FRI Sensing: 2D Localization from 1D Mobile Sensor Data

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***Abstract*—Sensor localization is a basic and important problem in many areas. It often relies on transmission-communication equipment to obtain the sensor geolocation information. However, in this work, on the contrary, our goal is to retrieve the 2D sensor location only from the 1D sensor data. We demonstrate that there is valuable 2D geometric information that can be unveiled hidden within the 1D sampled signal. We investigate the hypotheses needed and propose a very efficient and robust algorithm to realize this 2D localization. This method can be possibly applied to a series of biomedical applications, like robotic endoscopic capsules, medicine tracking, and biological tissue detection. For example, people inject tiny sensors about the size of a grain of sand to monitor human biometrics (like blood ph, etc) and accurate localization plays an essential role in pathological diagnosis.**

***Index Terms*—Mobile sensing, data visualization, sampling technique, curve estimation, biomedical microrobot.**

I. Introduction

Recently, developments in circuit integration, semiconductor and material sciences have facilitated powerful new technologies for the design of biomedical devices. As a result, medical instruments with lower cost, minimally-invasive procedures and more comfortable patient experience have attracted the attention of researchers of healthcare companies, universities, institutes and physicians. Miniaturization of large electronic circuits has especially made the sufficiently small implantable or wearable microrobotic sensor systems possible, such as smart pills or capsules, and body sensor networks. Wearing or implanting inside the human body, these wireless systems efficiently provide valuable data of the body part being diagnosed, such as gastrointestinal (temperature, pH, pressure, etc) parameters, blood glucose and pressure levels. With smaller size and lower cost, these wireless microrobotic sensors are more convenient and effective tools for medical diagnosis, visualization and treatment of various circumstances.

Accurate localization of the microrobotic sensor unit (e.g., robotic capsule, catheter, needle) inside the human body is a key topic, with application examples ranging from reliable closed-loop control of active-locomotion capsules to lesions localization and targeting of pathologies [2], [16], [12]. For example, when the microrobotic device moves along the gastrointestinal tract and captures images, accurate knowledge of the position and orientation is crucial for physicians to better localize pathologies, implement follow-up diagnosis and drug delivery, and guide navigation of wireless capsule endoscopy [16]. Basically, the existing solutions can be divided into two categories [2], [16]: localization with respect to an external reference frame (i.e. external localization), such as antennas and magnetic sources; localization with respect to the surrounding deformable anatomical environment (i.e. internal localization), such as landmarks and organs. However, despite several available solutions, ranging from the use of magnetic fields to ultrasounds and computer vision technologies (e.g. CNN-based classifiers), accurate real-time localization of microrobotic devices still remains a challenging field due to the small device dimension and sophisticated signal propagation model [2].

In this work, contrary to positioning the microrobotic unit through geometric measurements (e.g. received signal strength, time-of-arrival, direction-of-arrival, etc), we aim to retrieve the trajectory of the mobile device (e.g. wireless endoscopic capsule, catheter, needle, etc) only from a sequence of 1D sensor data (e.g. magnetic field strength) without any location information. This allows us to position the microrobotic device accurately in real time without other restrictions, such as transmission/battery energy limitations, pre-computed signal propagation model, and costly and unwieldy hardware configurations (e.g. synchronization system between transmitter and receiver, array of antenna patterns, etc). This can be used to either localize the microrobotic unit directly, or cooperate with other trajectory localization techniques as a hybrid solution to give higher precision.

Despite these merits, yet it sounds impossible to achieve trajectory retrieval only from 1D sensor samples in the absence of any position information. At the first glance, it seems that most of the high-dimensional information is seriously lost after the sampling process. However, in [7], [8] we demonstrated that, by constructing appropriate hypotheses on the 2D physical field sampled and the trajectory, actually there is adequate 2D geometric information entangled within the resultant 1D temporal samples that can be fully revealed by sufficient processing—using the “FRI Sensing” algorithm. Specially, our setting is as follows: a mobile sensor (e.g. electromagnetic intensity sensor) samples an unknown 2D physical filed (e.g. electromagnetic filed) along some unknown trajectory. During sampling, we obtain the 1D sensor measurements transmitted from the mobile device inside the human body, and our goal is to retrieve the trajectory of the mobile device.

In order to make our program feasible, we impose the following hypotheses: the physical field is a finite sum of 2D sinusoids and the trajectory is continuous that is sufficiently straight locally. While, these hypotheses can be easily met in real-world applications. By providing extra sources (e.g. electromagnetic fields), the resultant physical field inside the human body can be well characterized as a sum of spatial sinusoids. By manipulating the velocity and accelerator of the mobile device, the sampling trajectory can satisfy the continuity and flatness requirements. Therefore, we can apply the sampling theorem and the associated algorithm directly to the collected 1D sensor measurements so as to retrieve the trajectory of the mobile sensor up to an affine transformation with 12 degree-of-freedom. One possible way to eliminate this uncertainty is to use the extra information that provide an access to estimate these 12 parameters. For example, the ground-truth positions of 4 distinct points of the sampling trajectory could be enough.

The rest of the paper is organized as follows: We present the basic framework of trajectory retrieval and related work in Sect.2. Then, the details of our experiments are presented in Sect.3. In Sect.4, we talk about the extensions and potential improvements of the proposed method. We conclude the paper in Sect. 5.

II. FRI Sensing

A. Related Work

Implantable microrobotic device is now an ubiquitous and promising solution for non-invasive diagnosis, such as gastrointestinal tract and stomach inspection. In most cases, the valuable diagnosed data are first transmitted from the microrobotic devices to an external receiver or detector, and then processed by a monitoring and control system for analysis, inspection and treatment by clinicians and physicians. Usually, most frequent microrobotic modules are typically in the form of wireless mobile capsules or implanted stationary devices. Among them, capsule-based systems have attracted significant research interest in a wide variety of applications, including endoscopy, microsurgery, drug delivery and biopsy. In particular, so far wireless endoscopic capsule is the unique solution for the diagnosis of the entire small bowel. In such implantable sensor applications, one of the most challenging problems is the accurate trajectory retrieval of the microrobotic sensor module inside the human body.

To address this issue, people have developed several solutions to trajectory retrieval of the implantable sensor system, ranging from the use of magnetic fields to ultrasounds and computer vision techniques [2]. At first, the use of magnetic fields for biomedical applications has captured lots of attention of many researchers interested to localize, or navigate microrobotic modules inside the human body [15], [11]. The advantages of this magnetic field-based localization strategies include low attenuation through the human body and flexible detection technology without the limitation of a line-of-sight [9]. However, such kind of methods suffer from the interferences between the localization system and contiguous ferromagnetic modules, such as surgical tools, but also the actuation module itself [2]. An alternative solution is the electromagnetic wave-based localization strategies, mainly using RF and visible wave spectrum due to the safety, complexity and attenuation reasons [17], [18]. By evaluating the strength of the RF signals directly transmitted by the capsule inside the human body, the sensor position can be obtained through the triangulation technique. However, due to the intrinsic features of the RF signal, the precision is in the order of centimeters [10], [5]. A possible way to improve localization accuracy could be to increase wave frequency (e.g. in microwave domain) so as to make the wavelength comparable to the capsule dimension; however, in that case, the signal attenuation rate through the human body would be too much high to ensure a sufficiently good signal-to-noise ratio (SNR). As a result, an inevitable trade-off between precision and attenuation rate have to be taken into account [14], [4]. Other techniques, such as computer vision-based methods and image analysis methods, in spite of the interesting and promising results in internal localization of pathologies, have not yet achieved high accuracy [1].

B. Problem Description

In this work, contrary to locate the implantable device relying on the geometric measurements, we focus on the non-positioning 1D sensor data sampled from a 2D physical field (e.g. electromagnetic field), and we demonstrate that there are valuable 2D geometric information entangled within this sequence of 1D samples. In order to investigate the principles and hypotheses of this program, we first limit ourselves to the 2D scenarios, where the 2D case is similar technically. Specially, assume that a mobile sensor is sampling an unknown image along an unknown trajectory without any position information. Based on the resultant 1D samples, our goal is to retrieve the sampling trajectory of the mobile device. The intuitive description of the proposed problem is presented in Fig. 1.

Notice that, in our problem formulation the 2D physical field is also unknown. The motivation behind is that the interferences between the sources and the transmission-communication systems or control equipment may affect the field distribution, resulting into measurement inaccuracy. Besides, the unknown image assumption provides more freedom for practical applications.

C. FRI Sensing Principles

In this subsection, we briefly describe the methodologies of “FRI Sensing”—retrieving the 2D trajectory from the 1D samples sampled from a 2D physical field along that curve. In order to validate the main principles and hypotheses, we stay in the 2D case, where the 2D scenario is quite similar technically but more realistic. The entire framework is based on the following observation: sampling a sum of 2D sinusoids along a straight line gives rise to a sum of 1D sinusoids as shown in Fig. 2.

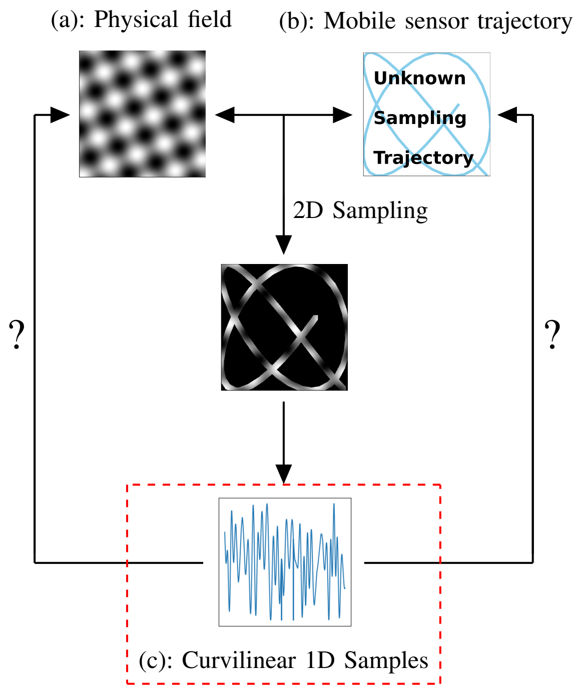
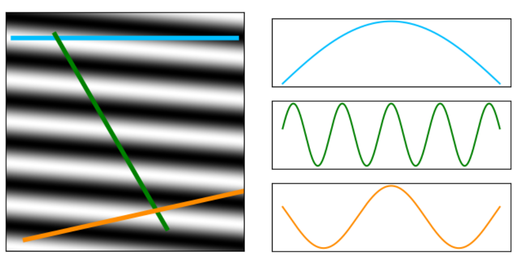


Fig. 1: Our goal is to retrieve the trajectory (b) of the mobile device from the 1D sensor measurements (c) (framed by a red box).



(a) (b)

Fig. 2: Sampling an 2D sinusoidal image along different straight lines (a) gives rise to distinct 1D sinusoids (b). For visual intuition, the 2D image is only made up of one spatial sinusoid ($K = 1$).

Obviously, the 1D sinusoids are closely related to the sampling line, which provides a clue to retrieve the trajectory geometric information—estimating frequencies of the received 1D samples. Actually, this result can be validated mathematically: Suppose the 2D image is a finite sum of sinusoids

$$I(\mathbf{r}) = \sum_{k=1}^{K} C_k e^{j\mathbf{u}_k^T \mathbf{r}} \tag{1}$$

and the trajectory is piecewise-linear consisting of L segments

$$r(t) = \mathbf{a}_l t + \mathbf{b}_l,\ l = 1, 2, \cdots L \tag{2}$$

and continuous

$$\mathbf{r}(t) = \mathbf{a}_l t + \mathbf{b}_l,\ t \in [t_{l-1}, t_l],\ l = 1, 2, \cdots L \tag{3}$$

while the sampling is uniform. Then, the obtained 1D samples $s_l(t),\ l = 1, 2, \cdots L$ along each of the L segments take the form of a sum of 1D sinusoids

$$s_l(t) = \sum_{k=1}^{K} C_{l,k} e^{j\omega_{l,k} t},\ l = 1, 2, \cdots L \tag{4}$$

where $C_{l,k} = C_k e^{j\mathbf{u}_k^T \mathbf{b}_l}$ and $\omega_{l,k} = \mathbf{u}_k^T \mathbf{a}_l$, for $l = 1, 2, \cdots L$. Then, the portion of 1D temporal signal in each segment is a sum of K sinusoids ($\{\omega_{l,k}\}_{k=1\ldots K}$) that can be exactly retrieved using Prony's method [3]. In practice, we use a very efficient FRI (Finite Rate of Innovation) algorithm [6], [13] so as to improve the robustness, accuracy and speed of frequency estimation, since Prony's method is susceptible to noise interference.

Notice that, by appropriate alignment, the following $K \times L$ matrix is at most rank-2

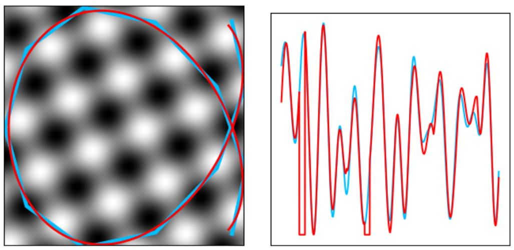
$$\mathbf{\Omega} = \begin{bmatrix} \omega_{1,1} & \omega_{2,1} & \cdots & \omega_{L,1} \\ \omega_{1,2} & \omega_{2,2} & \cdots & \omega_{L,2} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{1,K} & \omega_{2,K} & \cdots & \omega_{L,K} \end{bmatrix} \tag{5}$$

$$= \underbrace{\left[\mathbf{u}_1, \mathbf{u}_2 \cdots \mathbf{u}_K\right]^T}_{K\times 2 \text{ matrix } \mathbf{U}} \cdot \underbrace{\left[\mathbf{a}_1, \mathbf{a}_2 \cdots \mathbf{a}_L\right]}_{2\times L \text{ matrix } \mathbf{A}} \tag{6}$$

this means that, the image is required to be made up of at least $K = 2$ distinct sinusoids, and the trajectory should consist of at least $L = 2$ segments so as to retrieve the trajectory slopes $\mathbf{A}$ by matrix factorization up to an 2×2 linear transformation, e.g. using Singular Value Decomposition. Since the trajectory is continuous, the parameters $\mathbf{b}_l$ in (2) can be reconstructed up to a unique shift, i.e. the trajectory can be retrieved up to an affine transformation. While, in practice, usually an image with richer 2D sinusoids ($K \geqslant 3$) would give rise to a more robust and accurate trajectory retrieval.

In the exact sampling theorem of [8], we explicitly describe the determination on model order K, signal segmentation L and the pairing of frequencies $\{\omega_{k,l}\}_{l=1\ldots,L,\ k=1\ldots K}$ with concrete mathematical justification. The key idea is to utilize the property that $2K+1$ uniform temporal samples of a sum of K sinusoids can be used to build a $(K+1) \times (K+1)$ Toeplitz matrix whose rank is exactly K [3]. Please refer to [8] for more details.

Although we model the trajectory as pieceswise-linear, the proposed method can still apply to the curved trajectory as long as it is sufficiently straight locally. Actually in another work being reviewed, given the curved trajectory and 2D image, we proposed the approximated sampling theorem and a tight error predictor depending on the "image conditions" and trajectory curvature. One important result being revealed is that, the curved trajectory can still be recovered with a reasonable accuracy. Basically, the idea is that, the original trajectory can be well approximated by sums of straight line segments if it is sufficiently straight locally (as shown in Fig. 3). Hence, by applying the aforementioned method, the trajectory slopes can still be estimated accurately, and then the curve can be retrieved.



(a) (b)

Fig. 3: Piecewise-linear approximation of the curved trajectory. (a) red: ground-truth trajectory, blue: piecewise-linear approximation. (b) red: ground-truth 1D samples, blue: piecewise-linear approximated 1D samples.

D. Frequency pairing

Throughout the whole process, pairing the frequencies of different segments is challenging. While, there are several clues that help determine which frequency in segment l corresponds to which frequency in segment $l^{'}$. A possible solution is the amplitude criterion, where the amplitude modulus $|C_{l,k}|$ attached to the same 2D sinusoid $\mathbf{u}_k$ should be invariant across all segments. However, this criterion usually is not robust enough due to the large uncertainties of amplitude estimation, especially when two sinusoids $\omega_{l,k}$ and $\omega_{l',k}$ are very close.

Another alternative solution is that when the 1D frequencies $\{\omega_{l,k}\}_{k,l}$ have been paired correctly, the matrix $\mathbf{\Omega}$ should be rank-2—or in the situation of curved trajectory cases, can be approximated accurately by a rank-2 matrix. This criterion is more stable against noise, which leads to an efficient and robust frequency pairing algorithm. This part has been concretely discussed in another paper being reviewed, which is beyond the scope of this paper.

E. Scheme Overview

With the aforementioned ingredients, we summarize the main procedures in the following algorithm to retrieve the 2D trajectory only from a sequence of 1D data sampled from an image along that trajectory (as presented in Alg. 1).

III. Experimental Results

In order to evaluate the performance of trajectory retrieval, we sample an image made up of 10 spatial sinusoids ($K = 10$) along a random trajectory. We conduct two experiments on the clean image and noisy image corrupted by white noise (10dB PSNR) to demonstrate the robustness of the proposed algorithm. Here, for the convenience of visual comparison, we have transformed the trajectory reconstruction into the same coordinate system as the ground-truth, taking the indeterminacy into consideration.

Algorithm 1: Reconstructing image and curve from 1D samples

Input: 1D uniform samples $s(t)$

1: Divide the samples into several sub-signals $s_l(t)$, $l = 1, 2, \cdots L$
2: Estimate local frequency components $C_{l,k}$ and $\omega_{l,k}$
3: Obtain the frequency matrix $\mathbf{\Omega}$ through pairing process
4: Estimate the matrix of spatial frequencies $\mathbf{U}$ and curve directions $\mathbf{A}$

Output: The reconstructed physical field $I(\mathbf{r})$ and the sampling curve $\mathbf{r}(t)$

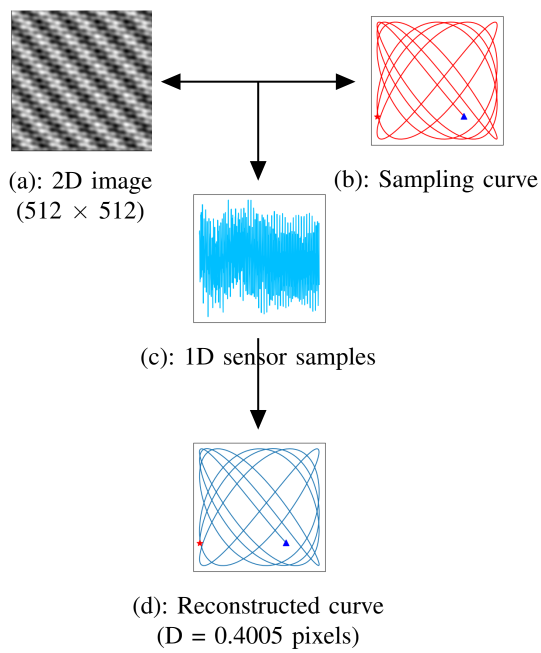


Fig. 4: Trajectory retrieval from 1D sensor measurements sampled from a noiseless sinusoidal image made up of 10 spatial sinusoids.

In order to measure the distortion between the final trajectory reconstruction and the ground-truth, we propose the following distance metric: let C_1 and C_2 denote the ground truth and the reconstructed curve, respectively. Then, we define

$$\mathrm{D} = \max_{z_1 \in C_1} \min_{z_2 \in C_2} ||z_1 - z_2|| \tag{7}$$

to characterize the distance between two curves. Its geometric meaning can be interpreted as the largest point-to-curve distance (z_1-to-C_2) over all possibilities on $z_1 \in C_1$ for which its point-to-curve distance is given by $\min_{z_2 \in C_2} ||z_1 - z_2||$. It well characterizes the largest distance between the trajectory reconstruction C_2 and the ground-truth C_1.

As shown in Fig. 4 and Fig. 5, the trajectory can be accurately reconstructed in all cases despite a sufficient long sampling period. In the noiseless case as shown in Fig. 4, we have an accuracy with D = 0.4005 pixels as a result of piecewise-linear approximation error of the trajectory. Thanks to the robust frequency estimation and pairing algorithm, the trajectory can still be retrieved with an accuracy D = 2.6581 pixels, in spite of the noisy 2D image. Moreover, the trajectory reconstruction error can be further improved by increasing the sample number along each segment or enlarging the 2D sinusoid modulus of the image. Of course, the image with richer distinct 2D sinusoids and a smoother trajectory will lead to a more accurate trajectory retrieval.

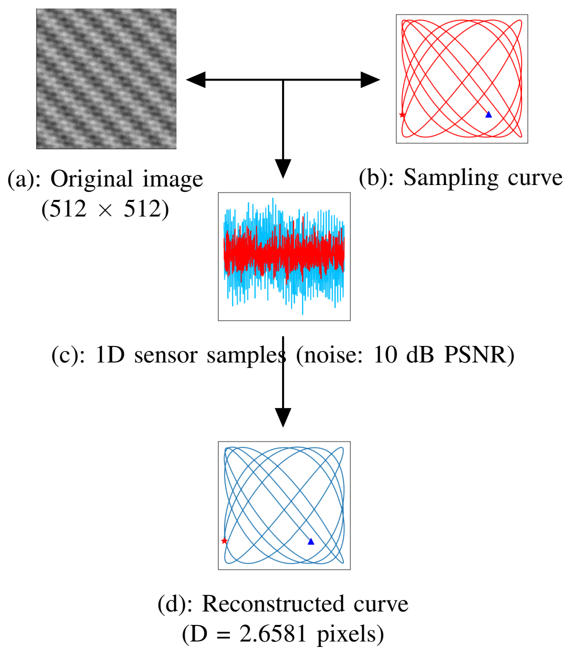


Fig. 5: Trajectory retrieval from 1D sensor measurements sampled from a noisy sinusoidal image ($\mathrm{PSNR} = 10\mathrm{dB}$) made up of 10 spatial sinusoids.

IV. Extension and Future Work

In the future, we plan to further extend the proposed method to the 3D case that is more realistic. Furthermore, we plan to apply the proposed algorithm to real clinical data to demonstrate its applicability. Fusing with other data, such as captured diagnosed images, gastrointestinal (temperature, pH, pressure) parameter values, blood glucose and pressure levels, we hope this method can help improve screening, diagnostic and therapeutic capabilities. In addition, we plan to develop a new image reconstruction algorithm based on the sample along the trajectory found. Eventually, our goal is to ultimately provide new tools for visualizing non-visual 1D data as meaningful trajectories and images.

V. Conclusions

In this paper we show that it is possible to retrieve the multidimensional geometric information that is hidden within a stream of one-dimensional sensor measurements. We demonstrate that this new trajectory retrieval method—"FRI Sensing", can be possibly applied to the localization of implantable sensor systems, which does not rely on any position information or prior knowledge of the sampled physical field. We show that the proposed algorithm based on high-resolution frequency estimation and frequency pairing, is very robust to noise that can accurately retrieve the ground-truth trajectory up to an affine transformation. Experimental results validate our theory, showing that the proposed algorithm is a very promising solution to the localization problem of microrobotic sensor units, such as wireless endoscopic capsule, catheter, needles, etc.

References

[1] G. Bao, K. Pahlavan, and L. Mi. Hybrid localization of microrobotic endoscopic capsule inside small intestine by data fusion of vision and rf sensors. *IEEE Sensors Journal*, 15(5):2669–2678, 2014.

[2] F. Bianchi, A. Masaracchia, E. Shojaei Barjuei, A. Menciassi, A. Arezzo, A. Koulaouzidis, D. Stoyanov, P. Dario, and G. Ciuti. Localization strategies for robotic endoscopic capsules: a review. *Expert review of medical devices*, 16(5):381–403, 2019.

[3] T. Blu, P. Dragotti, M. Vetterli, P. Marziliano, and L. Coulot. Sparse sampling of signal innovations. *IEEE Signal Processing Magazine*, 25(2):31–40, March 2008.

[4] R. Chandra, A. J. Johansson, M. Gustafsson, and F. Tufvesson. A microwave imaging-based technique to localize an in-body rf source for biomedical applications. *IEEE Transactions on Biomedical Engineering*, 62(5):1231–1241, 2014.

[5] Y. Geng and K. Pahlavan. On the accuracy of RF and image processing based hybrid localization for wireless capsule endoscopy. In *2015 IEEE Wireless Communications and Networking Conference (WCNC)*, pages 452–457. IEEE, 2015.

[6] C. Gilliam and T. Blu. Fitting instead of annihilation: Improved recovery of noisy FRI signals. In *2014 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pages 51–55, May 2014.

[7] R. Guo and T. Blu. FRI Sensing: Sampling images along unknown curves. In *ICASSP 2019 - 2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pages 5132–5136, May 2019.

[8] R. Guo and T. Blu. FRI Sensing: Retrieving the trajectory of a mobile sensor from its temporal samples. *IEEE Transactions on Signal Processing*, 68:5533–5545, 2020.

[9] C. Hu, Y. Ren, X. You, W. Yang, S. Song, S. Xiang, X. He, Z. Zhang, and M. Q.-H. Meng. Locating intra-body capsule object by three-magnet sensing system. *IEEE Sensors Journal*, 16(13):5167–5176, 2016.

[10] U. I. Khan, K. Pahlavan, and S. Makarov. Comparison of TOA and RSS based techniques for RF localization inside human tissue. In *2011 Annual International Conference of the IEEE Engineering in Medicine and Biology Society*, pages 5602–5607. IEEE, 2011.

[11] J. Li, E. S. Barjuei, G. Ciuti, Y. Hao, P. Zhang, A. Menciassi, Q. Huang, and P. Dario. Magnetically-driven medical robots: An analytical magnetic model for endoscopic capsules design. *Journal of Magnetism and Magnetic Materials*, 452:278–287, 2018.

[12] L. Liberman, E. A. Morris, D. D. Dershaw, C. M. Thornton, K. J. Van Zee, and L. K. Tan. Fast mri-guided vacuum-assisted breast biopsy: initial experience. *American Journal of Roentgenology*, 181(5):1283–1293, 2003.

[13] H. Pan, T. Blu, and M. Vetterli. Towards generalized fri sampling with an application to source resolution in radioastronomy. *IEEE Transactions on Signal Processing*, 65(4):821–835, Feb 2017.

[14] T. Shah, S. M. Aziz, and T. Vaithianathan. Development of a tracking algorithm for an in-vivo rf capsule prototype. In *2006 International Conference on Electrical and Computer Engineering*, pages 173–176. IEEE, 2006.

[15] L. Sliker, G. Ciuti, M. Rentschler, and A. Menciassi. Magnetically driven medical devices: a review. *Expert review of medical devices*, 12(6):737–752, 2015.

[16] I. Umay, B. Fidan, and B. Barshan. Localization and tracking of implantable biomedical sensors. *Sensors*, 17(3):583, 2017.

[17] L. Wang, C. Hu, L. Tian, M. Li, and M. Q.-H. Meng. A novel radio propagation radiation model for location of the capsule in GI tract. In *2009 IEEE International Conference on Robotics and Biomimetics (ROBIO)*, pages 2332–2337. IEEE, 2009.

[18] Y. Ye, P. Swar, K. Pahlavan, and K. Ghaboosi. Accuracy of rss-based rf localization in multi-capsule endoscopy. *International Journal of Wireless Information Networks*, 19(3):229–238, 2012.