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Main Objective

Using Machine Learning to beat the odds

Our Solution

- Easy to obtain data
- Optimized for betting

Easy to Obtain Data

- Use **Betting odds** as features
 - Betting odds are accurate in general
 - Odds are widely available

- Use Odds movements as features
 - Odds movements are not noises but informative
 - Odds change from time to time
 - Odds are becoming more accurate in general



Optimize for Betting

- Kelly Formula a simple and optimal betting strategy
 - Relates probability of winning and payoffs (odds) together
 - Compute the optimal bet size that maximizes the growth of profit
- Payoffs are known, probability is unknown
- Build machine learning model to map features into probability

Model Requirements

- 1. Activation and Loss Function
- 2. Ensemble model

Activation and Loss Function Ensemble model

- Objective: Mapping features into probability
- Activation in output layer: Sigmoid
 - To produce values ranging from 0-1



- Loss Function in training: Binary Cross Entropy
 - The optimal loss function to use with Kelly Betting



$$H_p(q) = -\frac{1}{N} \sum_{i=1}^N y_i \cdot log(p(y_i)) + (1 - y_i) \cdot log(1 - p(y_i))$$

Binary Cross-Entropy / Log Loss

Activation and Loss Function Ensemble model

- Kelly Formula assumes an unbiased probability
 - Overestimation causes bankruptcy quickly!
- To improve robustness
 - Train multiple models
 - Group them into an ensemble
 - Output Ensemble Mean
- 300 1000 models in an ensemble



Betting strategies

- Kelly & Fractional Kelly
- Improved Kelly

Betting strategies

- Kelly Betting (Full Kelly):
 - Use Kelly Formula to compute the optimal bet size
- Fractional Kelly:
 - Multiply a fraction (e.g. 20%, 30%) to the Kelly Formula
- Improved Kelly:
 - Solve for the optimal fraction under given uncertainty estimation

	Input	Output			
Kelly Formula	Deterministic Probability, Odds	Optimal bet			
Improved Kelly	PDF of Probability, Odds	Optimal fraction + Optimal bet			
PDF of Probability: Uncertainty in prediction					

Improved Kelly

	Input	Output			
Original Kelly	Predicted Probability, Odds	Optimal bet			
Improved Kelly	PDF of Probability, Odds	Optimal fraction + Optimal bet			

- Assume the **uncertainty = ensemble variance**
- We obtain the PDF by performing Beta fit on ensemble members' predictions.



Proposed Models

- 1. Closing Model
- 2. Continuous Model

Overview

1. Closing Model

- Model that gives predictions only just before closing
 - Closing (收盤): the moment that bookmakers do not accept bets anymore
- 2. Continuous Model
 - Model that gives predictions continuously until closing

Overview

- Select an odds considering period
 - Every period is in 1-minute interval
- Use the odds-implied probabilities in that period to form features
 - For example: 0 min 29 min before closing
 - Odds-implied probabilities: [P(t=0min), P(t=1min), ..., P(t=28min), P(t=29min)]

Closing Model Continuous Model

- Regression-based Closing Model (Term 1)
- LSTM-based Closing Model
- LSTM-based Continuous Model
- Convolution-based Continuous Model

Application in Horse Racing – Forming Records

- Aim: Predict Pr(Win) for each horse
- Pick an Odds Considering Period
 - Sequence of Odds-Implied Probabilities (P)
- Create a record for each horse:
 - lastP, minP, maxP
 - Coefficients from Polynomial Regression

1. Regression-based Closing Model

2. LSTM-based Closing Model

- 3. LSTM-based Continuous Model
- 4. Convolution-based Continuous Model

Application in Horse Racing – Forming Records

1. Regression-based Closing Model

2. LSTM-based Closing Model

3. LSTM-based Continuous Model

- Training Set: Data from 2017/01/01 2018/12/31 (19647 records)
 - 2-Year Data
- Testing Set: Data from 2019/01/01 2019/12/31 (9827 records)
 - 1-Year Data

Application in Horse Racing – Results:

- Kelly Betting
 - Initial capital: \$10000
 - Betting against the highest closing odds among \approx 10 bookmakers
- We tried models with different configurations
- The **best** model is �
- Problem:
 - The returns are very sensitive to the choice of degree in polynomial regression

Model Return 0-39-4deg 21287 0-39-6deg 20941 24307 (highest) 0-39-8deg 0-39-10deg 19255 0-39-12deg 17903 0-39-14deg 17472 0-39-16deg 10185 0-39-18deg 213

1. Regression-based Closing Model

2. LSTM-based Closing Model

3. LSTM-based Continuous Model

Application in Horse Racing – Improvement: LSTM-based

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1. Regression-based Closing Model

2. LSTM-based Closing Model

3. LSTM-based Continuous Model



Application in Horse Racing – Improvement: LSTM-based

1. Regression-based Closing Model

2. LSTM-based Closing Model

3. LSTM-based Continuous Model

Application in Horse Racing – Results

- Best regression-based model
 - 0-39-8deg
- Initial capital: \$10000
- Betting against the highest closing odds among ≈ 10 bookmakers

• Every LSTM-based model can outperform the Regression-based 1. Regression-based Closing Model

2. LSTM-based Closing Model

- 3. LSTM-based Continuous Model
- 4. Convolution-based Continuous Model



Application in Horse Racing – Results- By Luck? NO

Return



Using the closing-odds-implied probabilities as estimations IS NOT profitable

3. LSTM-based Continuous Model 4. Convolution-based Continuous Model 42500 40000 37500 35000 32500 30000 27500 25000 22500 12 14 10 Parameter k Improved Kelly Maximum Fractional Kelly 0-39-8deg: Return in Improved Kelly 0-39-8deg: Maximum Return in Fractional Kelly

0-39-8deg: Return in Full Kelly

1. Regression-based Closing Model

2. LSTM-based Closing Model

Application in Soccer

- Tested in the Over/Under (入球大細) market
 - Guess the total goal is Over(大) or Under(細) a Line
 - Models predict the probability of Over
- Using � to compute features
 - Odds Considering Period:
- Data
 - Training Set: games before 2019/07/01 (18847 lines)
 - Testing Set: games from 2019/07/01 2020/03/08 (8567 lines)

1. Regression-based Closing Model

2. LSTM-based Closing Model

3. LSTM-based Continuous Model



1. Regression-based Closing Model

2. LSTM-based Closing Model

- 3. LSTM-based Continuous Model
- 4. Convolution-based Continuous Model

Application in Soccer – Results

- Kelly Betting
 - Initial capital: \$10000
 - Betting against the highest closing odds
- All models are profitable



1. Regression-based Closing Model

2. LSTM-based Closing Model

- 3. LSTM-based Continuous Model
- 4. Convolution-based Continuous Model

Application in Soccer – Results- By Luck? NO





Closing Model Continuous Model

- Regression-based Closing Model
- LSTM-based Closing Model
- LSTM-based Continuous Model
- Convolution-based Continuous Model

Application in Horse Racing – LSTM-based Continuous Model

Regression-based Closing Model
LSTM-based Closing Model
LSTM-based Continuous Model

4. Convolution-based Continuous Model

- Similar structure to the LSTM-based Closing Model
- Continuous Prediction can be achieved by replicating data records
- Model:
 - Give minute-by-minute predictions for period 0 min 29 min before closing

Basic Features		Sequence of odds-implied probability <i>P</i> avg					
lastP	Minute before closing	$P_{avg}(t_0)$	$P_{avg}(t_1)$	$P_{\text{avg}}(t_2)$		$P_{avg}(t_{29})$	
$P_{avg}(t_0)$	0	0.1080	0.1101	0.1117		0.1378	
$P_{avg}(t_1)$	1	-1	0.1101	0.1117		0.1378	
$P_{avg}(t_2)$	2	-1	-1	0.1117		0.1378	
$P_{avg}(t_{29})$	29	-1	-1	-1		0.1378	

Records formed by a single horse

Application in Horse Racing – Results

- Since it is difficult to achieve optimal in continuous betting
 - We compute the **Binary Cross Entropy** on the **testing set** instead



Regression-based Closing Model
LSTM-based Closing Model
LSTM-based Continuous Model
Convolution-based Continuous Model

Regression-based Closing Model
LSTM-based Closing Model
LSTM-based Continuous Model

4. Convolution-based Continuous Model

Limitation

- Data size grows with number of timesteps
 - Original: 19647 records
 - After replicating records for every horse: 19647 x 30 = 589 410
- Not suitable for long period prediction
 - For example: Soccer games
 - Bookmakers offer odds several days or even a month before kickoff

1. Regression-based Closing Model

2. LSTM-based Closing Model

3. LSTM-based Continuous Model

4. Convolution-based Continuous Model

Convolution-based

- Designed for long period prediction
- Input a sequence of odds-implied probability
- output a sequence of predicted probability with time dependency preserved

1. Regression-based Closing Model

2. LSTM-based Closing Model

3. LSTM-based Continuous Model

4. Convolution-based Continuous Model

Convolution-based

- Casual Convolution
 - Preserves time dependency in a Sequence
 - Example: Window Size = 2



Application in Soccer – Data & Model

- Over/Under (入球大細)
 - Predict Pr(Over) minute by minute in period 0 – 1439 mins before closing

1. Regression-based Closing Model

2. LSTM-based Closing Model

3. LSTM-based Continuous Model

Application in Soccer – Results: BCE Test



1. Regression-based Closing Model

2. LSTM-based Closing Model

3. LSTM-based Continuous Model

4. Convolution-based Continuous Model

• Smaller window is better

Application in Horse Racing – Results – BCE Test

- Which one is better?
 - At Closing: LSTM-Based
 - At Beginning: Convolution-Based
- Both models show they are possible to outperform the odds

- 1. Regression-based Closing Model
- 2. LSTM-based Closing Model
- 3. LSTM-based Continuous Model
- 4. Convolution-based Continuous Model



Conclusion

We created the following models in this project

- 1. Regression-based Closing Model
- 2. LSTM-based Closing Model
- 3. LSTM-based Continuous Model
- 4. Convolution-based Continuous Model
- Above models are shown to be potentially profitable and able to outperform the betting odds
- Using to odds to beat the odds!