CENG 4480
Embedded System Development & Applications

Lecture 07: Kalman Filter–1

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Overview

1. Introduction
2. Complementary Filter
3. Kalman Filter Basic

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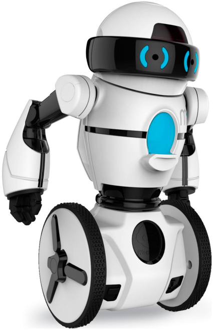
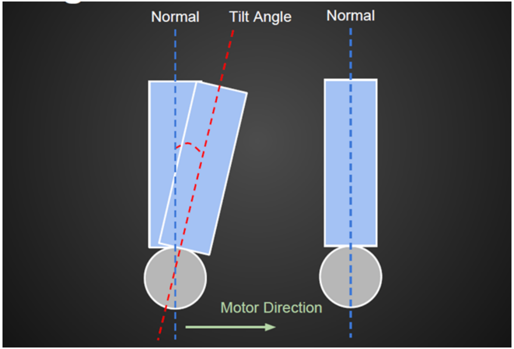
Self Balance Vehicle / Robot

- http://www.segway.com/
- http://wowwee.com/mip/

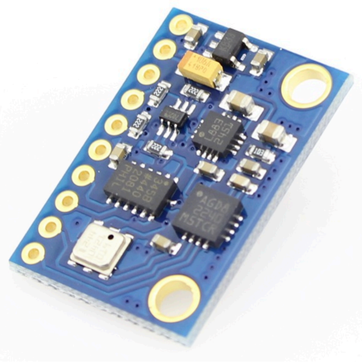


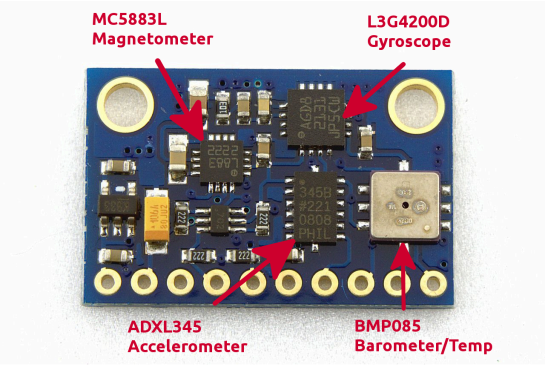
Basic Idea



Motion against the tilt angle, so it can stand upright.

IMU Board





http://www.hotmcu.com/imu-10dof-l3g4200dadxl345hmc5883lbmp180-p-190.html

- L3G4200D: gyroscope, measure angular rate (relative value)
- ADXL345: accelerometer, measure acceleration

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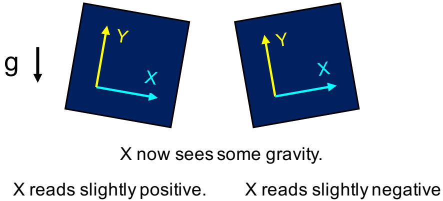
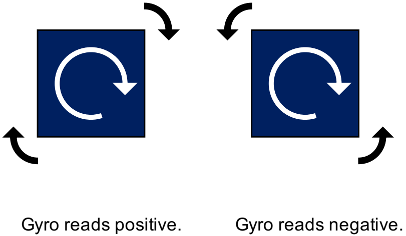
Complementary Filter



Accelerometer

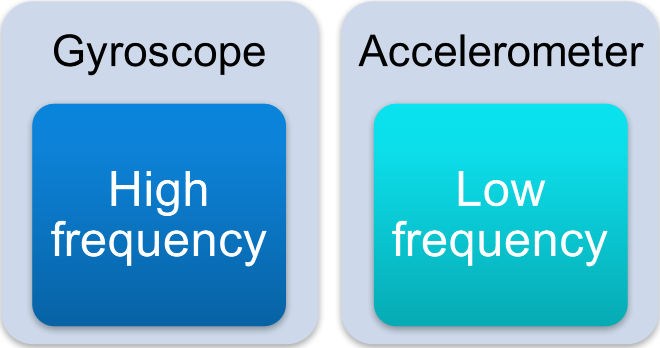


Gyroscope

- Give accurate reading of tilt angle
- Slower to respond than Gyro's
- prone to vibration/noise

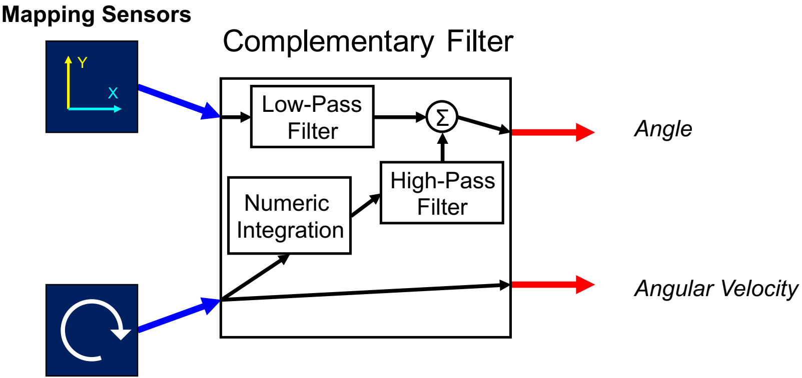
- response faster
- but has drift over time

- Since



- Combine two sensors to find output

Complementary Filter (cont.)




```
Read_acc();
Read_gyro();
Ayz=atan2(RwAcc[1],RwAcc[2])*180/PI;                    //angle by accelerometer
Ayz-=offset;                                            //adjust to correct
Angy = 0.98*(Angy+GyroIN[0]*interval/1000)+0.02*Ayz;  //complement filter
```

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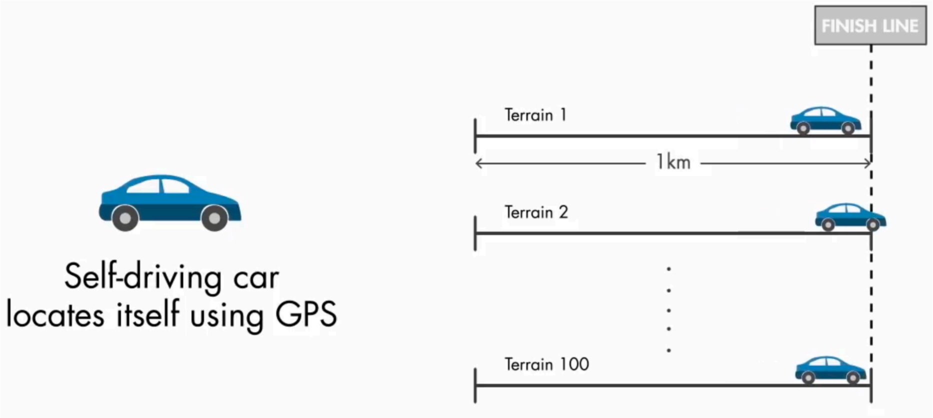
Rudolf Kalman (1930 – 2016)

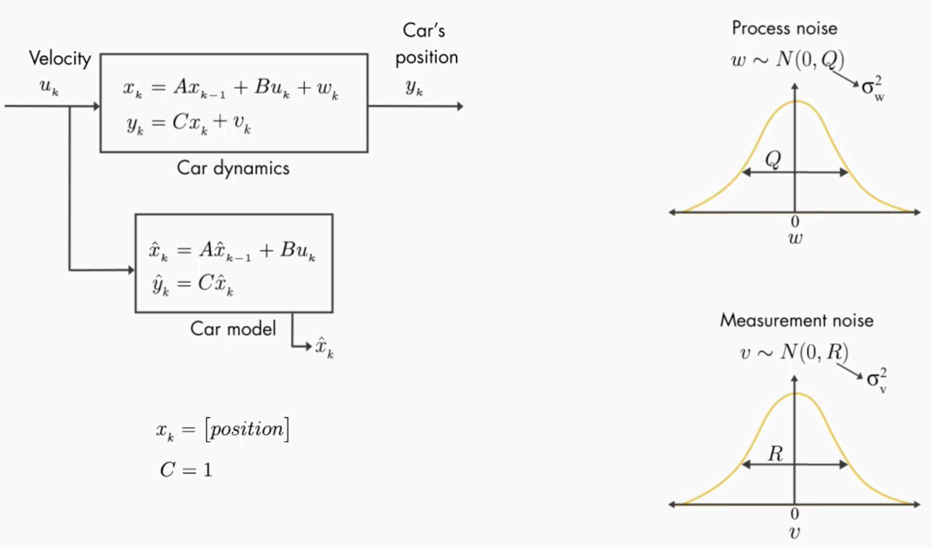
- Born in Budapest, Hungary
- BS in 1953 and MS in 1954 from MIT electrical engineering
- PhD in 1957 from Columbia University.

- Famous for his co-invention of the Kalman filter – widely used in control systems to extract a signal from a series of incomplete and noisy measurements.
- Convince NASA Ames Research Center 1960
- Kalman filter was used during Apollo program

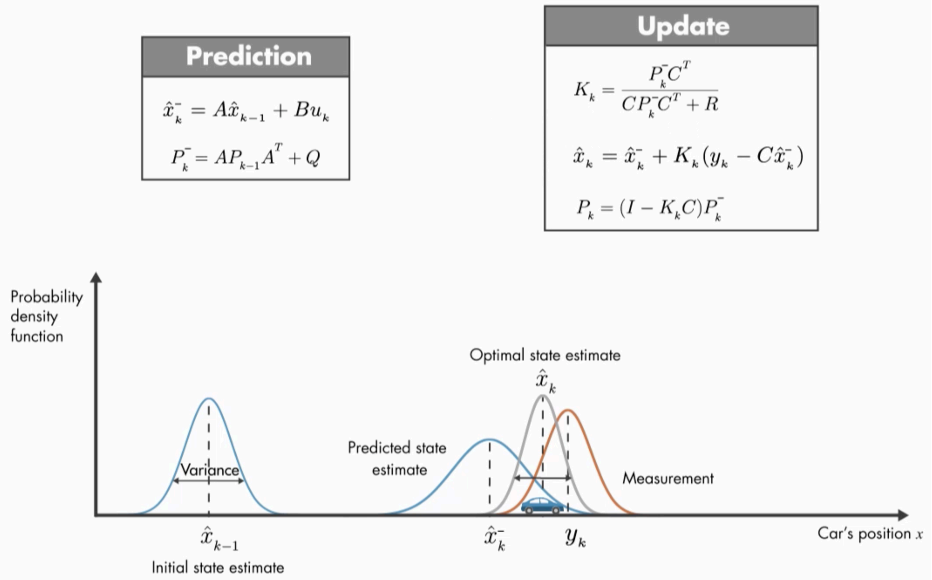
Self-Driving Car Location Problem



Self-Driving Car Location Problem



Self-Driving Car Location Problem



Exercise: Analyse Kalman Gain

What is Kalman Gain $\boldsymbol{K}_k$, if measurement noise $\boldsymbol{R}$ is very small? What if $\boldsymbol{R}$ is very big?

Problem Example 2

Angle Measurement System

$$\boldsymbol{x}_t = \boldsymbol{A}_t \boldsymbol{x}_{t-1} + \boldsymbol{B}_t \boldsymbol{u}_t + \boldsymbol{w}_t$$

- $\boldsymbol{x}_t$: state in time t
- $\boldsymbol{A}_t$: state transition matrix from time $t-1$ to time t
- $\boldsymbol{u}_t$: input parameter vector at time t
- $\boldsymbol{B}_t$: control input matrix – apply the effort of $\boldsymbol{u}_t$
- $\boldsymbol{w}_t$: process noise, $\boldsymbol{w}_t \sim N(0, Q_t)$[1]

[1] $\boldsymbol{w}_t$ assumes zero mean multivariate normal distribution, covariance matrix Q_t

Angle Measurement System

$$\boldsymbol{x}_t = \boldsymbol{A}_t \boldsymbol{x}_{t-1} + \boldsymbol{B}_t \boldsymbol{u}_t + \boldsymbol{w}_t$$

- $\boldsymbol{x}_t = [x_t, \dot{x}_t]^\top$: x_t is current angle, while $\dot{x}_t$ is current rate
- $\boldsymbol{A}_t = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}$
- $\boldsymbol{B}_t = [\frac{(\Delta t)^2}{2}, \Delta t]^\top$
- $\boldsymbol{u}_t = \Delta \dot{x}_t$

Problem Example 2

System Measurement

$$\boldsymbol{z}_t = \boldsymbol{C}\boldsymbol{x}_t + \boldsymbol{v}_t$$

- $\boldsymbol{z}_t$: measurement vector
- $\boldsymbol{C}$: transformation matrix mapping state vector to measurement
- $\boldsymbol{v}_t$: measurement noise, $\boldsymbol{v}_t \sim N(0, \boldsymbol{R}_t)$[2]

[2] $\boldsymbol{v}_t$ assumes zero mean multivariate normal distribution, covariance matrix $\boldsymbol{R}_t$

Exercise

In angle measurement lab, what is the transformation matrix $\boldsymbol{C}$?

$$\boldsymbol{z}_t = \boldsymbol{C}\boldsymbol{x}_t + \boldsymbol{v}_t$$