CENG 4480 Midterm (Fall 2021)

Name: __________
ID: __________

Solutions

Q1 **(30%)** Check or fill the correct answer:

1. A circuit where the input signal power is less than the output signal power is called **amplifier/attenuator**.
2. In an ideal op amplifier, V_{in+} $\underline{> / = / <}$ V_{in-}, since it has **infinite/finite** open-loop gain.
3. An amplifier with input voltage of 1mv and output voltage 1V has gain ___ dB.
4. A capacitor can be regarded as a short circuit when a **high/low** frequency signal is input.
5. A 5-bit D/A converter has a voltage output. For a binary input of 10100, an output voltage of 12 mV is produced. When the binary input is 11100, the output voltage is ___ mV
6. ___ is the minimum number of bits required to digitize an analog signal with a resolution of 0.5%. (Resolution is the ratio between minimum voltage that can be sensed and the input voltage range.)
7. Given a differentiator circuit and a triangular wave as input signal, the waveform of the output is **cosine/square**.
8. Bandwidth is defined as the frequency range over which the voltage gain of the amplifier is above 70.7% or __dB of its maximum output value.
9. The sampling rate of the ADC used depends on the maximum frequency f_{max} of input signal. According to Nyquist sampling theorem, the sampling frequency f_s should be greater or equal to **two/three** times f_{max}.
10. In PID control, decreasing proportional gain will lead to the **faster/slower** response. And we will get **faster/slower** elimination of steady state error by adding integral gain, while **increasing/decreasing** settling time and overshot with a larger derivative gain.

A1

1. amplifier
2. =, infinite
3. 60
4. high
5. 16.8
6. 8
7. square
8. -3

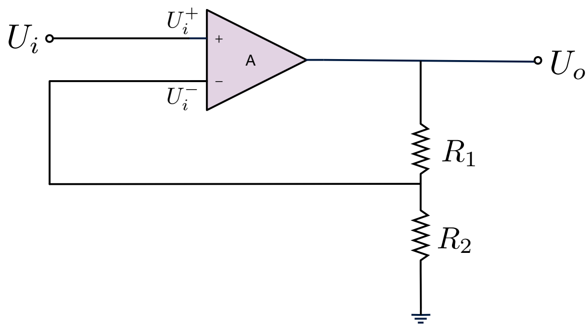

Figure 1: Non-inverting Amplifier.

9. two
10. slower, faster, decreasing

Q2 (20%) Given a non-inverting amplifier as shown in Figure 1. $R_1 = 3R_2$ and $A = 1000$.
(1) Calculate the voltage gain $G = \frac{U_o}{U_i}$.
(2) If the amplifier is ideal($A = \infty$), compute the voltage gain defined in (1).

A2 From the proterties of Op Amplifier,

$$U_o = A_0(U_i^+ - U_i^-) \tag{1}$$

Given that,

$$U_i^- = \frac{R_2}{R_1 + R_2} V_o \tag{2}$$

Substituting into (1) we have,

$$G_{real} = \frac{U_o}{U_i} = \frac{A}{1 + \frac{R_2}{R_1+R_2} A} \tag{3}$$

Besides,

$$G_{ideal} = (1 + \frac{R_1}{R_2}) \tag{4}$$

Substituting data into Eqs. (3) and (4),

$$G_{real} = 3.98, G_{ideal} = 4 \tag{5}$$

Q3 (10%) In the circuit of Figure 2, $R_1 = R_2 = 10\text{k}\Omega$, $R_3 = R_4 = 3\text{k}\Omega$, $C_1 = 0.1\mu\text{F}$, $C_2 = 0.05\mu\text{F}$. Calculate the bandwidth of the RC band pass filter.

1.

$$f_l = \frac{1}{2\pi(R_1//R_2)C_1} = 318.3Hz \tag{6}$$

$$f_h = \frac{1}{2\pi(R_3 + R_4)C_2} = 530.5Hz \tag{7}$$

$$\text{bandwidth} = f_h - f_l = 212.2Hz \tag{8}$$

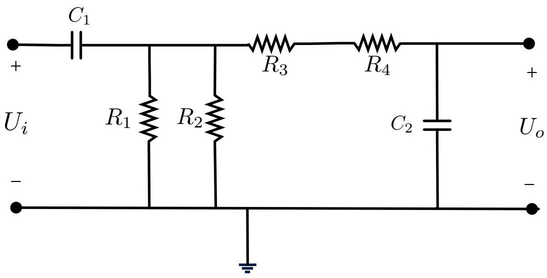

Figure 2: Band Pass Filter

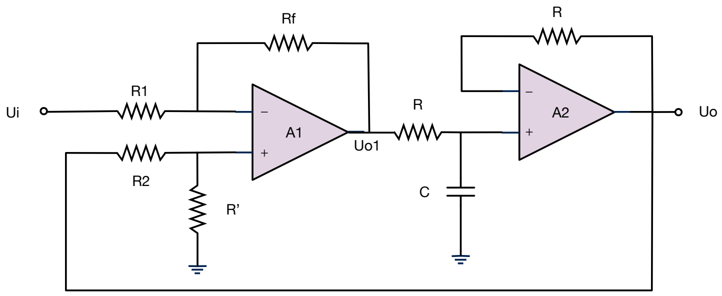

Figure 3: Voltage Follower.

Q4 (20%) In the circuit of Figure 3, $R_1 = R = R' = 100\text{k}\Omega$, $R_2 = R_f = 200\text{k}\Omega$ and $C = 1\mu\text{F}$. Assume the op-amps are ideal.

1. The relationship between U_i and U_o (U_{o1} is unknown).
2. Assume that when the time $t = 0$, $U_o = 0V$ and U_i jumps from $0V$ to $-1V$. How long will the U_o take to change from $0V$ to $6V$?

1. Because A_2 is a voltage follower (the voltage on the capacitance $U_c = U_o$) , so we can obtain U_{o1}

$$U_{o1} = -\frac{R_f}{R_1}U_i + \left(1 + \frac{R_f}{R_1}\right)\frac{R'}{R_2 + R'}U_o = U_o - 2U_i \tag{9}$$

The voltage on the capacitance $U_c = U_o$, so the current of the capacitance is

$$i_C = \frac{U_{o1} - U_o}{R} = -\frac{2U_i}{R} \tag{10}$$

And

$$U_o = \frac{1}{C}\int_{-\infty}^{t} i_c \text{d}t = -\frac{2}{RC}\int_{-\infty}^{t} U_i \text{d}t = -20\int_{-\infty}^{t} U_i \text{d}t \tag{11}$$

2. We can get $U_o = -20\int_0^t U_i \text{d}t = 20t = 6$. So $t = 0.3sec$

Q5 (20%) Try to use discrete incremental PID formulations to calculate $\Delta u(t)$. Some notations and values of parameters are given:

- $u(t)$ is the output of a controller in the tth measurement interval.

- $e(t)$ is the error between the target value and measurement value in the tth measurement interval. The error is measured every T time interval ($T = 0.001$).
  And $e(t) = 2$, $e(t-2) = 5$ and $e(t-1) = 3$.

- The numerical values of PID parameters, K_p, K_i and K_d, are 1, 50, 0.001 respectively.
  (Hint: The formulation of continuous PID is $u(t) = K_p e(t) + K_i \int_0^t e(t)\,\mathrm{d}t + K_d \frac{\mathrm{de}(t)}{\mathrm{d}t}$)

A5

$$u(t) = K_p * e(t) + K_i * \sum e(t) * T + K_d * \frac{e(t) - e(t-1)}{T} \tag{12}$$

$$u(t-1) = K_p * e(t-1) + K_i * \sum e(t-1) * T + K_d * \frac{e(t-1) - e(t-2)}{T} \tag{13}$$

$$\Delta u(t) = K_p * (e(t) - e(t-1)) + K_i * e(t) * T + K_d * \frac{e(t) - 2e(t-1) + e(t-2)}{T} \tag{14}$$

So $\Delta u(t) = 0.1$