On Modeling Clustering Indexes of BT-like Systems

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***Abstract*—In this paper, we explore the "clustering" phenomenon in BT-like systems. A high clustering implies peers have a high tendency to exchange information with peers of the similar bandwidth type. We first show the clustering does exist in BT-like systems. Although high clustering is desirable for file sharing application, it may not be appropriate for multimedia streaming applications. We provide analytical models to calculate the clustering index and illustrate how one can control the clustering index for different P2P applications.**

I. Introduction

In recent years, there is a significant increase of P2P traffic, in particular, due to the BitTorrent (BT) protocol. In fact, we have seen a growing applications, for example, file sharing, content distribution, multimedia streaming and video-on-demand,..etc, all use the BT-derivative protocols to provide high performance and scalable service.

The BT protocol adopts the well known tit-for-tat policy within the *choking algorithm* for peer selection. Peers in such a system will reward other peers who contribute more and minimize the upload service to the *free-riding* peers. Thus, every peer has incentives to upload in order to enjoy a better service. There are a number of papers [1], [6] which provide the quantitative analysis on BT's performance and fairness. However, most of the work do not capture the detail of the choking algorithm, which influence how peers select other peers for information exchange. Since the tit-for-tat policy affects the preference of peer selection, it can induce the *clustering phenomenon* within a BT-like system, e.g., peers which are of similar bandwidth type may prefer to exchange information with each other. When a BT-like file distribution system has a high clustering index, it implies that peers of type k may prefer to exchange information with peers of type k, and intuitively, this provides certain degree of fairness (since similar bandwidth peers exchange information). However, for some other BT-like systems, e.g., multimedia streaming or video-on-demand service, one may not want the system to have high clustering since we want a more resourceful peers to stay in the system to help other less resourceful peers to receive the information on time.

The goal of this paper is to investigate the clustering phenomenon of BT-like system. We provide an analytical model to evaluate the *clustering index* of a BT-like system which uses the *greedy selection algorithm* (a derivative of the tit-for-tat policy). In particular, we examine in detail the choking algorithm and analyze the peer's behavior in a heterogeneous system. Unlike the work in [3], which investigate the clustering via measurement, we provide a general analytical model to understand this phenomenon and we also show how to use our model to *control* the degree of clustering so as to adapt to the requirements of different P2P applications. The contributions of this paper are:

- We develop two analytical models, one for BT-like systems with two types (or groups) of peers, while the other for BT-like system with many types, in estimating the clustering index among peers.
- We validate our models extensively via simulation.
- We propose simple knobs to control the clustering index of a BT-like system.

In Section II, we introduce the peer selection algorithm, in Section III, we present our analytical models. Performance evaluation and validation are presented in Section IV and Section V concludes.

II. Peer Selection Algorithm

In this paper, we consider a particular peer selection algorithm, or the *Greedy Selection* (GS) algorithm, which is based on the classification of peer's uploading bandwidth. This algorithm will help us to understand how peer selection may impact the clustering phenomenon in BT-like systems. We assume that there are $n > 1$ different groups of peers, with group k denoted as G_k and $G = \bigcup_{k=1}^{n} G_k$, with $G_k \bigcap G_l = \emptyset$ for $k \neq l$. Peers in G_k has an upload capacity of U_k. Without loss of generality, we assume $U_1 > U_2 > \cdots > U_n$. Each peer in the system can provide K maximum uploads to other peers. i.e, in the BT protocol, K is set to five. We assume throughout this paper that the capacity of an uploading connection is limited to $u_k = U_k/K$ for any peer in group G_k. Our system follows the uplink sharing model [5] wherein the bandwidth constraint is in the uplink connections rather than the downlink connections. Note that this assumption is true for most broadband access technologies.

The *Greedy Selection* algorithm, which is also known as the *tit-for-tat* in the official BitTorrent protocol, is an effective policy to discourage free riding. A peer using this algorithm will unchoke (which means to provide upload service) the top $K-1$ peers who contribute the most downloading service to this peer. These $K-1$ connections are also called the *regular unchoking connections* and this operation is carried out every 10 sec . The remaining upload connection will *randomly* select another peer in the system to unchoke. This occurs every 30 sec and it is called the *optimistic unchoking*. The purpose of the optimistic unchoking operation is to discover peers which may be able to provide better downloading service to this peer. The regular unchoke is greedy oriented since it only reciprocates peers who contribute more service to this peer, while the optimistic unchoke serves as a discovery mechanism to find a more contributive peer [1], [3]. We model the GS algorithm as follows. Each round there is a regular unchoke event, while every $\omega \geq 1$ rounds there is an optimistic unchoke event. At the end of each round, the following operations will be carried out:

1) A peer sorts all its contributing peers in the current round according to their group number (e.g., for bandwidth differentiation). If there are several contributing peers belonging to the same group, then the contributing peer which provides upload service for a longer period will have a higher priority.
2) The peer will unchoke at most $K-1$ contributing peers which have the highest $K-1$ priority.
3) Every ω rounds, this peer randomly picks one neighboring peer, independent of its group number or service contribution, to provide optimistic unchoking service.

For example, consider the system with two groups of peer, G_1 and G_2. A peer belongs to G_1 and it has $K = 5$ upload connections. If there are two contributing peers from G_1 and one from G_2, then this peer will provide three regular unchoke connections: two to the contributing peers in G_1 and one to the contributing peer in G_2. This peer also uses one upload connection for optimistic unchoke, and one of its upload connections will remain idle for the current round.

It is important for us to point out that for the GS algorithm, when a peer provides a regular unchoke to another contributing peer, then a *bi-directional* connection is established since both peers provide upload service to each other. On the other hand, for the optimistic unchoke service, we only have a *uni-directional* connection.

III. Analytical Models

In this section, we first provide the definition of clustering index, then we will show how to formulate the mathematical model to estimate the clustering index of BT-like systems. To simplify our presentation, we first show how one can use a Markov chain to model the Greedy Selection algorithm with two groups of peers in a *close system*, that is, all the peers arrive at the same time and they will not leave. In this BT-like system, we have N peers with N_1 and N_2 being the number of peers in group G_1 and G_2 respectively. When extending the model for more than two groups, the state space and correspondingly, the one-step transition probability matrix becomes large. To resolve the curse of state space explosion, we simplify the state space and propose another Markovian model to represent the Greedy Selection algorithm so that we can derive the clustering effect for BT-like systems with more than two peer groups. Throughout this paper, we denote $B(N, p, k)$ as the binomial probability $\binom{N}{k} p^k (1-p)^{(N-k)}$. Unless we state otherwise, we set $\omega = 1$ (i.e., at each round, each peer will have an optimistic unchoke operation).

A. Clustering Index

Clustering index is a measure of the fraction (or steady state probability) of bi-directional connections between peers of the *same* group. For example, given that K is the number of upload connections of a peer and $K-1$ of these upload connections are used for regular unchokes, and if all of these $K-1$ upload connections are used to connect to other peers of the same group, then the clustering index will be 1. There are two important points to note: (1) regular unchokes will create a bi-direction connection between two peers from the same group; (2) a higher clustering index implies a higher tendency for peers of similar bandwidth type to exchange information.

Assume that a peer m of group G_k, we define c_m to be the clustering index for peer m as:

$$c_m = \frac{\text{\# of bi-directional connections to peers in } G_k}{K-1}. \quad (1)$$

The clustering index for peers in G_k, where $1 \leq k \leq n$ is:

$$C_k = \frac{\sum_{m \in G_k} c_m}{|G_k|} \quad (2)$$

where $|G_k|$ is the number of peers in group G_k.

It is easy to verify that $C_k \in [0, 1]$. When C_k is larger than the fraction of G_k peers in the system, then a tighter cluster forms. This means that the unchokes from peers of G_k will have a higher priority and peers in G_k tend to unchoke their *compeers* (e.g. other peers in G_k). On the other hand, if C_k is smaller than the fraction of G_k's peers in the system, then the unchokes from G_k is less competitive and peers tend to unchoke other groups which may provide a better service.

B. Analysis of the Greedy Selection Algorithm

Given the description of the GS algorithm from Section II, we can construct a discrete time Markov chain $\mathcal{M}_G$ with the following state space S_G:

$$S_G = \{(i,j) | i \geq 0, j \geq 0, i + j \leq K-1\},$$

where i is the number of bi-directional connections (or regular unchokes) to group G_1 and j is the number of bi-directional connection to group G_2. We also define the following probability vectors:

$$\boldsymbol{\pi}^{(1)} = \{\pi^{(1)}_{i,j} | (i,j) \in S_G\}, \quad \boldsymbol{\pi}^{(2)} = \{\pi^{(2)}_{i,j} | (i,j) \in S_G\},$$

where $\pi^{(k)}_{i,j}$ represents the fraction of group G_k peers in state (i, j), for $k = \{1, 2\}$. Follow the definition of clustering above, the following expressions are the clustering index for G_1 and G_2:

$$C_1 = \frac{\sum_{(i,j) \in S_G} i\, \pi^{(1)}_{i,j}}{K-1}, \quad C_2 = \frac{\sum_{(i,j) \in S_G} j\, \pi^{(2)}_{i,j}}{K-1}. \quad (3)$$

In the GS algorithm, a bi-directional connection between two peers in G_1 will hold once it is established since they have the highest selection priority under the GS algorithm. A bi-directional connection between a peer in G_1 and a peer in G_2 can be terminated when the peer in G_1 receives a new optimistic unchoke request from a compeer (e.g., another peer in G_1). While the bi-directional connection between two peers in G_2 can be terminated if a peer from G_1 has an optimistic unchoke request to either of these two peers. Lastly, there are other factors that will terminate a bi-directional connection, e.g., when the file is not available or when network errors occur. Hence, we denote $\gamma^{k,l} \in [0,1]$, for $k,l \in \{1,2\}$, be the probability that a bi-directional connection between G_k and G_l be terminated by the peer from the G_l side. We use $\gamma^{1,1}$ and $\gamma^{1,2}$ as inputs to derive the possible events that cause the termination of a bi-directional connection in G_1.

There are three possible processes that can cause a state transition in our Markov chain $\mathcal{M}_G$. These processes are: *cut process*, *search process* and the *match process*. Let us describe these processes in detail.

Cut Process: for this process, bi-directional connections are terminated based on $\gamma^{k,l}$. Let us focus on the derivation of $\gamma^{2,1}$ and $\gamma^{2,2}$, with $\gamma^{1,1}$ and $\gamma^{1,2}$ being input values. Define the $\boldsymbol{\delta}^{(2)} = \{\delta^{(2)}_{i,j} | (i,j) \in S_G\}$ as the probability vector where $\delta^{(2)}_{i,j}$ is the probability that one of G_2 peer's bi-directional connection is terminated by a peer in state (i,j), which can be expressed as:

$$\delta^{(2)}_{i,j} = \sum_{k=u(i,j)}^{N_1} \min\left\{\frac{k-u(i,j)}{j}, 1\right\} B(N_1, \frac{1}{N}, k),$$

where $u(i,j) = K-(i+j)-1$ is the number of idle uploading connections for a peer in state (i,j). Assuming that the number of bi-directional connections are uniformly distributed among peers in the same group, we have,

$$\gamma^{2,1} = \frac{\sum_{(i,j)\in S_G} j\pi^{(1)}_{i,j}\delta^{(2)}_{i,j}}{\sum_{(i,j)\in S_G} j\pi^{(1)}_{i,j}}; \gamma^{2,2} = \frac{\sum_{(i,j)\in S_G} j\pi^{(2)}_{i,j}\delta^{(2)}_{i,j}}{\sum_{(i,j)\in S_G} j\pi^{(2)}_{i,j}}. \quad (4)$$

Let $\boldsymbol{Q}^{(k)}_C$ be the one-step transition probability matrix of the cut process for a G_k peer, where $k \in \{1,2\}$, then the transition probability Prob$\{(i,j)|(i',j')\}$ for $\boldsymbol{Q}^{(k)}_C$ is:

$$B(i', \gamma^{k,1}, i'-i)B(j', \gamma^{k,2}, j'-j)\mathbf{1}_{\{(i\le i')\wedge(j\le j')\}},$$

with $(i',j') \in S_G, (i,j) \in S_G$, and $\mathbf{1}_{\{x\}}$ is an indicator function that $\mathbf{1}_{\{x\}} = 1$ if condition x is true and 0 otherwise.

Search process: for this process, the peer performs optimistic unchoke and randomly selects another peer at the end of each round. We define $\boldsymbol{\alpha}^{(k)} = \{\alpha^{(k)}_{i,j} | (i,j) \in S_G\}$ as the probability vector for group G_k, $k = \{1,2\}$, where $\alpha^{(k)}_{i,j}$ is the probability that a peer of type G_k randomly unchokes and the receiving peer, which is in state (i,j), decides to provide a reciprocative upload. Note that an optimistic unchoke from a peer in group G_1 have a higher priority than any unchoke operations (both for regular and optimistic) for peers from G_2. We have:

$$\alpha^{(1)}_{i,j} = \sum_{k=0}^{N_1} \min\left\{\frac{K-i-1}{k+1}, 1\right\} B(N_1, \frac{1}{N}, k).$$

On the other hand, optimistic unchokes from a peer in G_2 are reciprocated only when the peer in G_1 has an idle upload connection. Therefore, $\alpha^{(2)}_{i,j}$ can be expressed as:

$$\alpha^{(2)}_{i,j} = \sum_{k=0}^{u(i,j)} \sum_{l=0}^{N_2} \left[\min\left\{\frac{u(i,j)-k}{l+1}, 1\right\} B(N_1, \frac{1}{N}, k)B(N_2, \frac{1}{N}, l)\right],$$

with $u(i,j) = K-(i+j)-1$ as mentioned above. Finally, the probability that an optimistic unchoke from a peer in G_k receives a reciprocation from a peer in G_l, which we denote as $\beta^{k,l}$, is:

$$\beta^{k,l} = \boldsymbol{\pi}^{(k)} \times (\boldsymbol{\alpha}^{(l)})^T \quad \text{with } k,l = \{1,2\}.$$

Table I summarizes the one-step transition probability matrix $\boldsymbol{Q}^{(k)}_S$ of the search process for peers from group G_k, $k = \{1,2\}$.

State	Probability	Condition
(i,j)	1	$i = K-1, j = 0$
(i,j)	$1 - \frac{N_2}{N}\beta^{k,2}$	$i+j = K-1, j > 0$
(i,j)	$1 - \frac{N_2}{N}\beta^{k,2} - \frac{N_1}{N}\beta^{k,1}$	$i+j < K-1$
$(i+1,j-1)$	$\frac{N_1}{N}\beta^{k,1}$	$i+j = K-1, j > 0$
$(i+1,j)$	$\frac{N_1}{N}\beta^{k,1}$	$i+j < K-1$
$(i,j+1)$	$\frac{N_2}{N}\beta^{k,2}$	$i+j < K-1$

TABLE I

TRANSITION PROBABILITIES OF SEARCH PROCESS WITH INITIAL STATE (i,j)

Match process: a peer uses the Greedy Selection algorithm to select contributing peers to unchoke. Let us consider a G_1 peer, say v. Peer $v's$ reciprocation to the optimistic unchoke can establish a bi-directional connection when the peer which initiates the optimistic unchoke is in state (i,j), where $i < K-1$. While a G_2 peer's reciprocation can be accepted only when the peer which initiates the optimistic unchoke is in state (i,j), where $i+j < K-1$. We define $E_{k,l}$ be the set of *candidate peers* in group G_k which can still establish a bi-directional connection from the point of view of peers in G_l, then we have:

$$E_{k,1} = \{v | v \in G_k, \text{with state}(i,j), i < K-1\},$$
$$E_{k,2} = \{v | v \in G_k, \text{with state}(i,j), i+j < K-1\}.$$

Also, define $\overline{E}_{k,l} = G_k - E_{k,l}$ as the complement set of $E_{k,l}$.

Given an initial state, the state transition is determined by the number of received optimistic unchokes from the set $E_{k,1}, \overline{E}_{k,1}, E_{k,2}, \overline{E}_{k,2}$ respectively. We define x,y,z,w, where $x \in \{0,..,|E_{k,1}|\}$, $y \in \{0,..,|\overline{E}_{k,1}|\}$, $z \in \{0,..,|E_{k,2}|\}$ and $w \in \{0,..,|\overline{E}_{k,2}|\}$, as the number of optimistic unchokes from the corresponding sets. Table II illustrates transition probability matrix $\boldsymbol{Q}^{(k)}_M$, for $k \in \{1,2\}$. Note that the existing bi-directional connection between G_1 peers can never be terminated in this process, thus we have the Prob$\{(i,j)|(i',j'), i < i'\} = 0$, as shown in the second row of Table II, and the function Select(a,b,c,d) is:

$$\text{Select}(a,b,c,d) = \frac{\binom{c}{a}\binom{d}{b}}{\binom{c+d}{a+b}}.$$

To compute the steady state probability vectors for $\boldsymbol{\pi}^{(1)}$ and $\boldsymbol{\pi}^{(2)}$, we use the following balanced equations:

$$\boldsymbol{\pi}^{(k)} = \boldsymbol{\pi}^{(k)} \times \boldsymbol{Q}^{(k)}_C \times \boldsymbol{Q}^{(k)}_S \times \boldsymbol{Q}^{(k)}_M \,;\, \boldsymbol{\pi}^{(k)}\boldsymbol{e} = 1, \text{ for } k = \{1,2\}. \quad (5)$$

State	Conditions	Next State
(i', j')	0	$(i, j),\ i < i'$
(i', j'), $j' > 0$	$(x = i - i') \wedge (y = u(i, j))$	(i, j), $i \geq i',\ 0 < j < j'$
(i', j'), $j' > 0$	$(x \geq (i - i')) \wedge (y \geq K - i - 1) \wedge \text{Select}(i - i', K - i - 1, x, y)$	$(i, 0)$, $i \geq i'$
(i', j')	$\{(x = i - i') \wedge (y \leq u(i, j))\} \wedge \{((w < u(i, j) - y) \wedge (z = j - j'))$ $\vee ((w \geq u(i, j) - y) \wedge (z \geq j - j') \wedge \text{Select}(j - j', u(i, j) - y, z, w))\}$	(i, j), $i \geq i', (j > j') \vee (j = j' > 0)$
$(i', 0)$	$\{(x \geq (i - i')) \wedge (y \geq K - i - 1) \wedge \text{Select}(i - i', K - i - 1, x, y)\}$ $\vee$ $\{(x = i - i') \wedge (y \leq u(i, j))\} \wedge \{((w < u(i, j) - y) \wedge (z = j - j'))$ $\vee ((w \geq u(i, j) - y) \wedge (z \geq j - j') \wedge \text{Select}(j - j', u(i, j) - y, z, w))]\}$	$(i, 0)$, $i \geq i'$

TABLE II
TRANSITION PROBABILITIES FOR A MATCH PROCESS

Note that the above steady state probability vector can be easily found by using standard numerical methods, e.g., power method.

C. Extending to Multi-Groups

Extending the above Markov chain to handle a BT-like system with more than two groups of peers can be prohibitive since the state space of the Markov chain will be large. To resolve this problem, we consider the following simplifications. Consider a BT-like system with $n > 2$ group of peers, we assume the GS algorithm operates as:

1) For a peer v in group G_1, besides the bi-directional connections to peers in G_1, v's regular unchokes are randomly distribute in G_k with $1 < k \leq n$, if there are enough receivers in such groups.
2) For a peer v in group G_k, with $1 < k \leq n$, besides v's bi-directional connections reciprocated to peers in groups G_l, where $1 \leq l \leq k$, v's regular unchokes are randomly distribute in G_m with $k < m \leq n$ if there are enough receivers in such groups.

With the above assumptions, we construct a discrete time Markov process $\mathcal{M}$ with the state space S_R for group G_k:

$$S_R = \{i | 0 \leq i \leq K - 1\},$$

where i is the number of *matched connections*, i.e., the bi-directional connections between peers of the same group. We also define the probability vector $\boldsymbol{\pi}^{(k)} = \{\pi_0^{(k)}, \ldots, \pi_{K-1}^{(k)}\}$, where $\pi_i^{(k)}$ represents the fraction of G_k peers holding i matched connections with $i \in S_R$. We can now express the clustering index for peers in G_k as:

$$C_k = \frac{1}{K-1} \sum_{l=1}^{K-1} l \pi_l^{(k)}. \tag{6}$$

Consider a G_k peer, the optimistic and regular unchokes from a peer in group G_l, where $1 \leq l < k$, have a higher priority than the unchokes from a G_k peer. We define O_k (R_k) as the set of such optimistic (regular) unchokes of group G_k. Obviously, we have $|O_k| = \sum_{l=1}^{k-1} N_l$. We can determine the cardinality of R_k as:

$$|R_k| = \min\left\{ N_k \sum_{l=1}^{k-1} \left[\frac{N_l (K-1)(1 - C_l) - |R_l|}{\sum_{m=l}^{n} N_m} \right], (K-1) N_k \right\},$$

where $|R_1| = 0$ as group G_1 has the highest bandwidth. Consider a G_k peer, because regular unchokes in R_k are uniformly distributed, so each of the connection has the probability of $p = |R_k| / (N_k \cdot (K-1))$ to respond to regular unchokes from a peer of high index group.

Let us consider group G_k for $1 \leq k \leq n$. Similar to the Markov model with two groups, there are three processes that cause the state transition. We describe them as follows.

Cut process: in this process bi-directional connections are terminated. Similar to the derivation of the previous Markov chain, the probability that a a state i peer terminates a bi-direction connection of G_k is:

$$\delta_i^{(k)} = \sum_{x=0}^{K-i-1} \sum_{y=u(x,i)}^{|O_k|} \min\left\{ \frac{y - u(x,i)}{i}, 1 \right\} \times B(K-i-1, p, x) B(|O_k|, \frac{1}{N}, y), \tag{7}$$

where $u(x, i) = K - x - i - 1$. Assuming that the bi-directional connections are uniformly distributed among G_k peers, we have,

$$\gamma^{(1)} = \text{input value}, \gamma^{(k)} = \frac{\sum_{i=0}^{K-1} i \pi_i^{(k)} \delta_i^{(k)}}{\sum_{i=0}^{K-1} i \pi_i^{(k)}} \text{ , for } k > 1. \tag{8}$$

Finally, the transition probability matrix $\boldsymbol{Q}_C^{(k)}$ has the following transition probability with $i, i' \in S_R$

$$\text{Prob}\{i | i'\} = B(i', \gamma^{(k)}, i' - i) 1_{\{i \leq i'\}}. \tag{9}$$

Search process: In this process, a peer randomly performs optimistic unchoke to search for available peers within the same group. Let $\beta^{(k)}$, for $1 \leq k \leq n$, be the probability that the optimistic unchoke finds an available peer and gets the reciprocated upload connection. We have:

$$\beta^{(k)} = \frac{N_k}{N} \sum_{i=0}^{K-1} \pi_i^{(k)} \alpha_i^{(k)}. \tag{10}$$

where,

$$\alpha_i^{(k)} = \sum_{x=0}^{K-i-1} \sum_{y=0}^{u(x,i)} \sum_{z=0}^{N_k} \min\left\{ \frac{u(x,i) - y}{z+1}, 1 \right\} B(K-i-1, p, x) \times B(|O_k|, \frac{1}{N_k}, y) B(N_k, \frac{1}{N}, z).$$

Therefore, the transition probability matrix $\boldsymbol{Q}_S^{(k)}$ has the following probability with $i \in S_R$:

$$\text{Prob}\{i|i\} = 1-\beta^{(k)} 1_{\{i \leq K-1\}}, \text{Prob}\{i+1|i\} = \beta^{(k)} 1_{\{i<K-1\}}.$$

Match process: in this process, a peer responds to the new optimistic unchokes from peers of the same group. Let us consider a group G_k with $1 \leq k \leq n$. Define (x, y, z) for a state i peer, where $x \in \{0, .., K-i-1\}$ is the number of bi-directional connection between G_k and G_l with $1 \leq l < k$; while $y \in \{0, .., |O_k|\}$ and $z \in \{0, .., N_k\}$ are the number of optimistic unchokes from G_l $(1 \leq l < k)$ and G_k respectively. Given the initial state of a peer, the state transition is determined by these three variables. Table III depicts the transition probability matrix $\boldsymbol{Q}_M^{(k)}$.

Transition	Condition
$p\{0\|0\}$	$\{(x \leq K-1) \wedge (y \geq u(0,x))\}$ $\vee$ $\{(x+y < K-1) \wedge (z=0)\}$
$p\{0\|i', i' > 0\}$	$(x \leq u(0,i')) \wedge (y \geq u(0,x))$
$p\{i\|i', i' > i > 0\}$	$(x \leq u(0,i')) \wedge (y = u(x,i))$
$p\{i\|i'\}$, $i > i' \vee i = i' > 0$	$\{(x+y < u(0,i)) \wedge (z = i-i')\}$ $\vee$ $\{(x+y = u(0,i)) \wedge (z \geq i-i')\}$

TABLE III
TRANSITION PROBABILITIES OF MATCH PROCESS WITH INITIAL STATE i'

Similar to the first Markov chain, we obtain the steady state vector $\boldsymbol{\pi}^{(k)}$ using Equation 5.

IV. Performance Evaluation and Validation

To validate our mathematical models of determing the clustering index of BT-like systems, we develop a discrete event simulator to perform peer selection and the related choking algorithm. Since we are interested in the clustering index of the GS algorithm, we implement this peer selection only and each peers will have some chunks that are of interested by other peers. All peers will arrive to the system at time $t = 0$ and they stay in the system forever to exchange chunk.

For our discrete event simulator, we set $N = 1000$. The other input parameters to our simulator are:

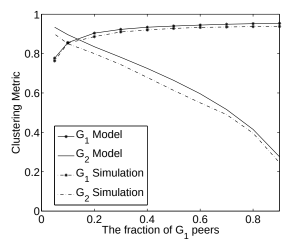
1) n, the number of groups,
2) the number of peers in each group G_k, $1 \leq k \leq n$,
3) $K = R + P$, R and P is the number of regular and optimistic unchokes for any peer,
4) ω, the number of time slots for each peer to perform the optimistic unchoke operation, where $\omega \in \{1, 2, \ldots\}$,
5) $\gamma^{1,k}$ for $1 \leq k \leq n$, the probability of terminating a bi-direction connection of G_1 by a peer in group G_k.

In our experiments, we first validate the accuracy of our mathematical models in estimating the clustering index, both for a BT-like system with two groups of peers and for systems with more than two groups of peers. We also carry out experiments to illustrate how one can change the clustering index of a system by varying the controllable system parameters, namely, R, P and ω. This is important since for some applications, e.g., P2P multimedia streaming, we may want to have a lower clustering index so that more resourceful peers may help other less resourceful peers in obtaining a satisfiable viewing service. It is important to point out that we set $R = 4, P = 1$ in the following experiments, unless we state otherwise.

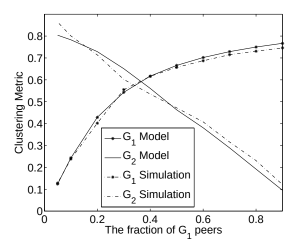
Experiment 1 (Validating the Two-Groups model): In this experiment, we consider a BT-like system with two different types of peers (or two groups). we set the input parameter $n = 2$ and $\omega = 1$. Therefore, each peer will perform the optimistic unchoke at that end of every round. In Table IV, we present our mathematical prediction of clustering index, the simulation results of clustering index and the corresponding errors when $\gamma^{1,1} = 0.01$. As we can observe, our model is very accurate in predicting the clustering indexes for both peer groups. Also Figure 1 illustrates the clustering index for both groups when we set $\gamma^{1,1}$ is equal to 0.01 or 0.2. One can conclude that our mathematical model can accurately predict the clustering index of each group in this scenario.

Fraction of	Model		Simulation		Error	
G_1	G_1	G_2	G_1	G_2	G_1	G_2
0.9	0.954	0.276	0.938	0.246	1.7%	12.1%
0.7	0.949	0.516	0.935	0.490	1.5%	5.2%
0.5	0.940	0.664	0.928	0.613	1.3%	8.3%
0.3	0.923	0.781	0.910	0.744	1.5%	4.9%
0.1	0.856	0.896	0.853	0.850	0.3%	5.3%

TABLE IV
COMPARING THE NUMERICAL & SIMULATION RESULTS OF CLUSTERING INDEX, FOR $n = 2$, $\gamma^{1,k} = 0.01$, k = {1,2}.



(a) $\gamma^{1,1} = \gamma^{1,2} = 0.01$

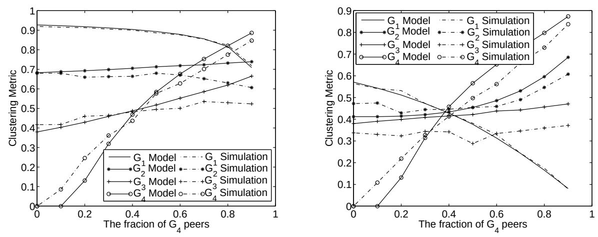


(b) $\gamma^{1,1} = \gamma^{1,2} = 0.2$

Fig. 1. Numerical solution vs. simulation result when $n = 2$

Experiment 2 (Validating the Multi-Groups model): In this experiment, we verify our mathematical models for multi-groups BT-like system. We consider the system has $n = 4$ different types of peers (or G_1 to G_4). Figure 2 illustrates the estimation from our mathematical model and the simulation results by varying the fraction of G_4 peers in the system. In this experiment, once we set the fraction of G_4 peers, peers of the other groups are uniformly distributed. In other words, $N_k = \frac{N-N_4}{3}$ for $k = 1, 2, 3$. We also set $\omega = 1$. From Figure 2, one can observe that the estimation of G_1 is accurate. While there is a small difference between the mathematical prediction of G_k, for $k = 2, 3, 4$, with that of the simulation result. This is due to our simplification assumption that the bi-directional connections from G_k to G_l are uniformly distributed for

$k < l$. Nevertheless, our mathematical is still quite accurate in estimating the clustering index of other groups.



(a) $\gamma^{1,k} = 0.01, k = \{1, .., 4\}$ (b) $\gamma^{1,k} = 0.2, k = \{1, .., 4\}$

Fig. 2. Numerical solutions vs. simulation results when $n = 4$

Experiment 3 (Varying the clustering index via ω): In this experiment, we consider how one can varying the clustering index by changing the value of ω. In Fig. 3, we vary the parameter ω to $1, 2, 4, 8$. One can observe that the clustering index of G_1 decrease when the ω is larger. Since ω is increased, it will take G_1 peers longer time to find a more resourceful peer via optimistic unchoke. On the other hand, the clustering index of G_2 persists as the average number of optimistic unchoke from G_1 remains relatively unchanged, independent on the values of ω.

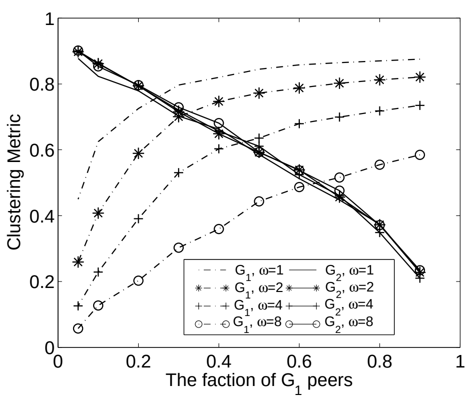


Fig. 3. varying the ω, $n = 2$, $\gamma^{1,1} = \gamma^{1,2} = 0.05$.

Experiment 4 (Varying the clustering index via R and P): In this experiment, we consider the effect of parameter R and P, i.e. the number of regular and optimistic unchokes. We set $\omega = 1$. In Fig. 4, we fix $P = 1$ and vary the number of regular unchokes R. We show that the clustering index of G_2 increases when R becomes larger. We know that a high clustering index of G_1 only gives optimistic unchokes to G_2 which is regardless of reciprocation. So the fraction of optimistic unchokes from G_1, which have a higher priority, becomes smaller compares to the regular unchokes when R is larger. Therefore, more G_2 regular unchokes are responding to their compeers, instead of responding to the optimistic unchoke from G_1. In Fig. 5, we fix $R = 4$ and vary the number of optimistic unchokes P to $1, 2, 4$. The clustering index of G_2 decrease as more optimistic unchokes from the peers in G_1 will select the peers in G_2.

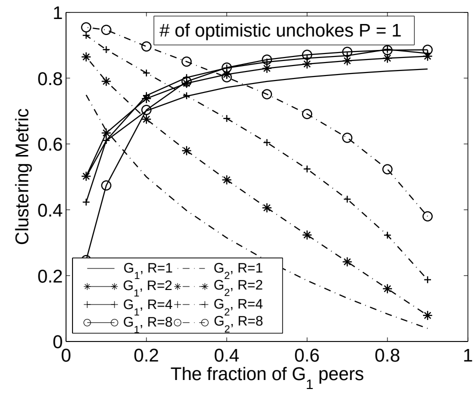


Fig. 4. Varying the R, $n = 2$, $\omega = 1$, $\gamma^{1,1} = \gamma^{1,2} = 0.05$.

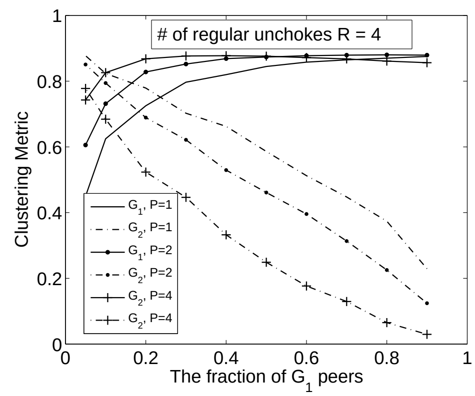


Fig. 5. varying the P, $n = 2$, $\omega = 1$, $\gamma^{1,1} = \gamma^{1,2} = 0.05$.

V. Conclusion

In this paper, we illustrate that BT protocol will generate a high clustering index system, i.e., peers will connect with other peers which are of similar bandwidth type. We provide two analytical models to accurate evaluate the clustering index of a BT-like system. The models are validated by extensive simulation. We also introduce design knobs to control clustering index so that BT-like streaming systems will have much better performance.

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