DiffusionRank: A Possible Penicillin for Web Spamming

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State of the Web

- Web is easily manipulated for commercial gains
  - About 70% of all pages in the .biz domain are spam [Alexandros Ntoulas et al., 2006]
  - About 35% of the pages in the .us domain belong to spam category [Alexandros Ntoulas et al., 2006]
- Web spamming techniques
  - Link Stuffing
  - Keyword Stuffing
- PageRank becomes the target of many spamming techniques

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PageRank

- Calculate the importance of a Web page based on the link structure
- Recursively defined by the in-coming links

$$x_i = \sum_{(j,i)\in E} a_{i,j} x_j \quad a_{ij} = 1/d^{+}(j)$$

$$x = Ax \quad x = [(1-\alpha)g1^T + \alpha A]x$$

- Issues

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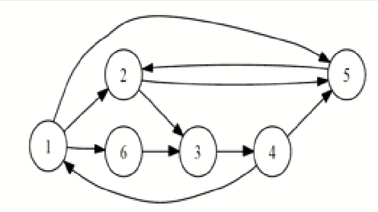
An Example of Web Manipulation

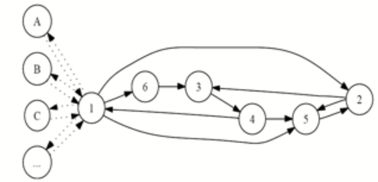
Perfect World



$$
\begin{aligned}
x_i &= \sum_{(j,i)\in E} 0.85 a_{i,j} x_j + 0.15/n \\
a_{ij} &= 1/d^{+}(j)
\end{aligned}
$$

PageRank Results:
2 > 5 > 3 > 4 > 1 > 6

Real World



Node 1's value can be increased greatly!

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Why Spamming Is Easy?

- Web is overly democratic–All pages are treated equal
- Input independent–For any given non-zero initial input, the iteration will converge to the same stable distribution

Web Spam Is Easy

PageRank can be easily manipulated by having link stuffing!

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- PageRank [L. Page et al., 1998]
- Ranking the Web frontier [N. Eiron et al., 2004]
- Generalize PageRank by damping functions [R. A. Baeza-Yates et al., 2006]
- TrustRank [Z. Gyöngyi et al., 2004]

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TrustRank

- Main characteristics
  - The seed set is selected according to the inverse PageRank
  - The biased PageRank is employed by setting **g** to be the distribution shared by all the trusted pages found in the first part
- Advantage–can combat Web spam
- Disadvantage–it does not follow the actual users' behaviors by setting a biased **g**

$$\mathbf{x} = [(1-\alpha)\mathbf{g}\mathbf{1}^T + \alpha\mathbf{A}]\mathbf{x} \quad (1-\alpha)\mathbf{g}\mathbf{1}^T + \alpha\mathbf{A}$$

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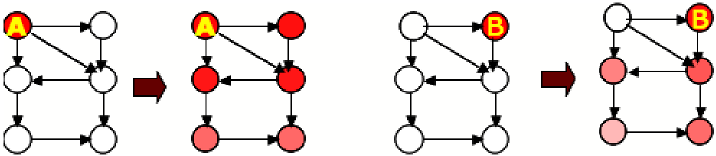
Heat Diffusion Model

Assumptions

- Pages are not equal
- Different initial temperature distributions will give rise to different temperature distributions after a fixed time period



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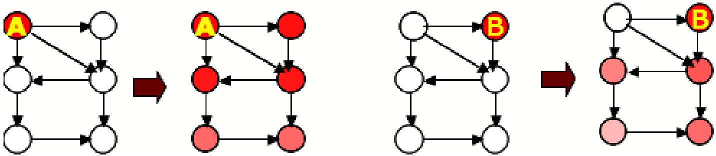

Our Contributions

- Propose a novel DiffusionRank
  - Provide a new viewpoint on ranking problems
  - Use random graphs
- Theoretically we show that DiffusionRank generalizes PageRank
  - When the thermal conductivity tends to infinity, DiffusionRank becomes PageRank
  - A finite thermal conductivity setting makes DiffusionRank have the effect of anti-spam

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DiffusionRank Defined

- Undirected Graph–the amount of the heat flow from j to i is proportional to the heat difference between i and j

$$\mathbf{f}(1) = e^{\gamma \mathbf{H}}\mathbf{f}(0), H_{ij} = \begin{cases} -d(v_j), & j = i, \\ 1, & (v_j, v_i) \in E, \\ 0, & otherwise. \end{cases}$$

- Directed Graph–there is extra energy imposed on the link (j, i) such that the heat flow only from j to i if there is no link (i, j)

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- Temperature distribution $\mathbf{f}(1)$ is the ranking vector

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- It is not over-democratic–Some pages will be born with a high temperature while others with a low temperature
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- It models actual users' behaviors–Heat diffusion model is established on a random graph describing actual users' behaviors
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$$(\mathbf{I} + \tfrac{\gamma}{N}\mathbf{R})^N \to e^{\gamma \mathbf{R}} \qquad \text{when } N \to \infty$$

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- How to set N?
  - When $\gamma = 1$, $N \geq 30$, the absolute value of real eigenvalues of $(\mathbf{I} + \frac{\gamma}{N}\mathbf{R})^N - e^{\gamma \mathbf{R}}$ are less than 0.01
  - When $\gamma = 1$, $N \geq 100$, they are less than 0.005
  - We use $N = 100$ in the paper

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The Thermal Conductivity, γ

1. $\gamma = 0$
   The ranking value is most robust to manipulation since no heat is diffused, but the Web structure is completely ignored
2. $\gamma = \infty$
   DiffusionRank becomes PageRank, it can be manipulated easily
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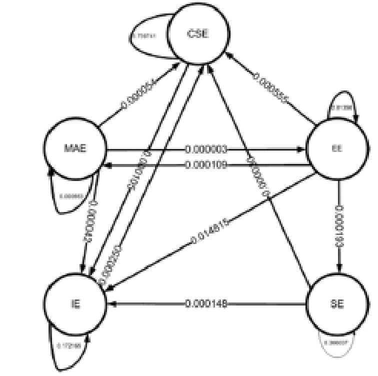
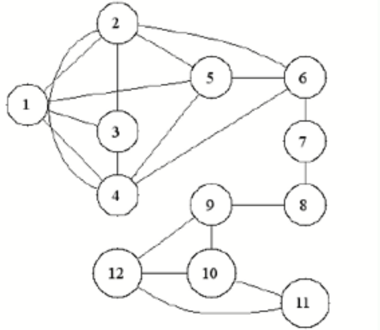


The amount of heat flow from all pages in one department to another

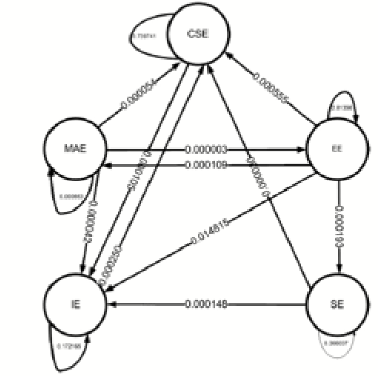
Classification



Temperature distribution at time 1:
(0.17, 0.16, 0.17, 0.16, 0.16, 0.12, 0.02, −0.07, −0.18, −0.22, −0.24, −0.24)

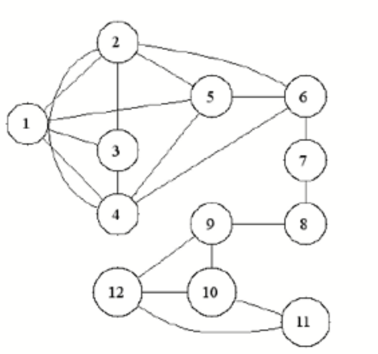
Applications of DiffusionRank

Group-to-group Relations



The amount of heat flow from all pages in one department to another

Classification



Temperature distribution at time 1:
(0.17, 0.16, 0.17, 0.16, 0.16, 0.12, 0.02, −0.07, −0.18, −0.22, −0.24, −0.24)

Experimental Set-Up

- Dataset
  - A toy graph (6 nodes)
  - A middle-size graph (18,542 nodes)
  - A large-size graph crawled from CUHK (607,170 nodes)
- Normalize the rank scores: the sum is the number of nodes
- Parameter settings

Symbol	Meaning	Setting
N	# iterations	100
γ	thermal conductivity	1 (best)
L	# trusted pages	1
$\mathbf{g}$	random jump distribution	uniformly (w/o a priori)
α	probability following actual links	0.85

Experiment I

- Tendency of DiffusionRank
  Rank value difference between $\{A_i\}$ and $\{B_i\}$: $\sum |A_i - B_i|$
- Compare with TrustRank and PageRank on variation of rank values
  When the number of newly added nodes for manipulation is increased
- Compare with TrustRank and PageRank on variation of order difference
  Order difference between $\{A_i\}$ and $\{B_i\}$ is measured by the number of all occurrences of the following cases:
  $|A_i - A_j| > 0.1 \,\&\, (A_i - A_j) * (B_i - B_j) < 0$
  $|B_i - B_j| > 0.1 \,\&\, (A_i - A_j) * (B_i - B_j) < 0$

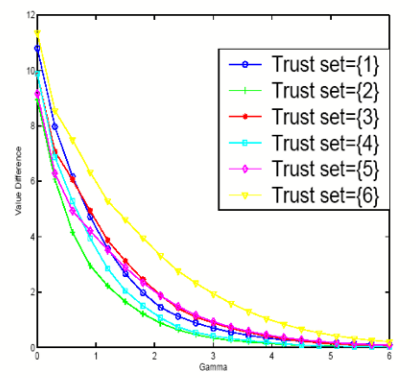
Experiment II

- Inverse PageRank scores:

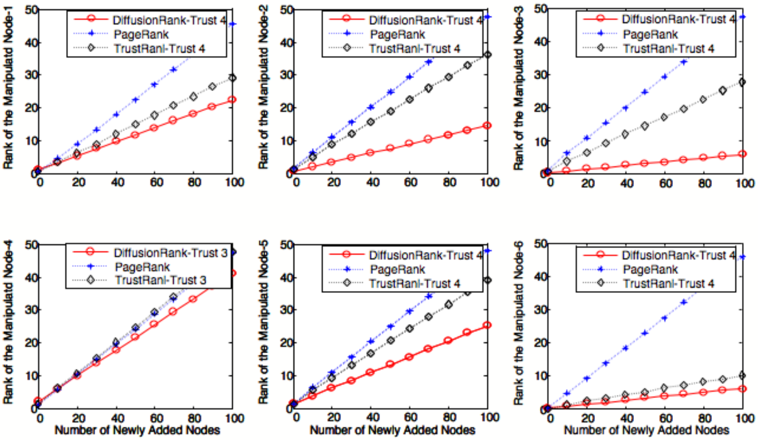
  $4 > 3 > 1 > 2 > 6 > 5$

- If node 4 has not been manipulated, then node 4 can be trusted, otherwise node 3 should be trusted

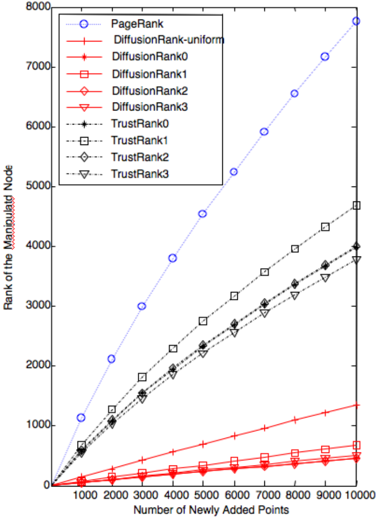
Value Difference Between PageRank and DiffusionRank

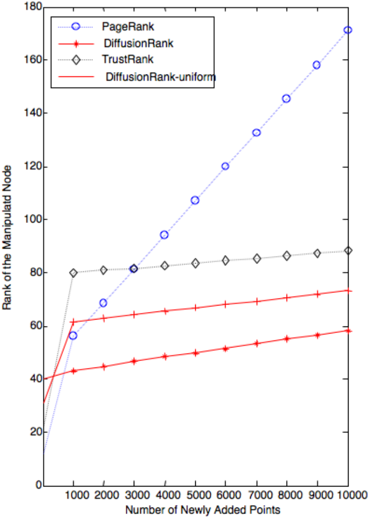


Variation of Rank Values on the Toy DataSet

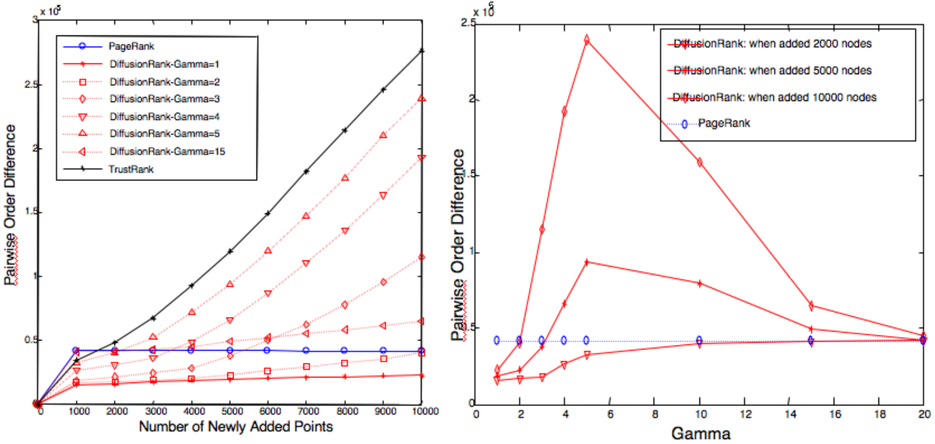


Variation of Rank Values on Two Larger Datasets

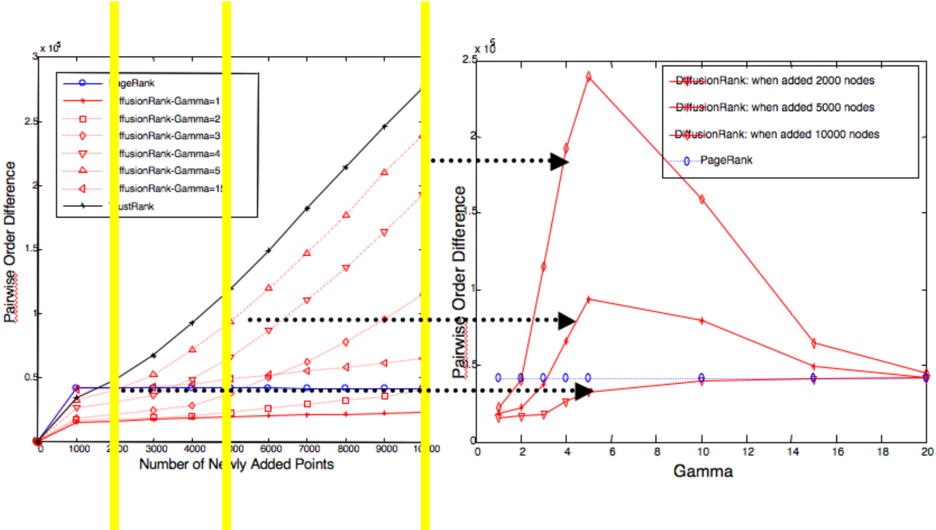




Variation of Order Difference on the Larger Dataset



Variation of Order Difference on the Larger Dataset



Looking Into the Crystal Ball...

Conclusion

- DiffusionRank combats Web spamming
- DiffusionRank is a generalization of PageRank when $\gamma = \infty$
- DiffusionRank can be employed to detect group-to-group relations
- DiffusionRank can be used for classification

Future Work

- Investigate the actual users' behaviors for random jumps, g
- What are the optimal values for L
- Commercial applications $$

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