CENG4480 2021F Homework 1

Due: Oct. 24, 2021

Solutions

All solutions should be submitted to the blackboard in the format of **PDF/MS Word**.

Q1 (10%) The following circuit Figure 1 is a difference amplifier circuit. $U_1 = 40V$, $U_2 = 25V$, $R_1 = 6\Omega$, $R_2 = 4\Omega$, $R_3 = 15\Omega$, $R_4 = 20\Omega$. Please calculate the output voltage of U_o.

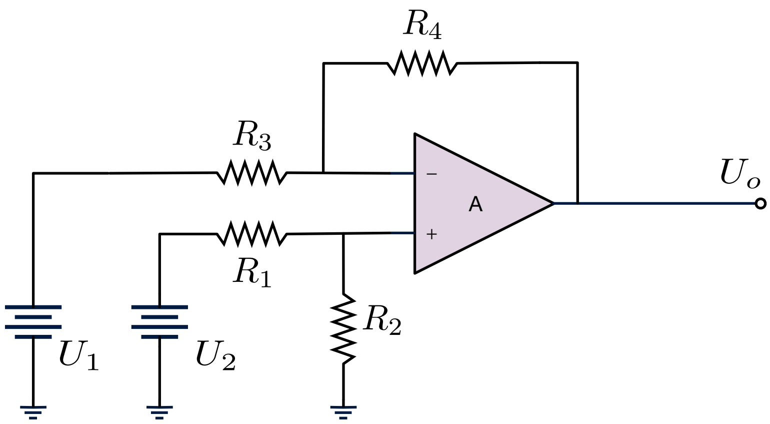

Figure 1: Q1 Circuit

A1 By voltage division,

$$U_{in+} = U_2 \times \frac{R_2}{R_1 + R_2} = 25V \times \frac{4\Omega}{6\Omega + 4\Omega} = 10V$$

By the virtual short circuit between the input terminals, $U_{in-} = U_{in+} = 10V$, the current through the 15Ω is

$$I_3 = \frac{U_1 - U_{in-}}{R_3} = \frac{40V - 10V}{15\Omega} = 2A$$

The input impedance is infinite; therefore, $I_{in-} = 0$ and $I_3 = I_4 = 2A$. Based on Kirchoff's voltage law to find the output voltage

$$U_o = U_{in-} - R_4 \times I_4 = 10V - 20\Omega \times 2A = -30V$$

Q2 (20%) Given an ideal op-amp circuit below where $R_1 = 1\text{k}\Omega$, $R_2 = 2\text{k}\Omega$, $R_3 = 3\text{k}\Omega$, if I want to obtain a gain of 5, what's the value of the resistor R_4 as shown in the circuit?

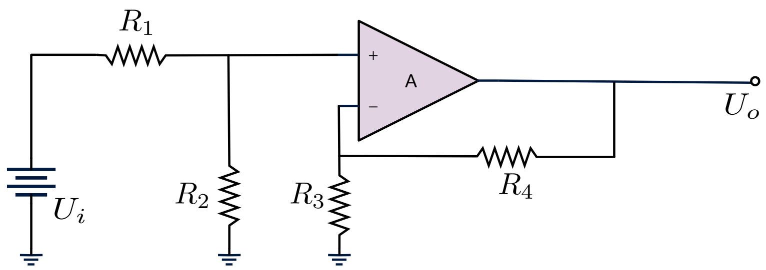

Figure 2: Q2 Circuit

A2 By voltage division,

$$U_{in+} = U_i \times \frac{R_2}{R_1 + R_2} = U_i \times \frac{2\text{k}\Omega}{3\text{k}\Omega} = \frac{2}{3}U_i \tag{1}$$

By the virtual short circuit,

$$U_{in-} = U_{in+} = \frac{2}{3}U_i \tag{2}$$

$$I_3 = \frac{U_{in-}}{3\text{k}\Omega} = \frac{\frac{2}{3}U_i}{3\text{k}\Omega} \tag{3}$$

And

$$\frac{U_o - U_{in-}}{R_4} = \frac{\frac{2}{3}U_i}{3\text{k}\Omega} \tag{4}$$

$$U_o = 5U_i \tag{5}$$

Therefore

$$R_4 = 19.5\text{k}\Omega$$

Q3 (20%) Given the following amplifier circuit as shown in Figure 3. $R_1 = 1\text{M}\Omega$, $R_2 = 500\text{k}\Omega$ and $R_3 = 100\Omega$. Also, I want to acquire a voltage gain $U_o/U_i = -120$, how to choose the value of R_4?

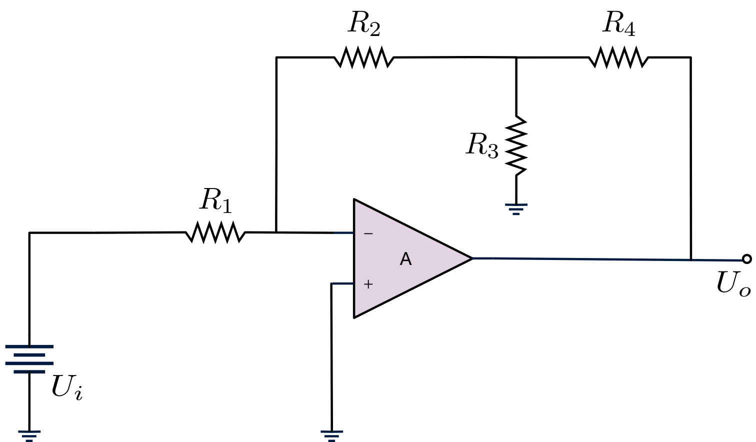

Figure 3: Q3 Circuit

A3 U_{in+} is grounded, so U_{in-} is also a virtual ground. So

$$U_{in-} = 0$$

Since $U_{in-} = 0$, $U_i = I_1R_1$ and $I_1 = U_i/R_1$. Since $U_{in+} = 0$, $U_x = -I_2R_2$ and $I_2 = -U_x/R_2$. Similarly

$$U_x = -I_3R_3 \tag{6}$$
$$U_x - U_o = I_4R_4 \tag{7}$$

From Kirchhoff's current law,

$$I_4 = I_2 + I_3 \tag{8}$$
$$\frac{U_x - U_o}{R_4} = \frac{-U_x}{R_2} + \frac{-U_x}{R_3} \tag{9}$$
$$U_o = -120U_i \tag{10}$$

Also,

$$I_1 = I_2 \tag{11}$$

So,

$$\frac{U_i}{R_1} = \frac{-U_x}{R_2} \tag{12}$$
$$U_x = -\frac{R_2}{R_1}U_i \tag{13}$$
$$\frac{-\frac{R_2}{R_1}U_i - (-120U_i)}{R_4} = \frac{\frac{R_2}{R_1}U_i}{R_2} + \frac{\frac{R_2}{R_1}U_i}{R_3} \tag{14}$$

Therefore

$$R_4 = \frac{120\frac{R_1}{R_2} - 1}{\frac{R_2+R_3}{R_2R_3}} = 2.39 \times 10^4\Omega \tag{15}$$

Q4 (20%) The diagram of amplifier circuit is shown in fig. 4. Given $R_1 = 4\text{k}\Omega$, $R_2 = 6\text{k}\Omega$ and $R_3 = 12\text{k}\Omega$, answer the following questions.

1. Find the output voltage U_o if $U_a = 4\text{V}$ and $U_b = -12\text{V}$.
2. When $U_b = -12V$, determine the range of U_a for linear operation if the saturation voltage of op-amp is $\pm 12V$.

A4 1. Obviously this is a summing amplifier.

$$U_o = -(\frac{R_3}{R_1}U_a + \frac{R_3}{R_2}U_b) \tag{16}$$
$$= -(\frac{12}{4} \times 4 + \frac{12}{6} \times (-12)) \tag{17}$$
$$= 12V \tag{18}$$

2. For linear operation,

$$\pm 12 = -(\frac{12}{4}U_a + \frac{12}{6} \times (-12)) \tag{19}$$
$$= -3U_a + 24 \tag{20}$$

Therefore, $4V < U_a < 12V$.

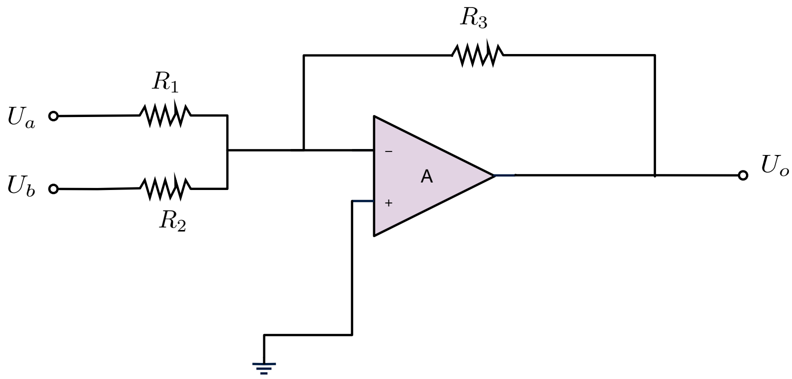

Figure 4: Q4 Circuit Diagram

Q5 (20%) In the circuit of Figure 5, $R_3 = R_4 = R_5 = R_6 = R_7 = R = 2R_1 = 2R_2 = 100\text{k}\Omega$ and $C = 1\mu\text{F}$. Assume the op-amps are ideal. What is the relationship between U_i and U_o?

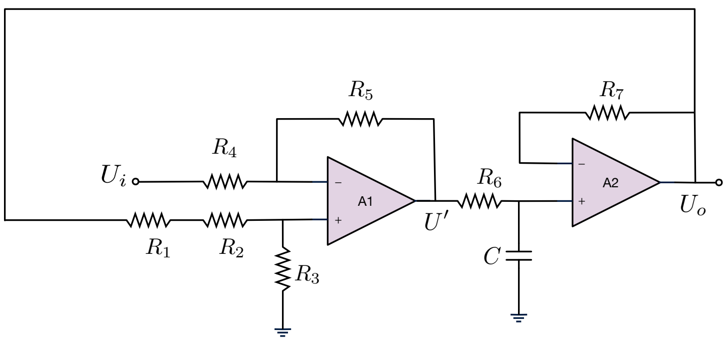

Figure 5: Q5 Circuit Diagram

A5 1. As the amplifier is ideal,

$$\frac{R_3}{R_1 + R2 + R3} \times U_o = U_i - \frac{U_i - U'}{R_4 + R_5} \times R_4 \tag{21}$$

Therefore,

$$\frac{U_o}{2} = U_1 - \frac{U_i - U'}{2} \tag{22}$$
$$U_o = U_i + U' \tag{23}$$

The voltage on the capacitance $U_c = U_o$, so the current of the capacitance is

$$i_C = \frac{U' - Uo}{R} = -\frac{U_i}{R} \tag{24}$$

And

$$U_o(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(\lambda)\mathrm{d}\lambda = -\frac{1}{RC}\int_{-\infty}^{t} U_i(\lambda)\mathrm{d}\lambda = -10\int_{-\infty}^{t} U_i(\lambda)\mathrm{d}\lambda \tag{25}$$

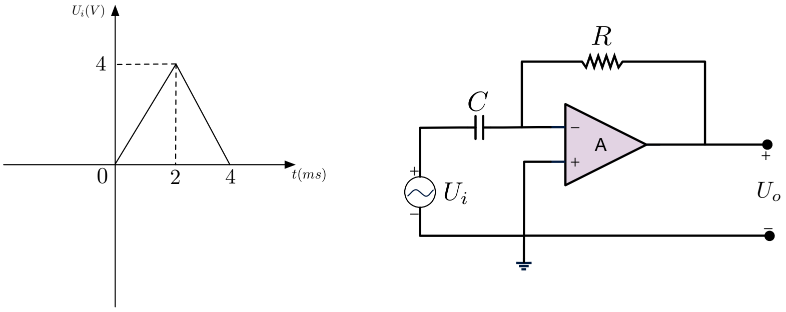

Figure 6: Q6 Circuit and Input Voltage Waveform

Q6 (10%) Draw the output voltage waveform given the input voltage waveform and the circuit in Figure 6. In the circuit, $R = 5k\Omega$ and $C = 0.2\mu F$.

A6 The input voltage is:

$$U_i = \begin{cases} 2000t, & 0 < t < 2 \text{ ms} \\ 8 - 2000t, & 2 \leq t < 4 \text{ ms} \end{cases}$$

The output voltage is given by:

$$U_o = -RC\frac{\mathrm{d}U_i}{\mathrm{d}t} = \begin{cases} -2V, & 0 < t < 2 \text{ ms} \\ 2V, & 2 \leq t < 4 \text{ ms} \end{cases}$$

The waveform is as follows:

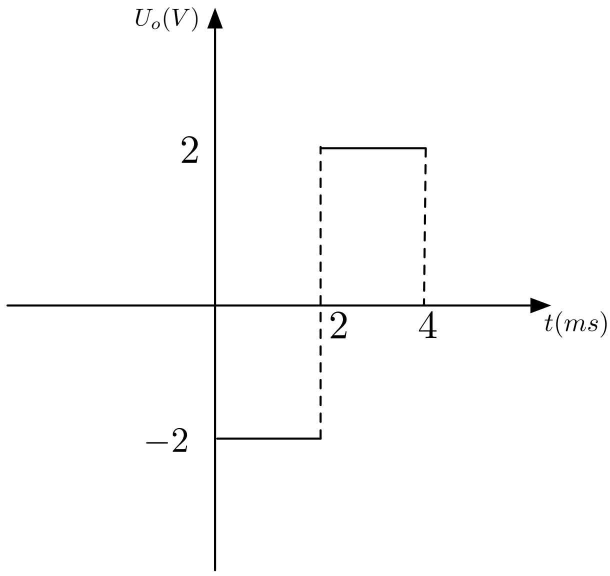

Figure 7: Q6 solution