Educational Research Journal《教育研究學報》, Vol. 22, No. 2, Winter 2007

A Preliminary Study of a Microworld for Learning Probability

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In this study, the researchers developed the Web-based Probability Explorer (WPE), a mathematical microworld and investigated the effectiveness of the microworld's constructivist learning environment in enhancing the learning of probability. The study also looked at students' satisfaction with the microworld's features. The quasi-experimental, non-random pre- and post-test static group research was conducted with 52 students from two secondary schools in Kuching, Sarawak, Malaysia. The findings showed that students' learning of probability was enhanced after interacting with the WPE. The students were also satisfied with the features of the WPE.

Key words: microworld, probability, attitudes toward mathematics, constructivist-learning environment

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Despite various initiatives implemented to improve the teaching and learning of mathematics, Malaysian students generally are still weak in mathematics, and mathematics anxiety is still prevalent among them (Rahmah bt. Murshidi, 1999). It has been suggested that technology can play an important role in improving students' learning of mathematics (Ferbar & Trkman, 2003; Godwin & Sutherland, 2000; Hoyles & Sutherland, 1999; National Council of Teachers of Mathematics [NCTM], 2000; Stager, 2004) and research has shown the potential of microworld as a learning environment that encourages and motivates learners to learn through actively interacting with real-life experiences (Alpers, 2002; Edwards, 1998; Noss & Hoyles, 2003; Roschelle, Kaput, & Stroup, 2000).

Microworlds are computer-based learning environments in which the learners do not simply respond to predetermined questions, but actually control when and how events will happen (Papert, 1980). Studies by Hoyles, Noss, and Adamson (2000), Jiang and Potter (1992), Noss, Healy, and Hoyles (1997), and Sinclair, Zazkis, and Liljedahl (2003) have shown that microworlds have the potential to improve students' learning and understanding of mathematical content.

Mathematics educators believe that probability is an important concept for school curriculum (NCTM, 2000). However, students tend to have misconceptions and difficulty in understanding probability (delMas, 2002) such as rational number concepts and proportional reasoning. Microworlds have been suggested as being able to help reduce students' difficulties in learning of mathematics (Edwards, 1998; Kynigos, 2001; Noss & Hoyles, 2003) in general and learning of probability (Drier, 2000; Jiang & Potter, 1992) in particular.

Purposes of the Study

In this study, the researchers developed the Web-based Probability Explorer (WPE), a Web-based mathematical microworld and investigated the effectiveness of the WPE in enhancing probability learning among secondary school students. The study also looked at students' satisfaction with the features of the microworld.

Research Questions

The specific research questions to be investigated in this study were:

1. Could the microworld learning environment enhance the students' learning of probability?
2. Would the students be satisfied with the features in the microworld?

Review of Related Literature

Pedagogical Considerations in Using ICT in Teaching and Learning

Building on the pioneering work of Jean Piaget, psychologists and educational researchers have come to the understanding that learning is not simply information transmission (Resnick, 2001). Thus, discussions and interactions between students, peers and teachers are vital in developing mathematical ideas and understandings. Students are required to discuss ideas, test things out and listen to how other students arrived at solutions (Barney, 2001; Becta, 2003; Hoyles, 2002). When students communicate their results orally or in writing, they learn to use the mathematical language clearly, convincingly and precisely. Learners use their own stories, metaphors and analogies when discussing abstract and complex ideas. They develop their own understanding when participating and listening to peer's explanations (NCTM, 2000).

Jonassen (1994, p. 35) states, "Social constructivists believe that meaning making is a process of negotiation among the participants through dialogues or conversations". According to Dalgarno (2001), peer discussions make ideas visible to others in the class and give learners opportunity to relate to their prior experience. Hence, teachers should stimulate discussions among students, invite students to make predictions and interpretations of what they have seen and encourage individual student to give instructions or responses while using ICT for modelling mathematical ideas and strategies.

Furthermore, social constructivist learning environment requires teachers to act as facilitators, providing materials and guiding students to focus on situations that will foster students making the necessary mental constructions. Teachers who shaped the technology-mediated learning opportunities to the students would require a substantial understanding and familiarity with a range of ICT resources as well as the appropriate skills to successfully integrate them into their mathematics teaching (Muellar, 1998; Thompson & Kersaint, 2002). The effective use of ICT should enable students to focus on reasoning rather than on the answers, develop significant mathematical strategies and connect mathematical ideas with the real world (Becta, 2003).

In this type of learning environment, students are no longer passive knowledge receivers, but are actively constructing their mental model through exploration and engagement with activities in the constructivist learning environments (Jonassen & Carr, 2000; Papert, 1984). Learners are in control of the learning tools of knowledge, engaging in social interaction and eventually realising the concepts in a constructive representation mode (Li, 2001).

Role of Technology in Constructivist Learning

In the constructivist learning environment, the role of technology shifts from the technology-as-teacher to technology-as-partner (Jonassen, 1999). Technology acts as tools to support knowledge construction, as an information vehicle for exploring knowledge and as a context to support learning-by-doing. Technology is used as knowledge construction and representation tools, which facilitate students to learn *with* technology rather than learn *from* technology (Jonassen, 1998). Technologies function as intellectual partners of the students when they are used as mindtools (Jonassen, Peck, & Wilson, 1999).

What is a Microworld?

The term microworld describe a computer-based learning environment in which the learners do not simply respond to predetermined questions, but actually control when and how events will happen (Papert, 1980). Microworld often uses images to represent a concept (semantic icons), which can be directly manipulated by the learner on the screen with a mouse click or point (Kearney, 2001).

Thus, microworld is an exploratory learning environment or discovery space where learners can navigate, manipulate or create object, and test their knowledge (Jonassen, 1998; Jonassen, Carr, & Yueh, 1998). Students learn by doing instead of just watching or listening to the description of the domain knowledge (Lawler, 1987). Students bridge their prior knowledge and what they are about to learn when actively interact with the microworlds to solve real-life problem. In other words, they are built based on the theory of constructivist learning environment (CLE) or discovery learning environment (Jonassen & Rohrer-Murphy, 1999).

Microworld have "its own set of tools and operations that are open for inspection and change" (Hoyles et al., 2000, p. 2). The best microworlds are those with an easy-to-understand set of operations that enable the learners to engage the tasks that are of value to them and in the process come to know the powerful underlying principles (diSessa, 2000). Edwards (1998, p. 66) suggests the following features of microworlds learning environments:

a. a set of "computational objects... created to reflect the structure of mathematical ... entities within some sub domains of mathematics...",
b. a linkage between "more than one representation of the underlying mathematical or scientific entities or objects" (external multiple representations in different forms such as symbolic, visual or graphics, sound and motion),
c. a possibility to construct new objects from existing object (synthetic and constructive activities), and
d. a set of activities, "in which the user is challenged to use the entities

and operations to reach a goal, solve a problem, duplicate a situation or pattern and so forth".

Some microworlds in mathematics teaching and learning reported by recent researchers are listed in Table 1.

Table 1 Examples of Microworlds in Mathematics Teaching and Learning

Microworld	Development environment or language	Researcher/ developers (year)	Focus area in mathematics
Chance	Macintosh	Jiang & Potter (1992)	Introductory probability concept
Formula 1	Maple (CAS)	Alpers (2002)	Algebra
Mathsticks	Microworlds Logo	Noss, Healy, & Hoyles (1997)	Number patterns as functional relationship
MathWorld (SimCalc project)	Macintosh	Roschelle & Kaput (1996)	Calculus and algebra
Number World	Java applet	Sinclair, Zazkis, & Liljedahl (2003)	Number structures and relationships
ODEWorld	Mathematics	Kent (2000)	Differential equation
Probability	Window	Drier (2000)	Probability reasoning
Explorer	Logo and	Yeh & Nason	3D geometry
VRMATH	hypermedia	(2003)	

Note: CAS = Computer Algebra System

Methods

This study used the non-randomised pre- and post-test static group of quasi-experimental research design. The independent variable in the study was the WPE microworld learning environment while the dependent variables were the students' mathematics achievements and students' attitudes toward mathematics. The treatment group of this study was the 52 students from two Form 4 Science classes randomly chosen from two secondary schools in Kuching city, Sarawak, Malaysia. The students were co-educational (male and female) and of multi-ethnicity (Malay, Chinese and Indigenous people of Sarawak). They have not learnt the topic of probability before the treatment was conducted. Three teachers were also interviewed to obtain their views on the WPE microworlds.

Research Instruments

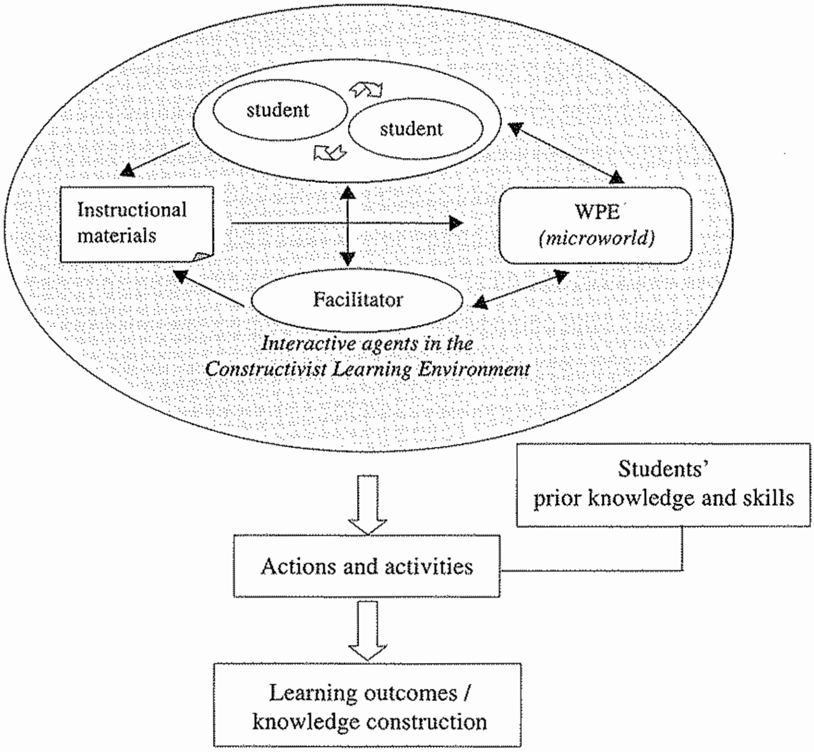
The instruments of this study were the probability microworld (WPE), probability pre-test (PRETEST), probability post-test (POSTTEST) and a rubric for evaluating the microworld (MWRUBRIC, modified from Dodge (2001).

The PRETEST and POSTTEST were identical in the format of the questions (multiple-choice questions), content tested, levels of taxonomy and number of questions. However, the sequencing and the numerical values/ sentence structure of the questions were modified. Both tests were based on the Malaysian Form 4 school syllabus for the topic of probability and covered various Bloom's taxonomy levels (knowledge, comprehension, application, analysis, synthesis, and evaluation). The multiple-choice questions format was chosen for the tests because the answers were unambiguous and it was the examination format used in the KBSM's (National Integrated Secondary School Curriculum of Malaysia) mathematics curriculum. Three mathematics teachers validated the PRETEST and POSTTEST and the research instruments was pilot tested with 71 Form 4 Sciences students who were not involved in the actual study resulting in Cronbach Alpha value of 0.797.

The WPE microworld fulfilled the basic characteristics of a microworld as suggested by Edwards (1998). It was designed in the Web-based environment due to the flexibility of development, interactivity, extensibility and extensity (Richard, 2005). The WPE incorporated content and learning scope of sample space, events and probability of an event according to the Malaysian Form 4 mathematics syllabus.

Figure 1 shows the framework for the design of the learning environment. The framework for the learning environment was based on the constructivist learning theory (Jonassen, 1994, 1999). The elements of the environment such as WPE microworld, meaningful instructional and challenging activities, students' schemes, social and computer interaction (Barney, 2001; Becta, 2003 ; Hoyles, 2002) were designed to operate interactively as a potential ground for students' construction of the learning concepts. The interactions between these agents became part of a complex process of each student constructing their own knowledge about probabilistic reasoning.

Figure 1 Framework for the Design of Learning Environment



As depicted in Figure 1, students learned the context guided by the instructional materials and have bi-directional communication with the WPE microworld and with the facilitator. The facilitator is responsible for designing the instructional materials according to the syllabus, considering the underlying activities to foster knowledge construction while exploring the WPE microworld and stimulating interaction of the existing agents in the learning environment. Knowledge construction occurred among the students through their actions and activities in the WPE microworld such as building, testing, interpreting feedback to verify their assumption and understanding which was affected by their prior knowledge and skills, also stated by Edwards (1998). Likewise, the active discussions among peers and with the facilitator contributed to the quality of the learning outcomes, also suggested by Barney (2001) and Hoyles (2002).

The WPE microworld consisted of a main page that functioned as a content page allowing users to navigate to other pages. It was divided into three parts (Figure 2). The static head section consisted of the title and pull down menus that were linked to subsequent pages, providing a consistent placement of navigation links. The body section provided the relevant information for reading and further understanding of the learning contents such as the microworlds and mathematics content. The bottom section listed the four microworlds (dices, coins, marbles and spinners) with their corresponding links, designed for students to learn probability with its underlying cognitive goals. The structure of the screen was consistent throughout the site. Each screen was identical in terms of background colour, template design and placement of Web elements. The four microworlds had identical templates with common functionalities (Figure 3) and consistent positions named as the button region, animation region, playful outcome region and moveable data region (Figure 4).

Figure 2 Content Page of the WPE Microworld

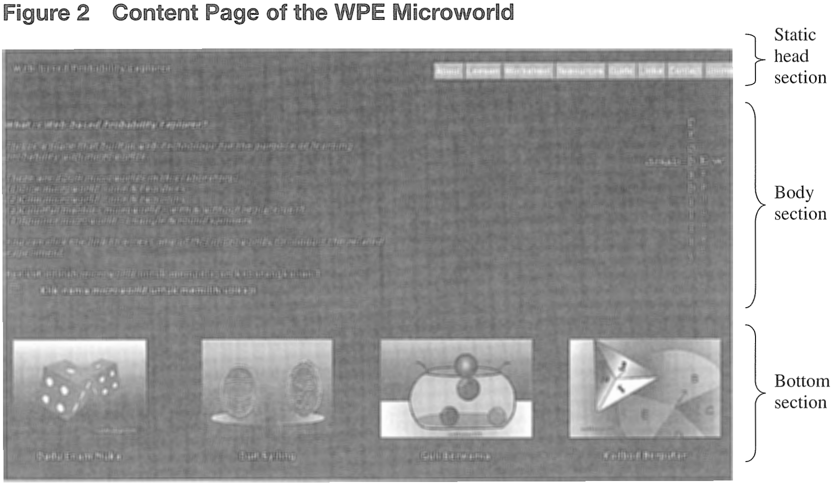


Figure 3 Top Menu with Common Functions of the Microworlds in WPE

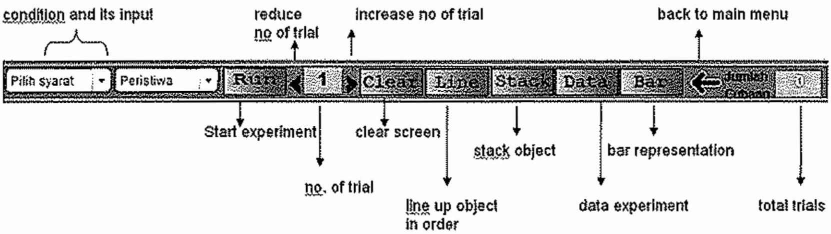
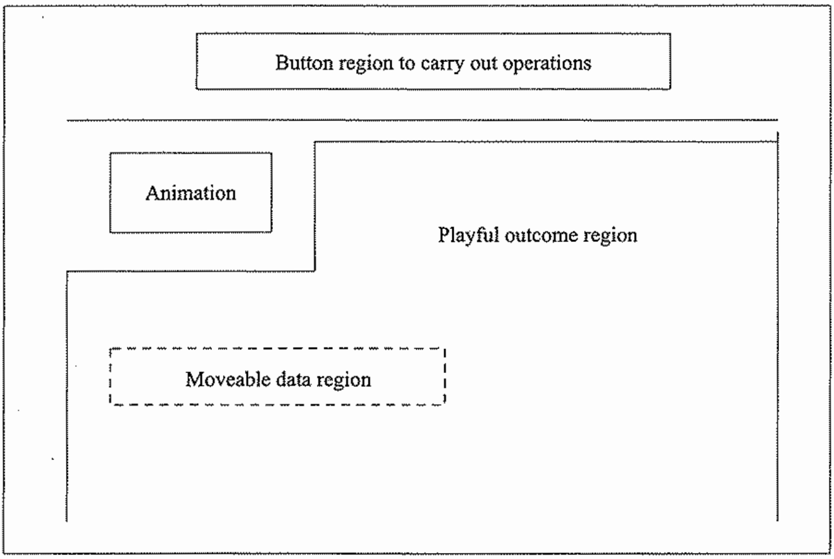


Figure 4 Identical Layout in Each Microworld of the WPE



Some appropriate multimedia elements such as animated, colourful 2D and 3D graphics were added in the microworld aimed to stimulate the students to learn through play and exploration. Students could drag and drop the outcomes freely in the outcome region and move the data window at the central part of the screen. The multiple representations (Alpers, 2002; Edwards, 1998; Goldin & Shteingold, 2001; Healy & Hoyles, 1999; Jonassen, 1999; Jonassen & Carr, 2000; NCTM, 2000) incorporated in the microworld were the tree structure representation (Figure 5), dynamically linked data representation in tabular form and graphical bar (Figure 6).

Figure 5 Tree Structure Representation

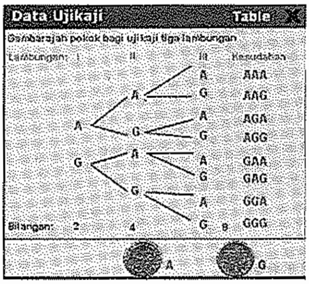
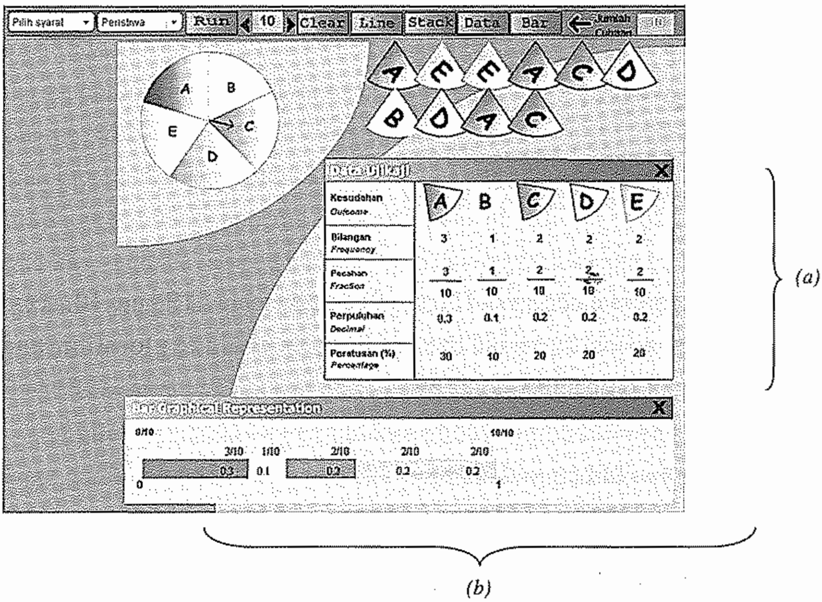


Figure 6 Dynamic Linked Data Representation in Tabular Form (a) and in Bar Graphical Form (b)



The "Line" cognitive feature (Figure 7) lined up the outcomes in order and was useful to introduce the outcomes in the randomness topic while the "Stack" cognitive feature (Figure 8) facilitated the learning process by illustrating the sample space for a repeated experiment.

Figure 7 Lining up the Outcomes in Triangle Spinner

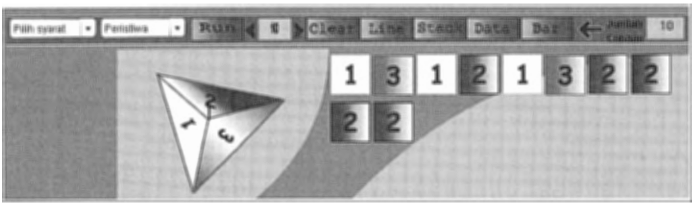
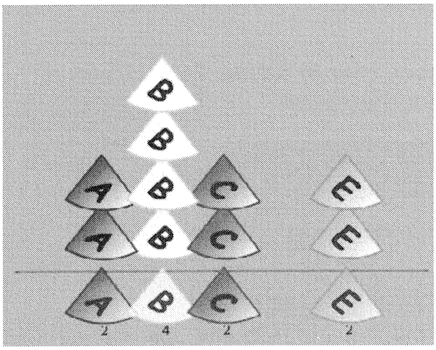


Figure 8 Stacking up the Outcomes in Round Spinner



The tree structure generator (Figure 9) and sketchpad (Figure 10) acted as useful resources to assist the students in their exploration and understanding of probability concepts.

Figure 9 The Tree Structure Generator

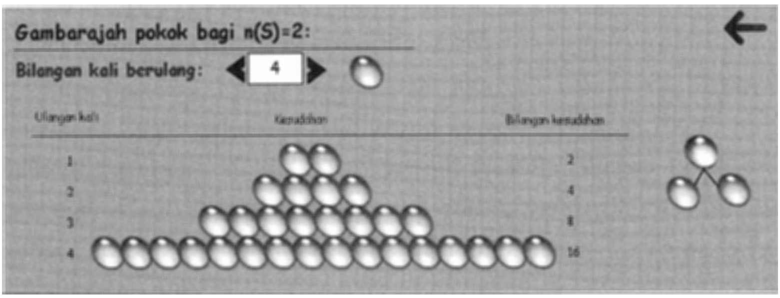
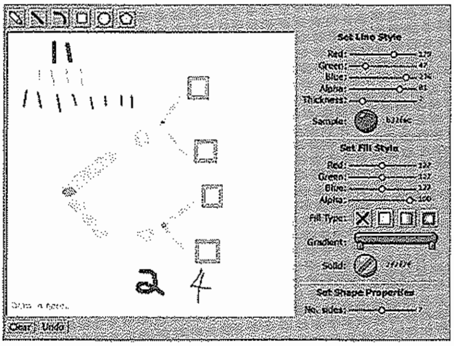


Figure 10 The Sketchpad



The WPE was developed on Microsoft Windows 2000 Professional platform, with Macromedia Flash 2004 in ActionScript 2.0. The Web-based environment was developed with Macromedia Dreamweaver MX 2004 with the support of cascading style sheet in defining the display of HTML elements on the web pages. The WPE consisted of four microworlds, which encompassed object-oriented concepts. The animated colourful 2D and 3D graphics elements and the four microworlds (dices, coins, spinners and marbles) were developed and authored using Macromedia Flash 2004. Each outcome and the data display windows of the experiment in the microworlds were in movie format in order to achieve the purpose of data retrieval and the effects of moveable and playful environment.

A pilot test on the WPE was carried out with four Form 4 students who were not involved in the actual study. Several minor bugs in the WPE were fixed after the pilot test. The size of the colourful marbles was scaled down to avoid over-crowding on the screen. The tree structure was added to help the students in getting the total number of outcomes for repeating experiments if the sample size was known.

Data Collection Procedures

At the initial stage of the study, the students completed the PRETEST. Before the treatment, the students attended a one-hour training session to familiarise with the microworld by engaging with a few activities in the WPE. The session was considered as an introduction to the learning topic of the probability. The treatment stage was carried out two weeks after the pre-test. The students paired up and learned probability with the WPE, guided by the researcher and the accompanying worksheets (Su, 2006) for eight weeks, two periods each week. The first author acted as a facilitator, rather than as a teacher in the learning environment with the WPE. Every learning session was monitored and observed to study the students' behaviours when they were engaged with the WPE microworlds. The worksheets for each lesson were monitored and assessed to determine their understanding of the learning topics and their abilities to relate the learning outcomes to the real-life problems. After the treatment, students answered the POSTTEST to assess their achievement after learning with the WPE microworlds. Twelve students and three teachers from both schools evaluated the features of the WPE microworlds in terms of the suitability and potential of microworld for learning purposes.

Data Analysis

Paired *t* test was conducted on the students' PREMATH and POSTMATH scores to determine the effectiveness of the WPE in enhancing the learning of probability. Students' comments, classroom observations and the interview data were analysed based on the MWRUBRIC using descriptive measures of frequencies.

Results

Students' Learning of Probability

Based on the paired *t* test results shown in Table 2, there was a significant difference in the students' achievements before and after the treatment ($t = 8.525$, $df = 51$, $p < 0.0005$). The students achieved higher score in the POSTTEST ($M = 12.52$, $SD = 2.31$) compared to the PRETEST ($M = 8.81$, *SD* = 2.89). The WPE microworld learning environment has enhanced the students' learning and understanding of probability.

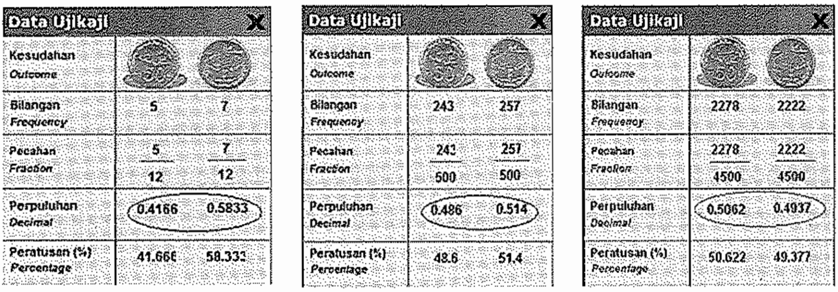
Table 2 Paired Sample *t* test on pre- and post-test (*n* = 52)

	M	*SD*	*t*	*df*	*p*
Pre-test	8.8077	2.8904			
Post-test	12.5192	2.3051	–8.525	51	< 0.0005

Six out of the twelve students interviewed also stated that the WPE assisted them to learn and understand probability concept better. The features they found to be useful were the tree structure generator (Figure 9), data display in tabular form (Figure 6) and the consistency in common operations (Figure 3, by clicking the button or the image) available in the WPE to manipulate the coins, dices, marbles and spinners on the screen. These dynamic multiple representations displayed experiment results in various forms in one single page.

The students viewed WPE as a practical learning tool that helped them to visualise and understand the probability concept. By comparing and observing the results of three experiments in the coins microworld with different number of tosses ($n = 12$, $n = 50$ and $n = 4500$) shown in Figure 11, they came to realise that when the number of trials (n value) is large (for example, $n = 4500$), the probability of each event (e.g., p (getting side A) = 0.5062, p (getting side B) = 0.4937) approaches the theoretical value (p (each event) = 0.5).

Figure 11 Probability of Getting Each Side of the Coin When *n* = 12, *n* = 50 and *n* = 4500



In another example, for a game of throwing two dices at the same time, probability to get both dices showing the same number of dots was lower than probability of getting both dices showing different number of dots. Students realised that this was an unfair game to compete between a person to get both dices showing same number of dots and another person to obtain both dices showing different number of dots.

The observations in the class also showed that the WPE helped the students in knowledge construction and it encouraged discussions among the students while exploring the microworlds and learning the probability concept. Students understood and constructed new knowledge as they listened to their peer's and their instructor's explanations during the discussions.

Students' Satisfaction with the Features in the Microworld

The students' evaluation of the WPE was related to various aspects such as visual appeal, navigation and flow sequencing, transactional and mechanical quality (technical, links, misplaced or missing images and grammar errors) as suggested by the MWRUBRIC evaluation rubric (Dodge, 2001).

Referring to the students' responses listed in Table 3, generally, they were satisfied with the WPE. Forty-seven percent of the students liked to learn probability using WPE. Another forty-four percent of them answered as sometimes while eight percent did not like to use the WPE. Generally, the students felt happy and excited when exploring WPE while learning. They found that the WPE was able to facilitate their learning and understanding. Though this was their first encounter with a microworld, they hoped to see microworld being integrated in the learning environment. Based on the interview responses, the students generally found the WPE microworld visually appealing, and the navigation, page flow, transitional and mechanical aspects of WPE were between developing (good) and accomplished (excellent) levels according to Dodge's (2001) evaluation criteria.

Table 3 Students' Evaluation on the WPE

No.	Statements	Responses		
		Yes	Sometimes	Not at all
9	I like to learn probability using the microworld called WPE.	18 (47.3%)	17 (44.7%)	3 (8.0%)
		Yes, very much	Some of them	No, terrible
10	Do you like the interface in the microworlds?	10 (26.3%)	28 (73.7%)	0 (0.0%)
11	What would you rate the colour, background and text in the microworlds?	Good	Average	Poor
	Colour	29 (76.3%)	9 (23.7%)	0 (0.0%)
	Background	13 (34.2%)	25 (65.8%)	0 (0.0%)
	Text	19 (50.5%)	15 (39.5%)	4 (10.0%)
		Yes	Some are not clear	No, I always get lost
12	The flow from one page to the other page is smooth	27 (71.1%)	11 (28.9%)	0 (0.0%)
		Yes, a lot	Yes, some	None at all
13	Is there any broken link in the microworlds?	0 (0.0%)	13 (34.2%)	25 (65.8%)
		Yes, a lot	Yes, some	None at all
14	Is there any spelling mistake found in the microworlds?	0 (0.0%)	7 (18.4%)	31 (81.6%)

Note: The table is based on responses from 38 students.

Three experienced secondary school mathematics teachers from three different schools used, evaluated and reflected on the microworld from the aspects of content and resources available in the WPE, WPE implementation strategies (inclusive of classroom implementation, understanding of microworlds, potential of microworlds in the classroom) and the possibilities of using microworlds in the teaching and learning process and its possible impact on the Malaysian education settings. They believed that WPE was sufficient in terms of probability content and was well designed for learning. One of the teachers commented that the WPE was a motivating way to learn mathematics and it was able to build on students' prior knowledge. The resources in the microworlds were relevant and sufficient to cater to the needs of students in the lessons. The teachers would choose microworld as a strategic courseware to be implemented in the classroom. The WPE was different from the typical educational software that they have used before because WPE assisted the students to construct the concept themselves and exemplified constructivism teaching and learning environment. There are possibilities for integrating microworlds in the Malaysian education setting.

Discussions

The Learning of Mathematical Content

Consistent with the studies in the literature on the use of technology and microworlds in learning mathematics (Drier, 2000; Jiang & Potter, 1992; Sinclair et al., 2003; Yeh & Nason, 2003), the findings of this research showed that the students could learn and understand the mathematical content in the constructivist-based microworld learning environment. The students obtained significantly higher scores in the post-test, compared to the pre-test after learning using the microworld environment. Students stated that the WPE microworld helped in their learning and understanding of probability concepts. The following responses during the interview exemplified the students' views (the responses were edited for grammatical purposes):

> *The interesting graphics can help me remember what I have learnt.*
>
> *When using the microworld, we can visually see the table, but in the classroom, we don't have the table and we have to do it verbally. So, we don't understand. Using the computer, the instructor can explain and at the same time show the ways to do it in theory and also practically.*

Consistent with Jiang's findings (1992), students in this study changed their misconceptions about probability and understood the concepts through the repeating experiments in the microworld. The probability concepts gained by the students from interacting with the activities in the WPE microworld include law of the large numbers, multiplicative and part-whole reasoning.

Similar findings were reported in the studies of Drier (2000). Some comments from the students were:

> *Probability involves reasoning, and the microworld helps me to reason and visualize the concept. The other features in the microworld also help me to do self-learning in mathematics.*
>
> *Using the microworld, we can actually perform the experiment. By clicking on certain buttons, we were able to do the experiment.*
>
> *For example, the probability lesson with marbles ... it is easier to try it on the computer. If we were to do it in the classroom, we'll need many things and can be difficult to prepare. Need a lot of time.*

In this study, the various visual representations in the WPE microworld assisted students in understanding the mathematical concepts, similar to the findings reported by Sinclair et al. (2003). The graphical elements assisted them to remember what they have learnt. The dynamically linked representations provided the students with the results of their experiments in multiple representations corresponding to their actions in the WPE microworld. Thus, the findings of this study was consistent with Chiappini and Bottino's (1999) contention that visualisation plays a central role in assisting students to comprehend the meaning of and the understanding and reasoning of probability concepts. A student commented:

> *I find that the WPE is a useful program and it helped me to study about probability better. I can get a visual diagram from the microworlds.*

The WPE microworld helped the students to bridge the gap between action and expression. The students in this study were able to construct new knowledge via the exploration and manipulation of objects in the microworld. Similar learning outcomes were also reported by Hoyles et al. (2000) in the MathSticks microworld and Yeh and Nason (2003) in the VRMATH microworld.

This research also found that active interactions were essential in knowledge construction. In the constructivist learning environment, interactions between students and the microworld, and interactions between instructor and students and among students contributed to the process of knowledge construction (Barney, 2001; Becta, 2003; Hoyles, 2002; NCTM, 2000). Students explored and learned from the dynamic feedback they received from the microworld via object manipulations (Hoyles et al., 2000).

Satisfaction with the Features of the WPE Microworld

Generally, the students were satisfied with the features of the WPE microworld encompassing the interface, graphic elements, colour, background and text. Only three of the students expressed negative reactions on using the microworld for learning. These three students were found to be lacking in basic computer skills. Novak (2003) suggest that students who lacked computer skills might require time to appreciate and benefit from constructivist learning environments. Handal and Herrington (2003) found that meaningful learning contributes to the building of an individual's knowledge structure and the effort of introducing new meaningful learning approach (e.g., constructivist learning methods) to supplement current behaviourist teaching and learning process should be pursued although resistance would occur along the way.

On the other hand, the teachers interviewed found that the WPE microworld had adequate lesson content. The WPE built on the students' prior knowledge and could motivate students to learn. They found that the WPE microworld was very different from the educational software they had used before and the WPE was a tool for students to construct their own knowledge. For example, some teachers commented:

> *I find it a motivating way to learn mathematics. I find it interesting and able to promote self learning.*

The microworld, I think in this case provides a hands-on experience for students to learn, rather than the conventional teaching of listen to "talk and chalk". It gives them a real life experience. They can do it by themselves. So, they do learn in a context and they can play around with the activities.

This was consistent with the teachers' views of microworlds reported by Jonassen (1998, 1999) and Hoyles (2002). Contrary to the studies by Tian Belawati (2003) and Ross, Hogaboam-Gray, McDougall, and Bruce (2002), the teachers in this study were willing to adopt new approaches for teaching mathematics. They perceived that the microworld has high potential to be integrated in the classroom setting and microworld reflect the underlying philosophy of the Malaysian smart schools.

Recommendations for Future Studies

The WPE, which was in a Web-based environment, could be extended to include some features for collaborative activities such as forum or discussion board. With the additional tools, students engaged in the WPE microworld can communicate and discuss their learning outcomes with peers locally or nationwide when it is connected to the wide area network. Other features such as the dynamic graphing tool, which is useful for statistical data collection and analysis, as suggested by NCTM (2000) and interactive games that encourage teamwork (Amory, Naicker, Vincent, & Adams, 1999; Kasvi, 2000; Rieber, 2005) should be added to the WPE. The drill and practise exercises should also be made available to the students in the WPE learning environment for the purpose of mastering basic skills or reviewing concepts that have been previously learned (Handal & Herrington, 2003).

Conclusions

Mathematics teachers should be encouraged to integrate the use of microworld in their mathematics classrooms to foster knowledge construction among the students. Teachers should change their roles from traditional "teacher-centred" approaches and knowledge deliverers to facilitators. Teachers should prepare supplementary materials and manage the microworld learning environments to maintain a balance between playing and learning activities for the students. According to Papert (1984) and Jonassen and Carr (2000), in constructivist learning environments such as microworlds, students should be active participants, engaging in explorations, discoveries and discussions.

Students should be exposed to learning with computer (including microworlds), so that they can be self-directed learners, building up their confidence to explore and interact with the learning tools in the microworlds. Students must learn to work in groups and participate in discussion with peers and listen to peers' explanations while constructing their mental model for better learning outcome (Becta, 2003).

Knowledge construction requires time and takes place both in the classroom and outside the classroom. Students require time for reasoning, discussions and explorations. Teaching in the classroom should integrate practical activities, which provide linkage between prior knowledge and real life experience. Students should have access to computers for exploring and enhancing their understandings of the concepts through multiple representations and active exploration in the microworlds.

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