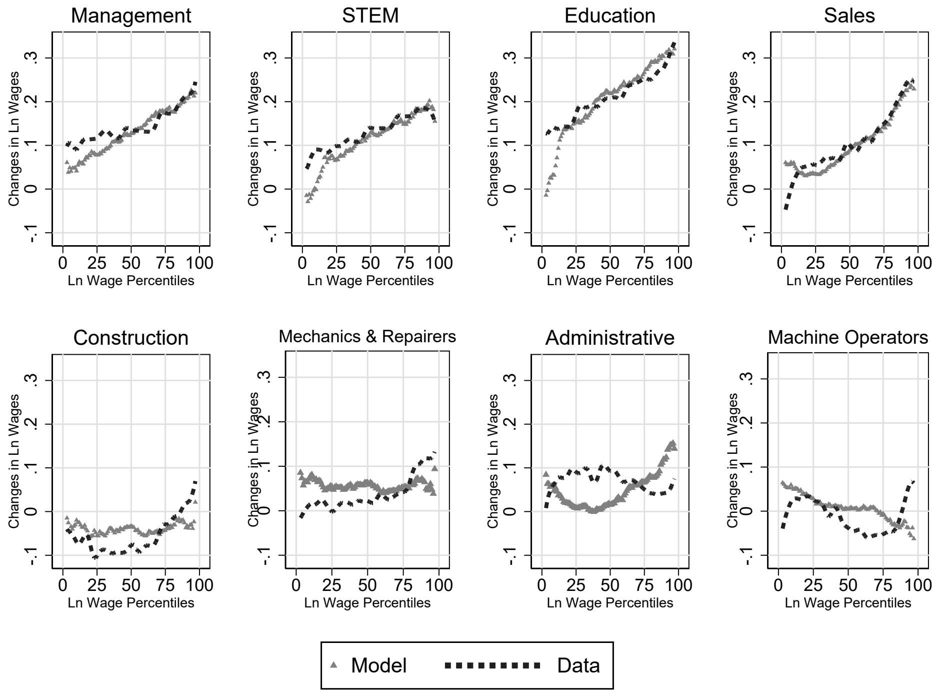

Figure D.1: Changes in Occupational Wages 1980-2000: Nested-CES Model $\eta = 2$

Notes: The changes are in real terms, for which I deflate the hourly wages by the PCE price deflator.

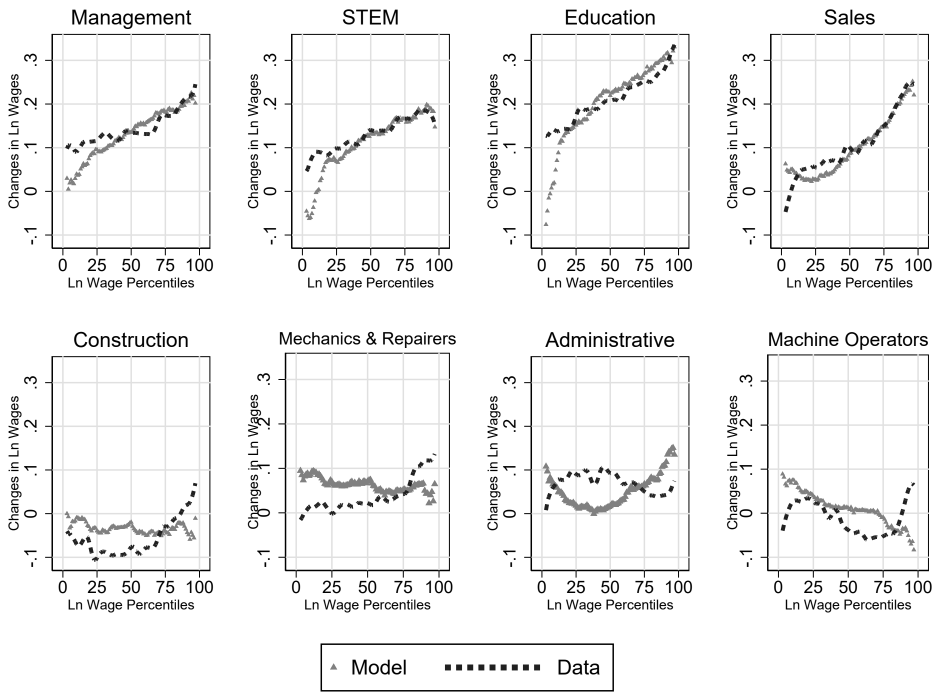

Figure D.2: Changes in Occupational Wages 1980-2000: Nested-CES Model $\eta = 3$

Notes: The changes are in real terms, for which I deflate the hourly wages by the PCE price deflator.

E Tables and Figures

Table E.1: Occupation Relative Task Demand in 1980 and 2000

Occupation	Cognitive	Social	Routine	Manual
Year 1980				
Executive Management	0.211	0.188	0.508	0.093
Management Related	0.178	0.111	0.648	0.063
STEM	0.197	0.107	0.594	0.102
Social Service, Lawyers	0.177	0.299	0.450	0.074
Education, Training, Library, legal support	0.231	0.212	0.474	0.083
Health Occupations	0.145	0.171	0.522	0.162
Technicians and Related Support	0.165	0.131	0.590	0.114
Financial Sales and Related Occupations	0.196	0.160	0.514	0.130
Retail Sales	0.147	0.191	0.535	0.126
Administrative Support	0.128	0.138	0.643	0.091
Housekeeping, Cleaning, Laundry	0.061	0.092	0.549	0.298
All Protective Service	0.124	0.143	0.469	0.264
Food Preparation and Service	0.050	0.137	0.570	0.243
Farm operators	0.064	0.074	0.472	0.391
Mechanics and Repairers	0.081	0.052	0.459	0.407
Construction	0.089	0.058	0.464	0.390
Precision production	0.099	0.072	0.567	0.263
Machine Operators, Assemblers, and Inspectors	0.043	0.020	0.638	0.299
Transportation and Material Moving	0.042	0.073	0.577	0.308
Handlers, Equipment Cleaners, and Helpers	0.023	0.021	0.560	0.396
Year 2000				
Executive Management	0.376	0.408	0.125	0.092
Management Related	0.408	0.308	0.205	0.079
STEM	0.425	0.278	0.176	0.121
Social Service, Lawyers	0.301	0.497	0.107	0.094
Education, Training, Library, legal support	0.407	0.365	0.117	0.110
Health Occupations	0.273	0.328	0.195	0.204
Technicians and Related Support	0.328	0.235	0.272	0.165
Financial Sales and Related Occupations	0.368	0.364	0.133	0.134
Retail Sales	0.275	0.322	0.232	0.171
Administrative Support	0.275	0.266	0.319	0.140
Housekeeping, Cleaning, Laundry	0.128	0.216	0.276	0.380
All Protective Service	0.224	0.287	0.202	0.288
Food Preparation and Service	0.104	0.313	0.280	0.303
Farm operators	0.153	0.205	0.252	0.389
Mechanics and Repairers	0.196	0.147	0.247	0.410
Construction	0.210	0.159	0.246	0.385
Precision production	0.236	0.199	0.303	0.262
Machine Operators, Assemblers, and Inspectors	0.128	0.070	0.428	0.374
Transportation and Material Moving	0.094	0.182	0.307	0.417
Handlers, Equipment Cleaners, and Helpers	0.067	0.073	0.371	0.490

Table E.2: Changes in Log Task Prices by Occupations Between 1980 and 2000

Occupation	Cognitive	Social	Routine	Manual
Executive Management	0.317	0.471	-0.310	0.111
Management Related	0.396	0.546	-0.234	0.188
STEM	0.345	0.492	-0.288	0.135
Social Service, Lawyers	0.395	0.483	-0.222	0.280
Education, Training, Library, legal support	0.365	0.452	-0.259	0.239
Health Occupations	0.451	0.552	-0.074	0.322
Technicians and Related Support	0.356	0.422	-0.126	0.243
Financial Sales and Related Occupations	0.123	0.273	-0.509	-0.084
Retail Sales	0.308	0.375	-0.167	0.201
Administrative Support	0.301	0.364	-0.168	0.194
Housekeeping, Cleaning, Laundry	0.172	0.320	-0.288	0.064
All Protective Service	0.311	0.437	-0.162	0.174
Food Preparation and Service	0.240	0.387	-0.218	0.123
Farm operators	0.257	0.412	-0.226	0.044
Mechanics and Repairers	0.281	0.432	-0.207	0.064
Construction	0.183	0.328	-0.305	-0.038
Precision production	0.219	0.359	-0.271	-0.022
Machine Operators, Assemblers, and Inspectors	0.324	0.486	-0.160	0.107
Transportation and Material Moving	0.155	0.312	-0.304	0.052
Handlers, Equipment Cleaners, and Helpers	0.188	0.377	-0.283	-0.006

Notes: Each value is calculated as $\ln p_{o,k}^{2000} - \ln p_{o,k}^{1980}$.

Table E.3: Average Earning Share by Tasks and Occupations, in 2000

Occupation	Cognitive	Social	Routine	Manual
Executive Management	0.365	0.292	0.161	0.181
Management Related	0.395	0.229	0.250	0.126
STEM	0.411	0.228	0.217	0.143
Social Service, Lawyers	0.281	0.323	0.137	0.259
Education, Training, Library, legal support	0.367	0.267	0.140	0.225
Health Occupations	0.248	0.240	0.233	0.279
Technicians and Related Support	0.308	0.187	0.325	0.179
Financial Sales and Related Occupations	0.358	0.279	0.173	0.190
Retail Sales	0.258	0.246	0.281	0.214
Administrative Support	0.244	0.215	0.357	0.185
Housekeeping, Cleaning, Laundry	0.122	0.185	0.318	0.375
All Protective Service	0.211	0.223	0.246	0.320
Food Preparation and Service	0.094	0.259	0.307	0.340
Farm operators	0.163	0.191	0.329	0.318
Mechanics and Repairers	0.205	0.140	0.323	0.333
Construction	0.218	0.149	0.322	0.311
Precision production	0.231	0.179	0.374	0.215
Machine Operators, Assemblers, and Inspectors	0.134	0.083	0.524	0.259
Transportation and Material Moving	0.099	0.178	0.378	0.345
Handlers, Equipment Cleaners, and Helpers	0.074	0.083	0.447	0.396

Notes: I use equation (20) to estimate $B_{k|o}$ for each pseudo worker, then obtain the aggregate moments using the predicted probability $\Pi_o(\nu^G)$ as the weight. When computing the average for $B_{k|o}$, I use the predicted probability defined in equation (11) as the weights.

Table E.4: Reduced-Form Evidence Using PDII Data

	A. Estimated Coefficients for Equation (1)					
		Social			Routine	
	(1)	(2)	(3)	(4)	(5)	(6)
HS graduate	-0.0225	-0.00765	-0.0150	-0.00572	-0.0222	-0.0103
	(0.0334)	(0.0332)	(0.0353)	(0.0276)	(0.0289)	(0.0304)
Some college	0.00703	0.00156	-0.0194	-0.0571	-0.0775	-0.0670
	(0.0353)	(0.0349)	(0.0368)	(0.0292)	(0.0304)	(0.0316)
College and above	0.00642	-0.0202	-0.0439	-0.117	-0.128	-0.115
	(0.0368)	(0.0362)	(0.0392)	(0.0305)	(0.0315)	(0.0337)
Age	0.0162	0.00989	0.0121	-0.0108	-0.0103	-0.00778
	(0.00474)	(0.00457)	(0.00487)	(0.00393)	(0.00398)	(0.00419)
Age^2	-0.000206	-0.000131	-0.000159	0.000116	0.000110	0.0000842
	(0.0000591)	(0.0000567)	(0.0000602)	(0.0000490)	(0.0000494)	(0.0000518)
Male	0.0521	0.0253	0.0115	-0.0819	-0.0612	-0.0509
	(0.0202)	(0.0201)	(0.0220)	(0.0167)	(0.0176)	(0.0189)
Black	0.0734	0.0921	0.0824	0.00229	0.00110	-0.00693
	(0.0287)	(0.0281)	(0.0296)	(0.0238)	(0.0245)	(0.0255)
2-digit Occup.	✓			✓		
3-digit Occup.		✓			✓	
5-digit Occup.			✓			✓

Notes: All reduced-form equations are estimated using PDII data. The omitted group is high school dropout females who are 41-60 years old. $N = 1333$ for all models. Standard errors are reported in the parenthesis.

Table E.5: The variance-covariance matrix of $\ln \Pi_{k|o}(z^G) - \ln \Pi_{4|o}(z^G)$, By Groups

	By Education Groups					
	Non-College			**College**		
	Cognitive	*Social*	*Routine*	*Cognitive*	*Social*	*Routine*
Cognitive	1.85			1.34		
Social	0.95	2.48		1.04	2.56	
Routine	0.60	0.62	1.46	0.81	0.71	1.94
	By Gender					
	Male			**Female**		
	Cognitive	*Social*	*Routine*	*Cognitive*	*Social*	*Routine*
Cognitive	1.57			1.87		
Social	0.88	2.35		1.14	2.71	
Routine	0.60	0.49	1.45	0.80	0.85	1.84
	By Age Groups					
	21-40 Years Old			**41-60 Years Old**		
	Cognitive	*Social*	*Routine*	*Cognitive*	*Social*	*Routine*
Cognitive	1.74			1.68		
Social	0.80	2.31		1.21	2.74	
Routine	0.63	0.51	1.46	0.75	0.81	1.82

Notes: The parameters are estimated using observations who belong to a particular group.

Table E.6: The SMM Estimates of Structural Parameters, Baseline Model

Education and Age Groups	$\mu^G_{z_1}$	$\mu^G_{z_2}$	$\mu^G_{z_3}$	$\mu^G_{z_4}$	**Cov**$(\ln z^G_1, \ln \nu^G_4)$	**Cov**$(\ln z^G_2, \ln \nu^G_4)$	**Cov**$(\ln z^G_3, \ln \nu^G_4)$	**Var**$(\ln \nu^G_4)$
				Females				
HS dropouts, 21-41 years Old	-.34 (.003)	-.06 (.072)	.024 (.059)	-.06 (.211)	-.12 (.021)	-.14 (.033)	-.13 (.006)	.141 (.000)
HS dropouts, 41-60 years Old	0 (0)	0 (0)	0 (0)	0 (0)	-.19 (.007)	-.15 (.008)	-.16 (.005)	.185 (.002)
HS graduates, 21-41 years Old	-.10 (.000)	.018 (.010)	.485 (.002)	-.10 (.019)	-.05 (.003)	-.14 (.002)	-.21 (.005)	.269 (.007)
HS graduates, 41-60 years Old	.131 (.001)	-.04 (.003)	.518 (.006)	-.01 (.000)	-.11 (.001)	-.11 (.003)	-.16 (.002)	.209 (.014)
Some college, 21-41 years Old	.379 (.002)	-.81 (.000)	.054 (.002)	.230 (.010)	-.12 (.002)	-.06 (.004)	-.17 (.005)	.229 (.011)
Some college, 41-60 years Old	.333 (.027)	-.46 (.006)	-.01 (.016)	.362 (.054)	-.15 (.005)	-.10 (.007)	-.18 (.003)	.253 (.004)
College and above, 21-41 years Old	.528 (.010)	-.29 (.005)	-.80 (.000)	.444 (.009)	-.19 (.004)	-.13 (.007)	-.19 (.006)	.302 (.006)
College and above, 41-60 years Old	.568 (.078)	-.10 (.031)	-.63 (.002)	.524 (.116)	-.20 (.003)	-.15 (.004)	-.21 (.001)	.317 (.002)
				Males				
HS dropouts, 21-41 years Old	-.42 (.000)	-.20 (.028)	-.24 (.012)	.190 (.119)	-.03 (.007)	-.09 (.013)	-.02 (.003)	.093 (.003)
HS dropouts, 41-60 years Old	-.42 (.003)	-.41 (.020)	.111 (.078)	.301 (.209)	-.06 (.004)	-.04 (.021)	-.06 (.002)	.110 (.010)
HS graduates, 21-41 years Old	.104 (.021)	-.55 (.023)	.171 (.088)	.353 (.283)	.034 (.026)	.046 (.040)	-.11 (.013)	.144 (.052)
HS graduates, 41-60 years Old	-.19 (.002)	-.66 (.004)	.398 (.076)	.375 (.129)	-.02 (.010)	.017 (.036)	-.08 (.009)	.142 (.047)
Some college, 21-41 years Old	.396 (.043)	-.84 (.000)	.038 (.021)	.413 (.126)	-.05 (.006)	.045 (.018)	-.10 (.006)	.163 (.037)
Some college ,41-60 years Old	.389 (.008)	-.58 (.001)	.303 (.005)	.501 (.001)	-.06 (.002)	.042 (.013)	-.13 (.003)	.198 (.013)
College and above, 21-41 years Old	.884 (.027)	.917 (.093)	.473 (.004)	-.13 (.123)	-.21 (.014)	-.30 (.020)	-.24 (.022)	.469 (.048)
College and above, 41-60 years Old	.738 (.030)	.666 (.056)	-.06 (.000)	.315 (.092)	-.22 (.009)	-.23 (.006)	-.17 (.012)	.416 (.038)

Notes: Standard errors are reported in parenthesis. Appendix C.3 details how I construct the standard errors of the parameter estimates.

Table E.7: The SMM Estimates of Structural Parameters, Alternative Model 2

Education and Age Groups	$\mu^G_{z_1}$	$\mu^G_{z_2}$	$\mu^G_{z_3}$	$\mu^G_{z_4}$	**Cov**$(\ln z^G_2, \ln \nu^G_4)$	**Cov**$(\ln z^G_2, \ln \nu^G_4)$	**Cov**$(\ln z^G_3, \ln \nu^G_4)$	**Var**$(\ln \nu^G_4)$
				Females				
HS dropouts21-41 years Old	-.34 (.007)	-.06 (.151)	.101 (.241)	-.06 (.616)	-.11 (.048)	-.08 (.081)	-.13 (.024)	.151 (.021)
HS dropouts41-60 years Old	0 (0)	0 (0)	0 (0)	0 (0)	-.21 (.052)	-.11 (.028)	-.20 (.044)	.264 (.004)
HS graduates, 21-41 years Old	.048 (.011)	.003 (.045)	.548 (.070)	-.11 (.134)	-.10 (.011)	-.12 (.018)	-.23 (.003)	.270 (.024)
HS graduates, 41-60 years Old	.292 (.030)	-.09 (.035)	.557 (.075)	-.04 (.149)	-.10 (.009)	-.09 (.022)	-.15 (.006)	.211 (.073)
Some college, 21-41 years Old	.332 (.042)	-.73 (.008)	-.00 (.001)	.238 (.081)	-.13 (.012)	-.08 (.018)	-.21 (.051)	.275 (.068)
Some college, 41-60 years Old	.356 (.187)	-.45 (.022)	-.04 (.075)	.309 (.307)	-.21 (.057)	-.09 (.057)	-.23 (.064)	.353 (.082)
College and above, 21-41 years Old	.358 (.045)	-.03 (.058)	-.81 (.001)	.494 (.050)	-.20 (.094)	-.14 (.113)	-.23 (.198)	.365 (.098)
College and above, 41-60 years Old	.493 (.004)	.234 (.037)	-.86 (.003)	.523 (.007)	-.19 (.030)	-.12 (.094)	-.23 (.085)	.323 (.098)
				Males				
HS dropouts, 21-41 years Old	-.48 (.001)	-.34 (.038)	-.13 (.041)	.230 (.251)	-.00 (.013)	-.04 (.020)	-.03 (.006)	.081 (.009)
HS dropouts, 41-60 years Old	-.47 (.006)	-.45 (.016)	.217 (.150)	.293 (.331)	-.04 (.009)	.007 (.043)	-.04 (.009)	.107 (.059)
HS graduates, 21-41 years Old	.190 (.053)	-.75 (.015)	.273 (.220)	.301 (.496)	-.01 (.019)	.015 (.014)	-.17 (.030)	.190 (.066)
HS graduates, 41-60 years Old	.301 (.082)	-.73 (.010)	.356 (.327)	.380 (.475)	-.03 (.135)	.027 (.311)	-.03 (.123)	.100 (.283)
Some college, 21-41 years Old	.345 (.168)	-.74 (.012)	.041 (.085)	.420 (.384)	-.04 (.049)	.029 (.086)	-.11 (.029)	.168 (.148)
Some college, 41-60 years Old	.455 (.165)	-.63 (.012)	.253 (.151)	.505 (.372)	-.06 (.059)	.043 (.137)	-.11 (.029)	.173 (.065)
College and above, 21-41 years Old	.511 (.085)	1.05 (.492)	.385 (.041)	.013 (.370)	-.21 (.395)	-.17 (1.02)	-.20 (.306)	.384 (.929)
College and above, 41-60 years Old	.477 (.124)	.867 (.052)	-.07 (.004)	.382 (.039)	-.23 (.277)	-.14 (.540)	-.18 (.221)	.387 (.595)

Notes: Standard errors are reported in parenthesis. Appendix C.3 details how I construct the standard errors of the parameter estimates.

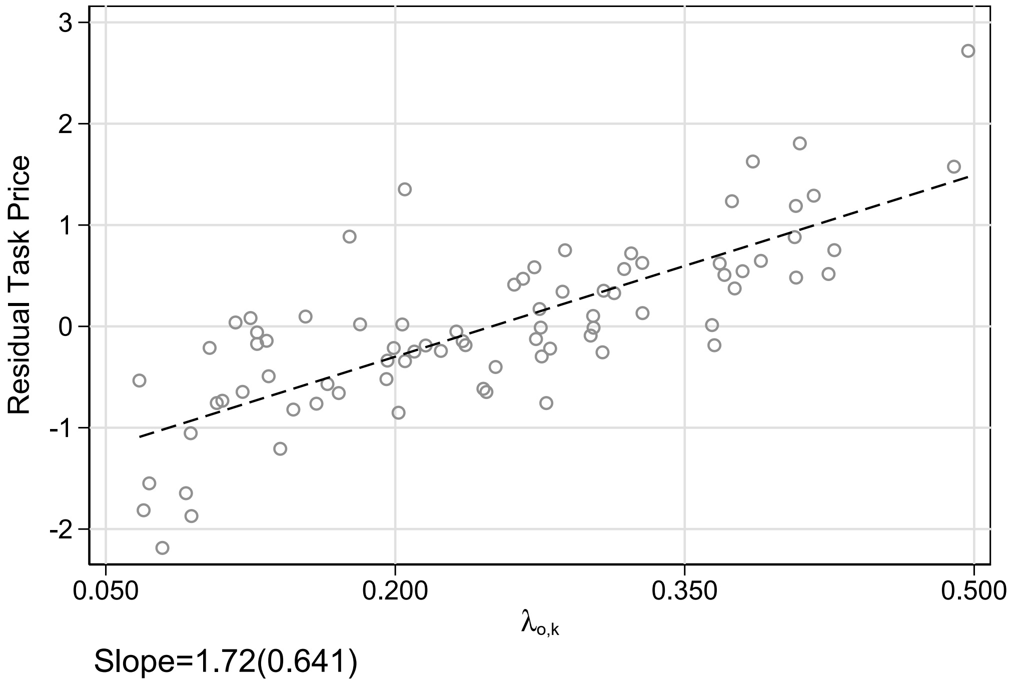

Figure E.1: Task Price Residuals and the Occupational Task Intensity, $\lambda_{o,k}$

Notes: The dependent variable is the residual of equilibrium task prices on 20 occupational dummies.

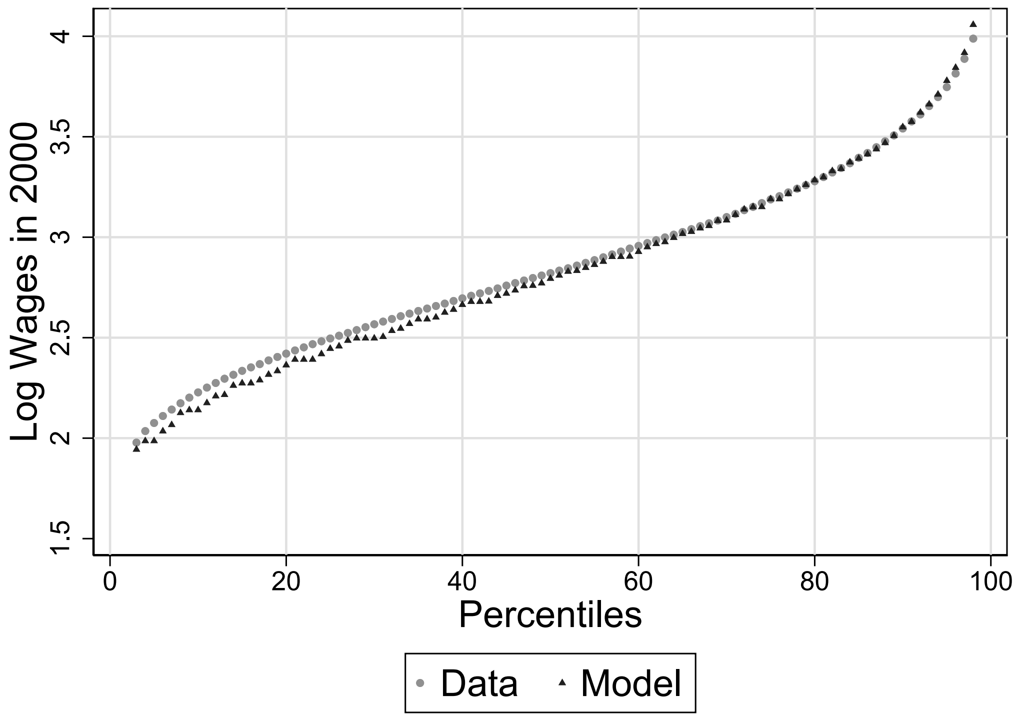

Figure E.2: The Percentile Values of Log Wage, Data, Benchmark Model

Notes: This figure plots the percentile on the horizontal axis and the value of log wage on the vertical axis.

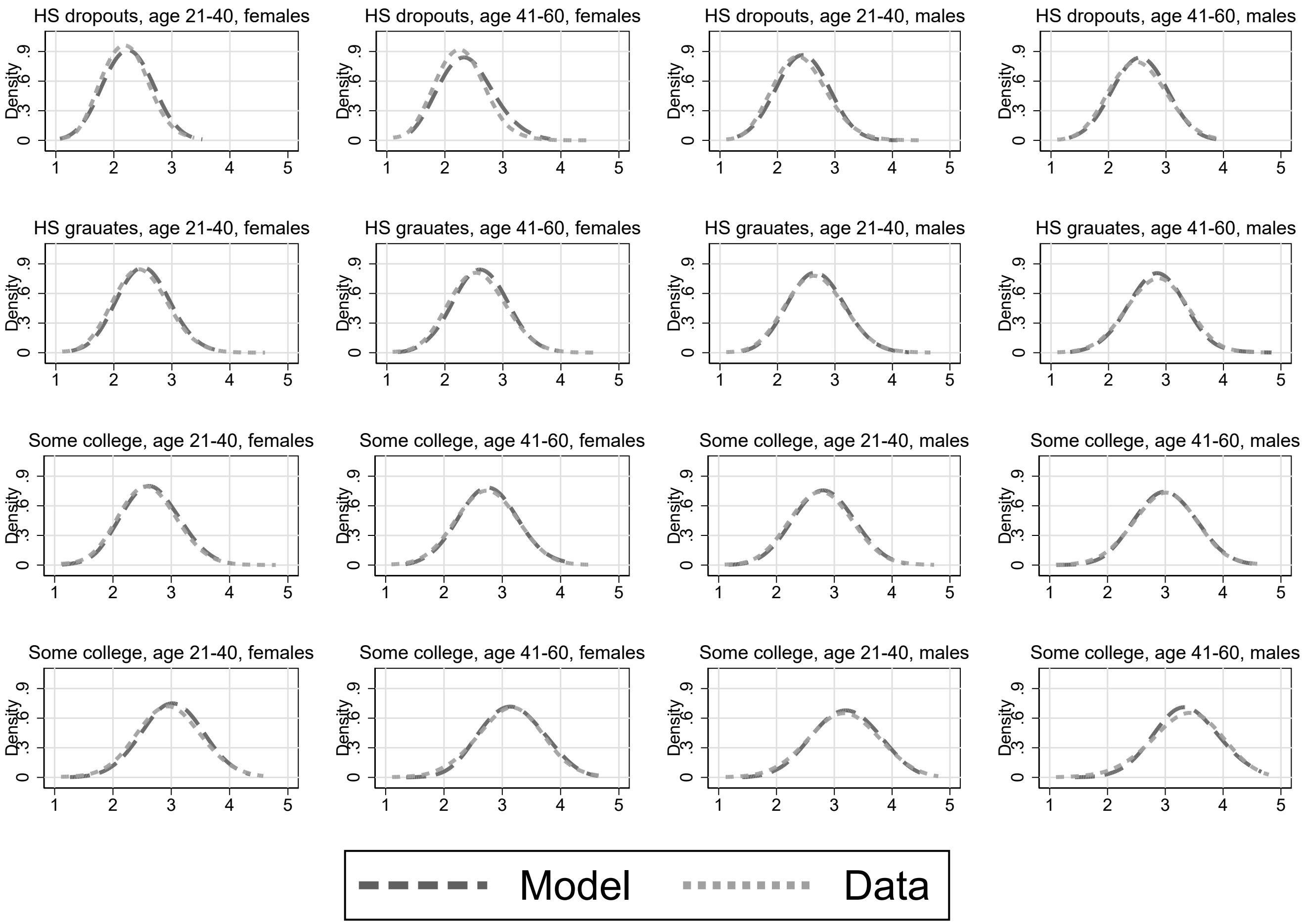

Figure E.3: The Kernel Density of the Empirical and Predicted Log Wages by Groups

Notes: I use the Epanechnikov kernel. To obtain a clear visualization in the log wage support, I set the bandwidth to be the default optimal bandwidth choice under normal density.

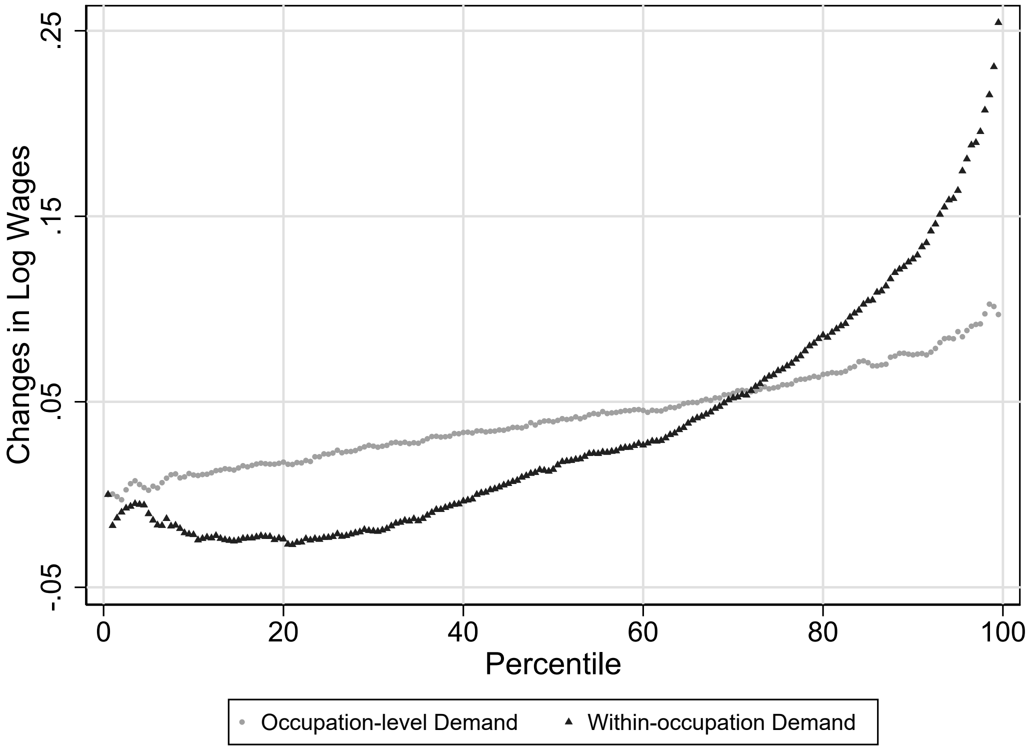

Figure E.4: The Relative Impact of Occupation-level Demand and Within-occupation Demand Changes on Wage Distribution. Both plots are normalized by the initial percentile value.

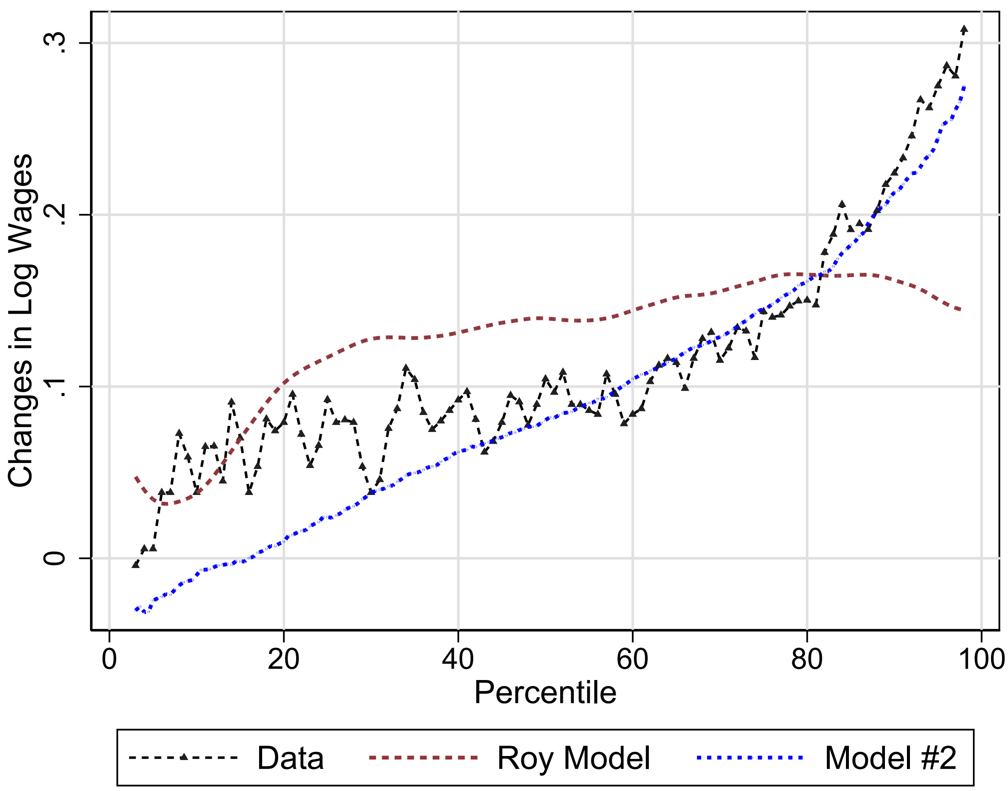

Figure E.5: Changes in the Log Wage By Percentiles: Alternative Models.