3D Motion Flow Estimation using Local All-Pass Filters

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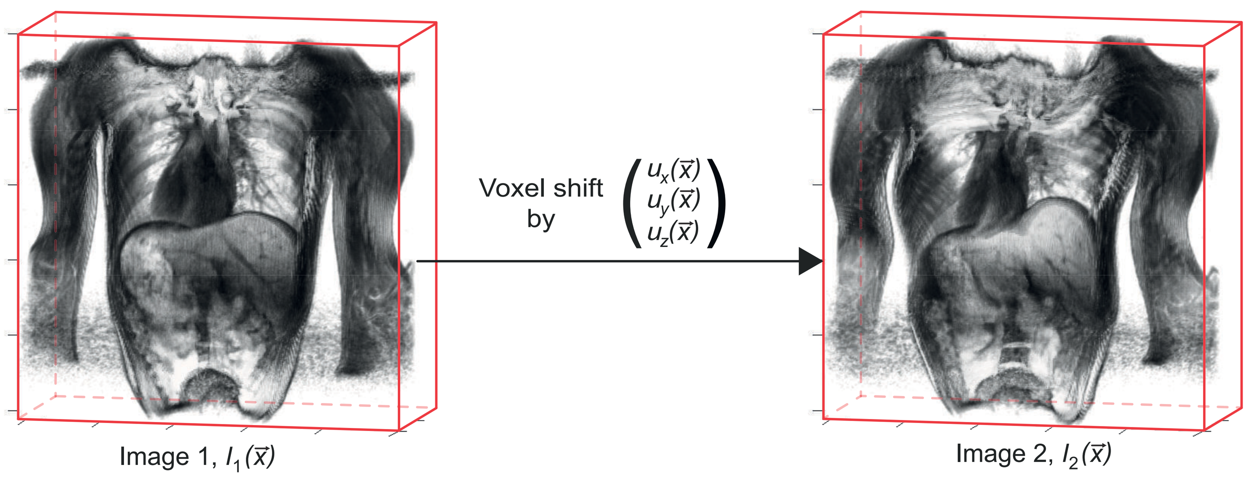
Summary

The estimation of motion from a sequence of volumetric images is an important task that has many applications in biological and medical imaging, e.g: image registration, cardiac analysis in 3D cine CT images and cell dynamics in confocal microscopy. In this work, we present a novel algorithm to estimate a dense 3D motion using local all-pass filters. We demonstrate the effectiveness of this algorithm on both synthetic motion flows and *in-vivo* MRI data involving respiratory motion. In particular, the algorithm obtains greater accuracy for significantly reduced computation time when compared to competing approaches.

Motion Flow Estimation

Problem: Find a velocity field $\vec{u} = (u_x(\vec{x}), u_y(\vec{x}), u_z(\vec{x}))^T$ based on the variation of intensities within a volumetric image sequence [1], where $\vec{x} = (x, y, z)^T$ is the voxel coordinates.



Optical Flow Point of View

Assume a voxel's intensity remains constants as it flows from one image to another:

$$\text{Brightness Constraint:} \quad \underbrace{I_2(\vec{x} + \vec{u}(\vec{x})) = I_1(\vec{x})}_{\text{Non-Linear}}$$

Standard algorithms [1,2,3] are based on linearising the constraint under the assumption that the displacement of the motion is small:

$$\text{Optical Flow Equation:} \quad \underbrace{I_2(\vec{x}) - I_1(\vec{x}) - \vec{u}^T \nabla I_1(\vec{x}) = 0}_{\text{1 Constraint for 3 Unknowns} \Rightarrow \text{Ill-posed}}$$

↬ Solve using regularisation [1] or assume motion is constant over a local window [2]

Our Approach

Instead of assuming small displacement and using the optical flow equation:

> Assume the motion is slowly varying ⇒ Treat as locally constant

Under this assumption:

- Relate local changes between two images via a filter that is **All-Pass** in nature
- Extract local estimate of motion flow from this all-pass filter

↬ No limit on the size of displacement of the motion

All-Pass Filtering Framework[4]

1. Shifting is All-Pass Filtering

Under brightness constraint:

Constant motion $\Longrightarrow$ Shifting by a displacement vector $\vec{u} = (u_x, u_y, u_z)^T$

Shifting in frequency domain:

$$\hat{I}_2(\vec{\omega}) = \underbrace{\hat{I}_1(\vec{\omega})\, e^{-j\vec{u}^T\vec{\omega}}}_{=\ \text{Filtering Operation}} \xRightarrow{\text{Define Filter}} \underbrace{\hat{h}(\vec{\omega}) = e^{-j\vec{u}^T\vec{\omega}}}_{=\ \text{All-Pass}}$$

where $\vec{\omega} = (\omega_x, \omega_y, \omega_z)^T$.

2. Linearising the All-Pass Filtering

Any all-pass filter can be expressed as $h[\vec{k}] = p[\vec{k}] * p^{-1}[-\vec{k}]$, where p is an arbitrary, real, digital filter and $\vec{k} = [k, l, m]^T$ is the discrete voxel coordinates:

> All-Pass Filtering Equation:
>
> $$I_2[\vec{k}] = h[\vec{k}] * I_1[\vec{k}] \iff p[-\vec{k}] * I_2[\vec{k}] = p[\vec{k}] * I_1[\vec{k}]$$

3. Filter Approximation - A Basis Representation

Approximate p using a linear combination of a few, known, real filters:

$$p_{\text{app}}[\vec{k}] = \sum_{n=0}^{N-1} c_n p_n[\vec{k}]$$

A good basis should span the derivatives of an isotropic filter [5]:

$$p_0[\vec{k}] = e^{-\frac{k^2+l^2+m^2}{2\sigma^2}}, \qquad p_1[\vec{k}] = k\, p_0[\vec{k}], \qquad p_2[\vec{k}] = l\, p_0[\vec{k}], \qquad p_3[\vec{k}] = m\, p_0[\vec{k}]$$

where $\sigma = (R+2)/4$ and R is the half-support of the filters.

↬ Extract estimate of displacement vector from all-pass filter h

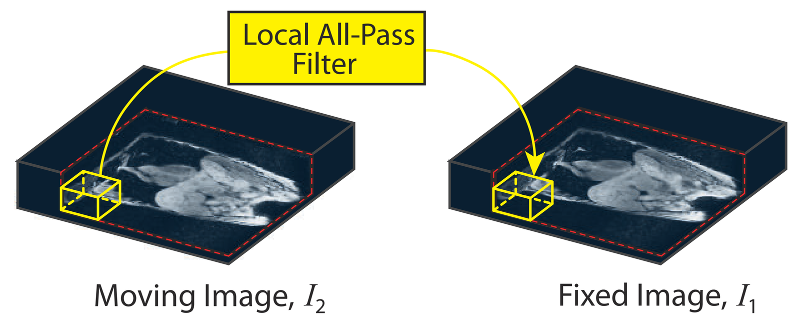
3D Local All-Pass Algorithm

Assume motion is constant within a window $\mathcal{W}$ and estimate a local all-pass filter. Thus, for $(2R+1)$ cubic window $\mathcal{W}$, solve at every voxel:

$$\min_{\{c_n\}} \sum_{\vec{k} \in \mathcal{W}} \left| p_{\text{app}}[\vec{k}] * I_1[\vec{k}] - p_{\text{app}}[-\vec{k}] * I_2[\vec{k}] \right|^2$$

↬ $c_0 = 1 \implies$ Solve linear system of equations with $N-1$ unknowns

- Efficient implementation using convolutions and pointwise multiplication
- Extract motion estimate from filters

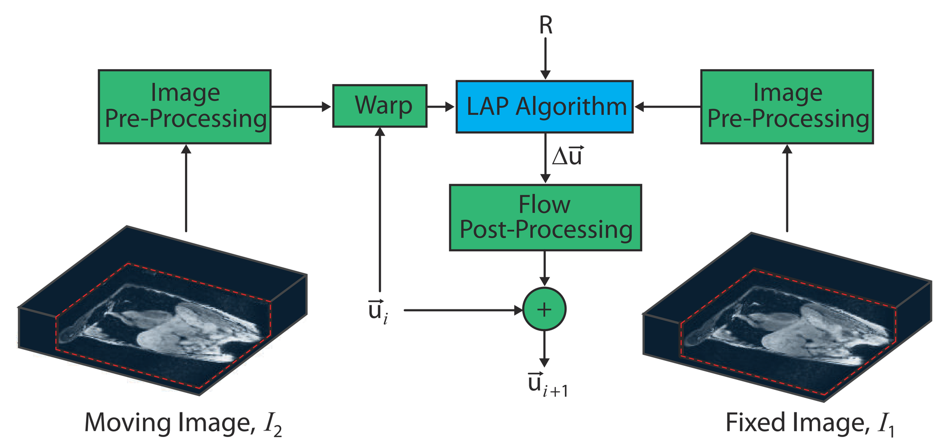


Poly-Filter Framework

Estimate the motion in a slow-to-fast varying manner by changing the filter parameter R; large values of R allow the estimation of large flow whilst small values allow faster variations.

Post-Processing:

- Remove erroneous flow estimates using inpainting
- Smooth estimate using Gaussian filtering



↬ Pre-process images using high-pass filter

Results

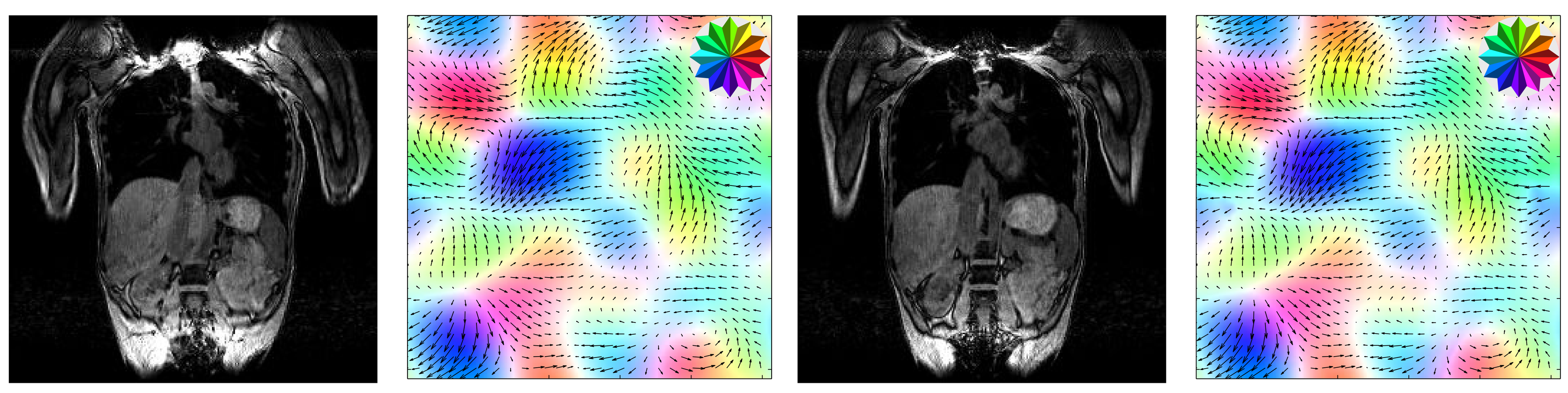
Synthetic Evaluation: Image I_1 is generated by warping image I_2 using a known ground truth motion - brightness constraint exactly satisfied.

	Noise Images ($128 \times 128 \times 64$ voxels)						MR Images ($256 \times 256 \times 72$ voxels)					
	Constant Flow			Smoothly Varying Flow			Constant Flow			Smoothly Varying Flow		
	AEE	AAE	Time	AEE	AAE	Time	AEE	AAE	Time	AEE	AAE	Time
3D LAP	**0.014**	**0.065**	**9.320**	**0.019**	**0.319**	**9.290**	**0.007**	**0.038**	**34.77**	**0.048**	**0.771**	**40.82**
Elastix [6]	0.174	0.558	47.20	0.223	4.400	49.80	0.196	0.914	69.42	0.494	7.809	76.00
Demons [7]	0.173	0.784	66.14	0.253	4.853	134.5	0.240	1.070	246.7	0.230	3.070	235.5

* AEE - Average End-point Error, $\|\vec{u} - \vec{u}_{\text{est}}\|_2$, (in voxels), AAE - Average Angular Error (in degrees) [3] and Time - computation time in seconds.
** Maximum displacement for each motion flow is 8 voxels.

↬ LAP computation times achieved using only a Matlab implementation (no C++ code)

Example estimating the smoothly varying motion flow



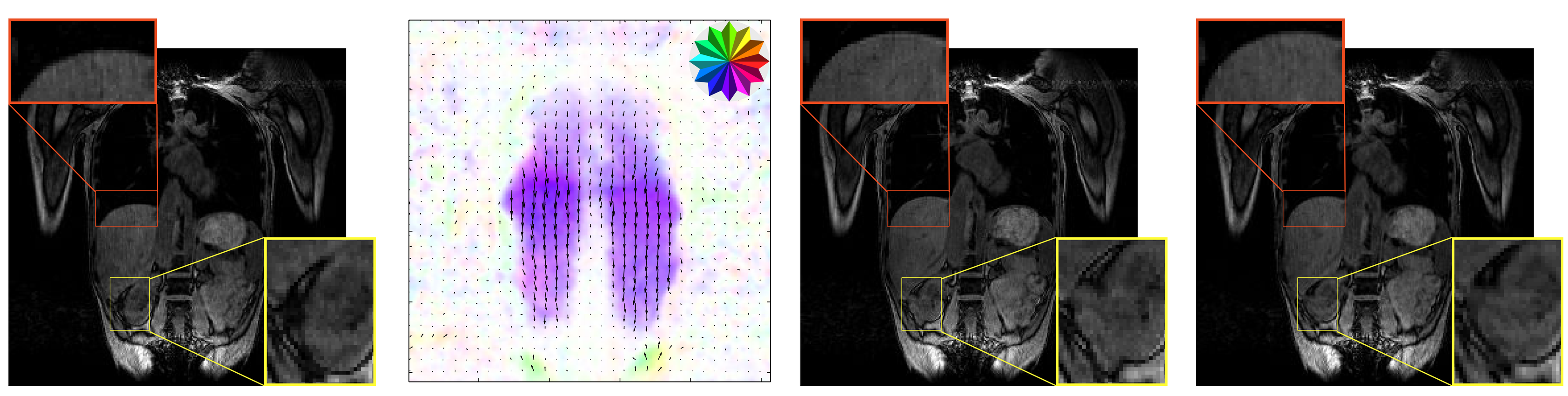
(a) xy-Slice of Image 1, I_1 (b) xy-Slice of Ground Truth Motion (c) xy-Slice of Image 2, I_2 (d) LAP Motion Estimate

Respiratory Motion Estimation on three *in-vivo* MRI: Noisy, real, conditions - unlikely that the brightness constraint is satisfied.

	Lung Segmentation (Dice Coefficient [8])	Image Registration Accuracy (dB)	Computation Time (seconds)
3D LAP	**0.90 (0.01)**	**39.93**	**36.28**
Elastix [6]	0.87 (0.02)	37.30	61.55
Demons [7]	0.73 (0.05)	38.23	434.6

* Lung Segmentation - perform automatic lung segmentation on both I_1 and the registered version of I_2 and then measure the overlap using Dice Coefficients [8]

Example estimating respiratory motion in MR images



(e) Moving Image, I_2 (f) LAP Estimate (coronal slice) (g) Fixed Image, I_1 (h) Registered Image, I_r

References

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