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Metaphorical Reasoning: Novices and Experts Solving and Understanding Negative Number Problems

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Do novices and experts use the same arithmetic metaphors for negative numbers? Twenty-four participants (12 middle school children and 12 post-secondary adults) computed arithmetic expressions involving negative numbers and described their understanding of six arithmetic expressions. The children and adults used the same metaphors to understand arithmetic expressions involving negative numbers. The adults showed a more integrated understanding of arithmetic expressions through multiple metaphors and multiple uses of each metaphor. In the problem-solving task however, the children reasoned metaphorically more often to perform computations and detect errors. When the children reasoned metaphorically, they computed more accurately, but they also used more time. The adults also showed less detail in their metaphors than the children did.

Key words: metaphors; negative numbers; theory-constitutive

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Researchers have begun viewing metaphors as an important learning mechanism rather than a mere literary device (Black, 1979; Gentner, 1989; Kuhn, 1979; Lakoff & Johnson, 1980; Ortony, 1993; Reddy, 1979). In addition to studies showing the effectiveness of pedagogical metaphors (e.g., Petrie & Oschlag, 1993), Boyd (1993) argued that there are theory-constitutive metaphors at the heart of disciplines. In particular, Lakoff & Nunez (1997) further argued that mathematics has theory-constitutive metaphors (see English [1997] for more on metaphors in mathematics). If experts continue using the same metaphors that novices use, these metaphors are candidates for theory-constitutive metaphors. Do they? In this study, I examine the metaphors that expert adults and novice children use to solve problems and understand arithmetic expressions involving negative numbers.

I begin with a discussion of metaphorical reasoning's benefits, limitations, and changing usage as a person develops expertise. Then, I describe the methodology for my study of middle-school children and post-secondary adults solving problems involving negative numbers. After analyzing the data, I conclude with a discussion of the results and their implications.

Theoretical Perspective

Past researchers have discussed the advantages, disadvantages, and changing uses of metaphorical reasoning. Given the disadvantages of a metaphorical reasoning, experts either stop using it or adapt to its disadvantages.

Advantages

Researchers have argued that students reason metaphorically to construct new concepts, interpret representations, connect concepts, improve recall, compute solutions, and detect and correct errors. Building on their prior knowledge, students can construct new concepts metaphorically (Lakoff & Johnson, 1980; Lakoff & Nunez, 1997). For example, they can project their understanding of space and motion on to arithmetic to make sense of it (ARITHMETIC IS MOTION ALONG A PATH, see Table 1). The familiar understood domain is often called the *source* while the unfamiliar domain to be understood is often called the *target*.

Table 1 A Partial Mapping of the Metaphor ARITHMETIC IS MOTION ALONG A PATH in which a Person Projects His or Her Understanding of Motion Entities, Properties, and Relationships to Arithmetic to Interpret Arithmetic Entities, Properties, and Relationships

Motion along a path	→	Arithmetic
Location relative to the origin	→	Number
Origin/starting point	→	0
Location A steps to the right of the origin	→	Positive number A
Location A steps to the left of the origin	→	Negative number (-A)
Distance from the origin (A steps away)	→	Absolute value $|A|$
Location A is to the right of location B	→	A > B
Location A is to the left of location B	→	A < B
Move A steps (if A=0, then hop in place). (- A) indicates A steps backwards. The default operation is to face right and move forward in that direction	→	Add A
The order of steps does not matter	→	A + B = B + A
Turn around and move B steps	→	Subtract B
Turning around at different times changes where one goes	→	A - B B - A
Walking backwards is the same as turning around and walking forward	→	A + (-B) = A - B
Moving forward is the opposite of moving backwards	→	Addition is the inverse of subtraction
Direction and steps needed to return to the origin	→	Additive inverse
Take M steps, each of size N. If M is negative, turn around first. If N is negative, take steps backward.	→	Multiply M x N
Switching the number of steps and the step size does not change the destination	→	M x N = N x M
Multiplication is repeating a particular step (N) several times (M)	→	M M x N = N
How do you reach location -20 using a step size of 2? Turn around (-) and make 10 steps, so the answer is -10	→	Division M/N -20/2 = -10
How many steps of size 0 are needed to go to 4? Impossible	→	4/0 =?
How many steps of size 0 are needed to go to 0? Any number of steps ...0,1, 2...	→	0/0 = ?
Asking how to get there is the opposite of asking where are we going?	→	Division is the inverse of multiplication

Many mathematical representations such as Venn diagrams, number lines, and graphs rely on metaphorical inferences from a person's prior understanding of space. (For the purpose of this article, a representation refers to any perceptually accessible stimuli such as drawings, gestures or talk that a person interprets.) Consider the LINES ARE PATHS metaphor that underlies ARITHMETIC IS MOTION. Because students are familiar with the source (motion), they know that travelers move along a path. Sfard (1994) argued that students can create new target entities, in this case, virtual travelers that move along the paths. Using this source understanding, students can view static lines as paths on which travelers move.

Students can build connections between concepts through single or multiple metaphors (Chiu, 1997). Consider three metaphors with a common source but different targets, ARITHMETIC IS MOTION, VARIABLES ARE UNKNOWN MOTIONS, EQUATIONS ARE JOURNEYS OF TWO TRAVELERS. A student can use these metaphors to solve the problem $x - 2 = -9$. A student solving this problem using these EQUATIONS and VARIABLES metaphors views the goal as finding the unknown motion (variable value) that will result in both travelers reaching the same destination from the same origin (0). Because moving both travelers in the same way moves them to the same place, having both travelers undoing the known motions of the left traveler reveals its unknown motion. For instance, both travelers can take two steps forward from their current location to reverse the two steps backward by the left traveler ($x - 2 + 2 = -9 + 2 \Rightarrow x = -7$). Thus, the right traveler moves to -7, so the left traveler's unknown motion must have been moving to -7 from the starting point (or origin). These additional vivid connections between concepts provide additional avenues for students to remember specific information and thereby improve recall (Ortony, 1975). For example, Reynolds and Schwartz's (1983) study showed that those participants who used metaphors remember more.

Students can metaphorically compute expressions (as shown in the above example) and metaphorically detect and correct errors (Chiu, 1994). In the above problem, a person who mistakenly computes "7" from -9 + 2 can detect the error by recognizing that taking two steps from -9 is insufficient to leave the negative region, so the answer can not be positive.

Disadvantages

However, metaphorical reasoning also has several potential disadvantages including: invalid metaphorical inferences, interference with later learning, unreliable metaphorical justifications, and inefficient metaphorical procedures. Because a metaphor's source and its target are inherently different phenomena, there must be omissions or invalid metaphorical inferences. For example, a person reasoning through PRIME NUMBERS ARE PRIMARY COLORS (Nolder, 1991) can infer that because there are a finite number of primary colors, there must be a finite number of primes. However, this metaphorical inference is false; there are an infinite number of primes. Furthermore, Spiro, Vispoel, Schmitz, Samarapungavan and Boerger's (1987) study showed that students who viewed a metaphor as literally true had difficulty learning concepts that contradicted inferences from that metaphor. Metaphorical inferences can be incorrect, so metaphorical reasoning is not a generally reliable mathematical method. As a result, experts do not justify their results through metaphors in formal mathematical discourse (e.g. journals such as *American Journal of Mathematics*). Finally, metaphorical reasoning is slower and less efficient than mathematical facts or algorithms (Chiu, 1994). For example, the metaphorical computation described above for "-9 + 2" requires more time than (a) simply recalling the memorized result "-7" or (b) computing the difference (9 – 2 = 7), and attaching the sign of the number with the greater magnitude (-from -9, so the answer is -7).

Changing Usage of a Metaphor

In this section, I compare how use of a metaphor changes with respect to use of earlier metaphors, source comprehension and adaptation to a metaphor's limitations.

Expert use of earlier metaphors? Some researchers have claimed that metaphors are only temporary scaffolds, but others have argued that experts retain access to them. According to Searle (1979), metaphorical expressions that people understand quickly must be "dead metaphors", in which the source is lost and unrecoverable. Likewise, Post, Wachsmuth, Lesh, and Behr (1985) argued that abstraction during developing expertise entails target understanding without any concrete metaphorical source. For metaphors (Lakoff & Nunez, 1997) in which the sources consist of basic intuitions (Chiu, 1996), students can easily recreate the metaphorical projections from the intuitive source on to the target (also known as *grounded* metaphors or *mundane* metaphors [Reyna, 1986]). Boyd (1993) and Lakoff and Nunez (1997) further argue that there are theory-constitutive metaphors at the heart of mathematics. In short, past research has not empirically resolved the question of whether experts continue using metaphors that they learned as novices.

Novice and expert differences in source comprehension. Metaphorical reasoning capabilities (and learning facility for metaphors) differ primarily along the dimension of source experience. Goswami's (1991) review of developmental research showed that metaphorical reasoning developed early and that knowledge differences were better predictors than age differences in metaphorical reasoning. In particular, Chen, Sanchez, and Campbell (1997) showed that 13-month-old infants can reason metaphorically. Furthermore, Gibbs's (1988, 1990) studies showed that the participants' metaphorical inferences about the target was limited by their knowledge of the source. Inadequate understanding of the source limits a person's capacity to reason through that metaphor. For example, when told that "ROOTS OF A NUMBER ARE COMPLEX PLANE ROTATIONS OF ONE ANOTHER," a mathematician can view a root of a number as a vector and the other roots as rotations of it, but a lay person who does not understand the "complex plane" source cannot. As source comprehension enables and limits reasoning through specific metaphors, adults have both greater potential to access more metaphors and greater use of specific metaphors. (Nesher's [1989] *exemplification* component and Post et al.'s [1985] *embodiment* are both subsets of a metaphor source. Whereas exemplification components are full subsystems, embodiments are much smaller pieces that students translate into mathematical symbols.)

Adapting to a metaphor's limitations. Adaptations to the limitations of metaphorical reasoning include using multiple metaphors, streamlining them, and limiting their use.

Experts can use multiple metaphors with different sources for the same target to overcome the limitations of individual metaphors (Chiu, 1997). For example, students can use VARIABLES ARE PLACEHOLDERS FOR NUMBERS to understand that a given variable has a specific number inside, but the metaphor suggests that variables can only assume one value (cf. y = 2x). Although students can use VARIABLES ARE TRAVELERS to understand that a variable may assume different values, using it to interpret a variable's motion in equations such as 3 / x + 2 = 5 is more difficult than using VARIABLES ARE PLACEHOLDERS. Learning both metaphors helps overcome the limitations of each individual metaphor.

Students can also streamline their metaphorical reasoning and eventually bypass it to create autonomous reasoning in the target (cf. Anderson's [1987] knowledge compilation). Projecting all source properties on to the target mathematics wastes time and effort because some metaphorical inferences are not relevant to the current problem. Consider solving the problem -2 + -1 with ARITHMETIC IS MOTION. Initially, a novice may draw a line, label it from -5 to 5, walk backwards two steps from the origin, walk one more step backwards, and then read off the answer, -3. As that person develops expertise, he or she recognizes that some source-to-target projections are unnecessary for a solution and omits them (e.g., labeling fewer locations on a line, starting at -2 rather than walking there from the origin [0], etc.). Eventually, experts can recognize the initial target condition -2 + -1 and simply recall the result, -3, bypassing metaphorical reasoning through the source entirely (hence, Post et al. [1985] claim that expertise entails expert understanding without a concrete metaphorical source). Theoretically, novices are expected to reference more source details compared to experts.

Finally, experts learn when not to reason metaphorically. As students develop expertise, they receive negative feedback on incorrect metaphorical inferences about the target and learn to avoid them. After learning that metaphorical inferences can be unreliable, experts also find alternate justifications (such as deductive proofs) for their results even if they initially generate their results through metaphorical reasoning (Schunn & Dunbar, 1996). Experts also use metaphors less often because they learn or create algorithms that are more efficient than cumbersome metaphorical procedures. As students acquire expertise, they do not need the scaffolding that was useful in building their initial understanding (Vygotsky, 1978). In short, experts avoid invalid metaphorical inferences, metaphorical justifications, and inefficient metaphorical procedures.

Method

In this study, children and adults solved arithmetic problems and showed their understanding of arithmetic expressions. Data sources included audio-taped semi-structured individual interviews of 24 participants, their written work, and field notes.

Participants

The 24 participants consisted of 12 children (12 to 13 years old, 7 males, 5 females) and 12 adults (18 to 25 years old, 7 males, 5 females). Six weeks before this study, the children's seventh-grade teacher self-reported using the Keedy and Bittinger (1987) textbook and classroom discussions of temperatures and debts to teach negative number arithmetic. The twelve adults were all mathematically-talented, studying mathematics or engineering at an elite university at the graduate or undergraduate level. This study was conducted in the US.

Tasks

An interviewer presented each participant with a description of the stock market, a two-part problem-solving task and an understanding task. Each participant read the following short written description of buying (+) and selling (-) precious metals and the consequences of the price subsequently increasing (+) or decreasing (-) for each transaction (technically, metals are traded in the commodities market):

> The stock market is a place for gambling, like a casino. In the stock market, you can buy and sell things like gold and silver.
> Suppose you BUY an ounce of gold for $100.
> If the price increases by $1 the next day, you have $101 and you win.
> If the price decreases (- $1), then you lose.
> On the other hand, you SELL some of your silver for $50 an ounce.
> If the price increases ($5) the next day, then you lose because you should have kept your silver, which is now worth $55 instead of $50.
> If the price decreases (- $3), you win by selling early and getting more money for it, $50 instead of $47.

Then, in the first half of this problem, the interviewer asked the participants to calculate the day's earnings or losses given a particular set of transactions. A report listed four transactions and the subsequent changes in price:

Stock Market Summary

	bought / sold(-) ounces	gain / loss(-) change per ounce	Total
Gold	10	$8	
Silver	-30	$5	
Platinum	-40	-$1	
Copper	10	-$4	

How much money did you win or lose?
[How can you tell if that's right?]

The solution entailed multiplying the ounces by the change in price per ounce for each transaction and totaling the results: (10 oz. x $8/oz.) + (30 oz. x -$5/oz.) + (-40 oz. x -$1/oz.) + (10 oz. x -$4/oz.) = $80 + -$150 + $40 + -$40 = -$70. On this particular day, the overall result was a loss (-$70). After each participant produced a result, the interviewer asked him or her to justify it.

In the second half of the problem, each participant could break into the computer account and change a single number by 5 (+5 or -5) to make a profit.

> It looks pretty bad, but fortunately, you're a computer expert. You can break into the computer account and change any one of the numbers by 5 (either +5 or -5). Which number should you change?
> [How can you tell if that's right?]

For example, subtracting 5 from the change in the price of platinum (-\$1/oz.) yields -\$6/oz. (-1 – 5 = -6), and so the new profit from the platinum transaction is -40 oz. x -\$6/oz. = \$240. The additional \$200 (\$240 – \$40 = \$200) changes the net loss, -\$70, to a net gain of \$130 (-\$70 + \$200 = \$130).

In the understanding task, the interviewer asked each participant for his or her understanding of six arithmetic expressions:

> How do you make sense of -5 + 8?
> [The interviewer follows with similar questions about -4 - 6, 7 - -2, -2 x 3, -7 x -4, -8 / -4]

Scoring

A colleague and I calculated the correctness and computation time of every participant's computation. Each participant's arithmetic operation between two numbers was a computation, so "30 + 50 – 20 is 80, 60" was two computations. Then, a colleague and I blind coded the transcripts, using the participants' written work and the interviewer's field notes as supporting evidence. For both tasks, we coded each computation for solution method. For instances of metaphorical reasoning, we also coded for metaphor type and level of detail. In the problem-solving task data, we also coded for type of metaphor function.

We classified the participants' solution methods using a decision tree into the following categories: answer only, mathematical rule, mathematical transformation, situation-based, metaphorical reasoning and other.

Does the person give an answer immediately?

- Yes, code as "result only"
- No, Does the person invoke a mathematical rule?
  - Yes, Does the person create another mathematical expression using the rule?
    - Yes, code as "mathematical transformation"
    - No, code as "mathematical rule"
  - No, Does the person invoke a situation different from the problem?
    - Yes, Does the person apply an inference from the invoked situation to the problem?
      - Yes, code as "metaphorical reasoning"
      - No, code as "result only"
    - No, code as "problem situation"

The coders took a conservative approach toward coding for metaphors. For the purpose of this article, using the broader problem context to understand the mathematical problem is not metaphorical. Also, the mathematical problem was part of the broader problem situation, so a person need not invoke another situation. I have coded problem situation separately to allow for both types of analysis. Similarly, the instruction sheet explained the problem through gambling, so we coded participants' use of gambling inferences as problem situation rather than metaphorical because participants may view the gambling scenario as part of the problem context.

In the same conservative spirit, referencing another situation does not necessarily imply metaphorical reasoning. A person may use metaphors for non-mathematical, non-problem-solving reasons. Consider the following example (drawn from an earlier pilot study).

AZ : [computes -4 + -3] negative seven.

Int : Did you say negative seven?

AZ : Yeah, because negative four plus three is negative seven. So, we end up at negative seven.

AZ may have understood the problem metaphorically through vertical space to compute the result of negative seven ("we *end up* at negative seven"). Or, he could have computed the answer first, then metaphorically marked this part of the problem-solving discourse as completed (reaching its destination). Additional evidence for the latter interpretation includes both the reference to motion after the result of the computation and the use of the temporal and effect marker "so" (Halliday & Hasan, 1976). AZ may have reasoned metaphorically, but he did not provide clear evidence of a metaphorical computational strategy. If the computational means was not visible, the computation was not coded as an instance of metaphorical reasoning.

We classified the participants' solution methods and explanations into the following categories: answer only, mathematical rule, mathematical transformation, situation reasoning, metaphorical reasoning and other. When solving a problem such as "Find the net gain or net loss of the following transactions: -$4 and -$2," people could give an answer without any visible work (answer only), saying "minus six" or writing "-6." The participants explained their solution methods as they solved the problem, so many of these "answer only" solutions may have been arithmetic facts. They could also apply a procedure (mathematical rule), "negative and negative is negative, so negative six," or change the problem into a different problem (mathematical transformation), "that's like the negative of four plus two [-(4+2)], so negative six." They could also use the constraint information from the problem (situation reasoning), "I lost four dollars and then I lost two more dollars, so altogether I lost six dollars." Finally, they could: (a) invoke a new situation different from the problem situation (and different from the gambling explanation) and (b) apply an inference pattern from the invoked situation to the problem (metaphorical reasoning); for example, "[draws horizontal line with hashmark and writes '-4' underneath] we're going left, so one [pen bounces left and hashmarks line], two [pen bounces left and hashmarks line; writes '-5' under previous hashmark and '-6' under the last hashmark], minus six." For instances of metaphorical reasoning, we classified them by typing into the following categories: motion, opposing object, social transaction, and other. Consider for example, the analyses of the following segments.

Transcript	Analysis
Use: Computing a solution	Metaphor: Arithmetic is motion along a path
AZ: Minus forty minus fifty [-40 - 50] do you add or subtract?	AZ does not give an immediate answer, and does use a solution procedure.
IN: How can you figure it out?	Interviewer prompts for a strategy
AZ: Umm (3) [draws vertical line, labels it from "-5" at the bottom, "-4, -3, -2, -1, 0, 1, 2, 3, 4, 5" at the top]	AZ invokes a new situation, space, and represents numbers as locations along a line.
So minus 40, minus fifty, so down. One [pen bounces from -4 to -5] Wait [extends line further down and writes "-6, -7, -8, -9, -10"] One, two, three, four, five [pen bounces down from -4 to -9 as she counts], minus ninety [writes "-90"]. Okay, let's see, ten minus eighty…	AZ interprets "minus 50" as going down, so he begins counting and moving down. However, AZ recognizes that the line does not include enough numbered locations for him to compute his answer and adds more. AZ metaphorically computes the answer with a moving count along the numbered locations.
Use: Detecting and correcting an error	Metaphor: Arithmetic is manipulating objects
EL: Minus five plus eight is minus three. No, wait, the pluses wipe out the minuses. Then, there are only pluses left over. It should be plus, positive three.	EL initially states a result, but changes her mind. She invokes a new situation of objects, characterizing each number as entities (pluses and minuses). The pluses eliminate the minuses, and by applying the inference metaphorically, the answer must be "plus" or positive three. EL does not explicitly articulate the mechanism by which "the pluses wipe out the minuses," (e.g. by pairwise mutual annihilation of one plus for each minus.)
Use: Justifying a result	Metaphor: Arithmetic is social transactions
AZ: Minus five plus eight is three	AZ gives an immediate answer.
IN: How do you know that's right?	Interviewer asks for a justification.
AZ: Because it's like I owe five dollars and then I earned eight dollars, so when I pay back the five dollars, I have three dollars left. Does that make sense?	AZ invokes the situation of a debt. After earning money, he metaphorically repays the debt from his earnings, thus, the resulting subtraction leaves three.

We also coded the level of detail for each instance of metaphorical reasoning (0-5), with one point for each of the following categories: (a) non-numerical representations for problem or solution numbers, (b) other numbers, (c) non-numerical representations for other numbers, (d) operations other than standard arithmetic (+, -, *, /), and (e) other details. The following example (-5 + 8) includes all of these details:

> MO: Um, [draws a horizontal line with 11 hashmarks and arrows at each end, writing numbers underneath each one from left to right (-5, -4… 5)] Minus five [puts pen at hashmark above "-5"], one, two, three, four, five, six, seven, eight [pen moves to adjacent right hashmarks with each number, ending at the hashmark above "3"]. Three.

MO used non-numerical representations for the problem and solution numbers (hashmarks), other numbers (-4 ... 5), non-numerical representations for the other numbers (additional hashmarks), other operations (counting), and other details (lines, arrows).

For the understanding task data, we also counted how often participants used (a) multiple explanations for each arithmetic expression, (b) multiple metaphors for each arithmetic expression, (c) the same explanation for multiple arithmetic expressions, and (d) the same metaphor for multiple arithmetic expressions.

Using Cohen's kappa, we tested for inter-coder reliability. Higher inter-coder reliability increases the likelihood that different observers would classify the instances according to the above classification scheme in the same way. Finally, we resolved differences in coding through consensus.

Analyses

These children and adults were tested for differences in computational accuracy, speed, instances of metaphor use, and each type of metaphor use in the problem-solving task. In the understanding task, I compared the groups for differences in uses of each type of explanation, instances of multiple explanations for single arithmetic expressions, and instances of a single explanation for multiple arithmetic expressions. I also tested for gender differences. All significant results are significant at the .05 level.

Results

The children reasoned metaphorically both to solve problems and to understand arithmetics. In contrast, the adults rarely reasoned metaphorically to solve routine arithmetic problems. Nevertheless, they used more metaphors than the children did to show their understanding. There were no significant gender differences across any of the following measures (all $p > .10$).

Overall, the adults computed arithmetic expressions more accurately (M =.99 accuracy, SD = .02) than the children did (M = .83, SD = .15, $t = 3.44$, p = .005).

Problem-Solving Task

Problem-solving method. The participants primarily computed arithmetic expressions in the problem-solving task either by giving answers only or by metaphorical reasoning (see Table 2; Cohen's kappa = 0.872, $z = 11.1$, $p < .001$). The children both computed more arithmetic expressions ($M = 22$) than the adults did ($M = 12$; $t = 2.52$, $p < .03$) and gave answers

Table 2 Mean Uses and Standard Deviations of Each Computational Method by Each Participant While Solving the Investment Problem and While Showing His or Her Understanding of Six Arithmetic Expressions Involving Negative Numbers

Task & Participants	Answer only		Metaphor		Mathematical rule		Mathematical transformation		Situation	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Problem-solving										
Children	16.25	6.34	3.67	2.99	0.58	0.47	0.42	0.75	0.67	0.54
Adults	10.17	1.67	0.25	0.62	0.33	0.51	0.17	0.39	0.67	0.29
t test	3.21**		3.88**		1.25		1.02		0.00	
Understanding										
Children	1.92	0.62	2.25	0.35	1.17	0.55	1.17	0.62	n/a	n/a
Adults	0.67	0.78	7.00	2.70	1.58	1.38	1.67	0.90	n/a	n/a
t test	4.35***		6.04***		0.96		1.58			

Note. Participants may use multiple methods for a single computation.
* $p < .05$. ** $p < .01$. *** $p < .001$.

only ($M = 16$) more often than the adults did ($M = 10$, $t = 3.21$, $p < .01$). In this task, the children also reasoned metaphorically more often ($M = 3.67$) than the adults did ($M = 0.25$, $t = 3.88$, $p < .01$).

Accuracy. In this task, the adults computed more accurately ($M = .99$, $SD = .04$) than the children did ($M = 0.85$, $SD = 0.15$, $t = 3.08$, $p < .01$). However, the children's accuracy improved when they reasoned metaphorically. The children metaphorically computed 93% of their arithmetic expressions correctly (41 / 44), but only 81% correctly when using other methods (174 / 215, $x(2, 2) = 3.93$, $p < .05$).

Time. The children also took more time per computation when reasoning metaphorically ($M = 5.83$ seconds, $SD = 2.23$) than when using other methods ($M = 3.03$, $SD = 0.83$, $t = 14.27$, $p < .001$). The children took more time per computation ($M = 3.51$, $SD = 0.83$) than the adults did ($M = 2.21$, $SD = 0.91$, $t = 3.58$, $p < .01$).

Types of metaphor function. The children used metaphors to compute, detect and correct errors, and justify their results, whereas most of the adults did not (see Table 3, Cohen's kappa = 0.966, $z = 3.98$, $p < .001$). In the

Table 3 Mean Metaphorical Reasoning Functions (and Standard Deviations) by Each Participant While Solving the Investment Problem

	Compute		Detect		Justify	
Participants	*M*	*SD*	*M*	*SD*	*M*	*SD*
Children	1.58	2.11	0.75	0.87	1.33	1.23
Adults	0.00	0.00	0.17	0.39	0.00	0.00
t test	2.60*		2.11		3.75**	

* $p < .05$. ** $p < .01$.

investment task, the children computed metaphorically more often ($M = 1.58$, $SD = 2.11$) than the adults did ($M = 0$, $SD = 0$, $t = 2.60$, $p < .03$). The children also detected 12 of their 45 errors (27%), 75% of them through metaphorical reasoning (9/12). The adults committed too few errors (3) in the investment task for any meaningful comparison. Finally, the children justified their answers more often metaphorically ($M = 1.33$, $SD = 1.23$) than the adults did ($M = 0$, $SD = 0$, $t = 3.75$, $p < .01$). When asked to justify, most participants repeated their computations without offering additional justification. Occasionally, the adults referred to mathematical rules ($M = 0.33$, $SD = 0.51$) or to the problem situation ($M = 0.67$, $SD = 0.51$).

Types of metaphors. Participants primarily reasoned through two types of metaphors, motion and object metaphors in this problem (see Table 4, Cohen's kappa = 0.937, $z = 3.73$, $p < .001$). The children used motion metaphors ($M = 2.67$, $SD = 1.83$) more often than opposing object metaphors ($M = .92$, $SD = 1.00$, $t = 2.91$, $p < .02$). They used motion and object metaphors more than the adults did ($M = 0.17$, $SD = 0.39$, $t = 4.84$, $p < .001$, and $M = 0$, $SD = 0$, $t = 3.19$, $p < .01$, respectively).

Table 4 Mean Uses and Standard Deviations of Each Type of Metaphor by Each Participant While Solving the Investment Problem and While Showing His or Her Understanding of Six Arithmetic Expressions Involving Negative Numbers

Task &	Motion		Opposing objects		Social transactions		Others	
Participants	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Problem-solving								
Children	2.67	1.11	0.92	0.86	0.00	0.00	0.09	0.23
Adults	0.08	0.29	0.00	0.00	0.08	0.29	0.00	0.00
t test	7.36***		3.71**		0.96		1.36	
Understanding								
Children	1.75	1.02	0.34	0.40	0.09	0.23	0.09	0.23
Adults	2.08	0.90	3.83	1.67	0.75	0.50	0.33	0.51
t test	0.84		7.04***		4.15**		1.49	

* $p < .05$. ** $p < .01$. *** $p < .001$.

Details. Finally, the children included more details ($M = 2.83$, $SD = .68$, on a scale of 0-5) than the adults did ($M = 1.67$, $SD = 0.58$), but the adults did not use metaphors often enough for a significant comparison in this task.

In short, children reasoned metaphorically during the problem-solving task whereas most adults did not. Furthermore, children improved their computational accuracy by metaphorically computing results, detecting errors and justifying their answers.

Understanding Task

During the understanding task however, the adults explained arithmetic expressions involving negative numbers with more metaphors than the children did (see Table 2 above, Cohen's kappa = 0.921, $z = 3.96$, $p < .001$).

Number of explanations and metaphors. For the six arithmetic expressions, the adults had fewer expressions that they could not explain (answer only: $M = 0.67$, $SD = 0.78$) than the children did ($M = 1.92$, $SD = 0.62$, $t = 4.35$, $p < .001$). In addition, the adults provided more explanations ($M = 10.25$, $SD = 2.75$) than the children did ($M = 4.50$, $SD = 1.24$, $t = 6.60$, $p < .001$). To show their understanding, both adults and children primarily used metaphors (68% of adults' explanations were metaphorical, 49% of children's). Furthermore, the adults showed their understanding with metaphors more often ($M = 7.00$, $SD = 2.70$) than the children did ($M = 2.25$, $SD = 0.35$, $t = 6.04$, $p < .001$).

Types of explanations and metaphors. The adults also showed their understanding through a greater variety of explanations. The adults used more types of explanations ($M = 4.92$, $SD = 2.35$) than the children did ($M = 2.42$, $SD = 0.62$, $t = 3.56$, $p < .01$). The adults also used more types of metaphors (adults: $M = 2.50$, $SD = .80$, children: $M = 1.33$, $SD = .49$, $t = 4.31$, p < .001; see Table 4, Cohen's kappa = 0.918, $z = 4.57$, $p < .001$). In particular, adults showed more object metaphors and social transaction metaphors than children did.

Integrated explanations. The adults also showed more integrated understanding through more integrated explanations (see Table 5). The adults showed their understanding of a specific arithmetic expression with multi-

Table 5 Each Participant's Mean (and Standard Deviations) of Multiple Explanations and Metaphors Per Arithmetic Expression and Mean Repeated Uses of the Same Explanation or Same Metaphor

	Multiple explanations /Expression		Multiple metaphors / Expression		Same explanation for multiple expressions		Same metaphor for multiple expressions	
Participants	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Children	0.50	0.80	0.17	0.39	0.92	0.79	0.75	0.45
Adults	3.17	1.78	2.00	1.28	2.83	1.67	1.92	1.00
t test	4.74***		4.75***		3.58**		3.69**	

*$p < .05$. **$p < .01$. ***$p < .001$.

ple explanations or multiple metaphors more often than children did. Likewise, adults showed their understanding of different arithmetic expressions with the same explanation or the same metaphor more often than children did.

Details. Finally, children showed more detail in their metaphors (M = 3.44, SD = 0.76) than adults did (M = 2.46, SD = 0.89, t = 2.92, $p < .01$). The children also used more metaphor detail in the understanding task than in the problem-solving task (t = 3.53, $p < .01$).

In short, adults showed a more integrated understanding of arithmetic using more explanations (primarily metaphorical ones), more types of explanations, more multiple explanations for a given expression, more explanations for multiple expressions with the same metaphor and less detailed metaphors.

Discussion

These results suggest that metaphorical reasoning is an important component of understanding arithmetic expressions involving negative numbers. Both the children and the adults used metaphors to understand the expressions. In addition, most of the children also reasoned metaphorically to compute more accurately than through other methods.

The children used metaphors to understand arithmetic expressions, compute answers, detect errors and justify their results. Likely as a result of these benefits, the children's metaphorical reasoning was also more accurate than their other computational methods. However, it was also slower than other methods, possibly accounting for its comparatively infrequent use despite its higher accuracy. The children also used less detail when reasoning metaphorically to solve problems than when showing their understanding, consistent with the view that people streamline their metaphorical reasoning during problem-solving.

Meanwhile, adults rarely reasoned metaphorically to solve this problem, but used more metaphors to show their understanding of negative numbers.

These adults never reasoned metaphorically to compute or justify their results while solving this problem, and they only detected three errors metaphorically. Instead, they recalled arithmetic facts to compute arithmetic expressions quickly and accurately. The adults may not have needed to reason metaphorically because this problem was relatively routine for them. (Schunn and Dunbar [1996] showed that adults do reason metaphorically when faced with difficult problems.) Nevertheless, their metaphorical reasoning did not fade with increased computational efficiency but continued to help them integrate their understanding of negative numbers through multiple metaphors. The adults retained access to their metaphors and used less detail compared to the children's metaphors, consistent with the streamlining view of metaphors during development of mathematical understanding. Moreover, their understanding of negative numbers included more explanations, mostly metaphorical ones. Through combinations of these explanations (again mostly metaphorical), adults showed a more integrated understanding of negative numbers than children did.

Future Research

This study raises additional questions for future research. Do mathematicians use these metaphors to understand arithmetics? Are there other cognitive resources that people find productive early in learning a domain but rarely use for routine problems as they acquire expertise? When do experts use metaphors? What are the learning trajectories for signed arithmetic? Precisely what roles do metaphorical reasoning play in those trajectories? How do people learn to connect multiple metaphors and multiple expressions? Answering these questions can help children optimally use metaphors and other resources for learning and development.

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