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On the credit evolution of credit-based incentive protocols in wireless mesh networks

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A B S T R A C T

In designing wireless mesh networks (WMNs), incentive mechanisms are often needed so to encourage nodes to relay or forward packets for other nodes. However, there is a lack of fundamental understanding on the *interactions* between the incentive mechanisms and the underlying protocols (e.g., shortest-path routing, ETX routing or back-pressure scheduling), and whether integration of these protocols will lead to a robust network, *i.e.*, networks can sustain a given traffic workload. The objective of this paper is to present a *general mathematical framework* via stochastic difference equations to model the interaction of incentive mechanisms and various underlying protocols. We first present a credit evolution model to quantify the expected credit variation of each node in WMN, then use the *norm of the expected credits variation* to quantify the credit disparity. We also propose the use of *differentiated pricing* and show how it can achieve credit equality among nodes, resulting in a more robust network under different traffic loading. Our analytical framework can help researchers to model other incentive/routing protocols so to analyze the robustness of the underlying networks.

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1. Introduction

In the past few years, wireless mesh networks (WMNs) have been a topic of intense research study. A WMN consists of a set of wireless and mobile nodes that can self-configure and do not necessarily need an infrastructure to communicate. A source node communicates with a distant destination node using intermediate nodes as relays. When nodes in a WMN are not under the control of a single authority, *cooperation* among nodes are paramount so packets can be relayed to the destination. Since selfish users only want to maximize their own welfare (e.g., monopolize the bandwidth or reduce their energy usage), therefore they may refuse to relay packets for other nodes. To overcome such problem, researchers propose different *incentive mechanisms* to encourage nodes to cooperate. These mechanisms can be broadly divided in two types: (a) reputation-based schemes [1,2] and (b) credit-based schemes [3–6]. In reputation-based schemes, a node's reputation is measured by its neighbors, and selfishness is deterred by the threat of partial or total disconnection from the network. Due to packet collisions and interference, nodes cannot always reliably detect if a given node actually forwarded a packet or not, so it is possible that cooperative nodes will be perceived as being selfish, and wrongfully trigger a retaliation by their neighbors. In credit-based schemes, nodes receive a payment every time they forward a packet, and credit can be used by these nodes to transmit their own packets. Compared with the reputation schemes, credits enable more flexible and fine-grained control.

Although credit-based schemes can encourage cooperation among nodes, it does not imply that the WMN can *sustain* all traffic workloads. To illustrate, let us consider a WMN in Fig. 1.1. Let α be the payment from the source

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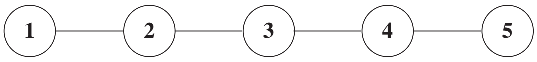


Fig. 1.1. An example of a linear WMN to illustrate credit imbalance.

node to relaying nodes for forwarding a message. When α is small, nodes have no incentive to forward packets. If α is sufficient to cover the cost of forwarding messages, nodes have incentive to forward messages. In this figure, node 3 is in the center of the network and it costs less credit to send data to other nodes (as compare to node 1 sending message to node 5). Furthermore, when each node has equal amount of traffic to send to all other four nodes in the network, then node 3 has more opportunities to earn credit since traffic of other nodes has to go through it. Node 1 or 5, on the other hand, has no chance to earn any credit because they are at the edge of this network. If α is not properly set, node 1, 2, 4, and 5 will eventually run out of credit and cannot transmit their own packets. Hence, even if an incentive mechanism can guarantee cooperation, it is possible that the WMN may not be able to *sustain* a given traffic workload in the long run because some nodes just run out of credit and they cannot transmit any packets.

The goal of this paper is *not* to advocate any incentive mechanism, but rather, our first contribution is in proposing a *general mathematical framework* to model and analyze the interaction of incentive protocols with the underlying routing protocols, and to reveal the *sustainability* of a WMN under different traffic workload. We quantitatively investigate the impact of credit inequality of credit-based incentive schemes in WMNs. We use Sprite [5] as an illustrative example of credit-based incentive protocol, and show how it interacts with a family of network protocols. We also consider the influence of link loss probability on the incentive scheme, and derive the closed-form expressions of the expected credit variation for various network topologies. Our second contribution is in proposing a *differentiated pricing* scheme to achieve *credit-equality*. Based on these results, our third contribution is in proposing the credit evolution model with link scheduling using the back-pressure algorithm to examine how the shared nature of wireless medium may effect the credit inequality and system performance.

This is the outline of this paper. In Section 2, we present the background of path-based routing protocols and the Sprite incentive mechanism. In Section 3, we present our mathematical framework to model the credit evolution. In Section 4, we introduce differentiated pricing. We present the credit evolution model with back-pressure algorithms in Section 5. Results on the performance evaluation are given in Section 6. Related work is given in Section 7 and conclusion is given in Section 8.

2. Background and technical preliminaries

We first provide the background of our work, then we formally state the mathematical model for WMNs, the traffic model, as well as a family of path-based routing protocols. To illustrate our framework, we consider the Sprite [5] incentive protocol. It is important to point out that our mathematical framework is general and can accommodate other routing and incentive protocols. In here, we simply use the path-based routing and Sprite to illustrate the utility of our analytical methodology.

- *Network model:* a WMN is modeled as a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{E}), \mathcal{N}$ is the set of wireless nodes with $|\mathcal{N}| = N$ and $\mathcal{E}$ is the set of wireless links. A link $(i,j) \in \mathcal{E}$ from node i to node j denotes that node j is within the transmission range of node i. If there is no link from i to j, then the message from node i has to be transmitted to another node, and this node has to forward the message, either directly or indirectly, to j. Let ϵ_{ij} be the loss probability of link (i, j). A message from i to j on (i, j) is correctly received with probability $1 - \epsilon_{ij}$.
  Denote r as a route, which is a sequence of ordered nodes or an non-empty subset of $\mathcal{E}$. Let $\mathcal{S}$ be the set of all traffic sources in the network. For a traffic source $s \in \mathcal{S}$, let $\mathcal{D}_s$ be the set of its destinations. For a given source s and its destination $d \in \mathcal{D}_s$, we define (s, d) as the flow between node s and d. Each flow has a set of routes, which is denoted by $\mathcal{R}_{sd}$ and $|\mathcal{R}_{sd}|$ represents the number of routes for flow (s, d). Let $A^r_{sd}(t)$ be the traffic for flow (s, d) on route r at time t, then the traffic of flow (s, d) at time t is $A_{sd}(t) = \sum_{r\in\mathcal{R}_{sd}} A^r_{sd}(t)$.
- *Traffic model:* the packet arrival process $\{A_s(t)\}_{t=1}^{\infty}$ are i.i.d. sequences of a random variable for all $s \in \mathcal{S}$. The arrival rate $E[A_s(t)]$ of source s is denote by λ_s, wherein the expected time between two consecutive arrivals is 1 (or a unit slot). The destination of a flow is chosen as follows: with probability h_{sd}, the packets generated by the source node will choose node d as the destination. Clearly, $\sum_{d\in\mathcal{D}_s} h_{sd} = 1$ for $s \in \mathcal{S}$.
- *Path-based routing:* we consider a family of path-based routing protocols, where the quality of a path can be chosen by (a) *hop count*, or (b) *expected transmission count* (ETX) [7]. Hop count assumes all links are homogeneous (*i.e.*, links have the same loss probability and channel fading characteristics). Therefore, for the flow (s, d), the *shortest-path hop-count routing* selects those paths with the smallest link distance between s and d. In ETX [7], it allows links to be heterogeneous. ETX minimizes the expected total number of packet transmissions required to successfully deliver a packet to the destination. ETX of a link is based on the delivery ratio: number of transmissions needed to successfully deliver a single packet. ETX of a path is the sum of the ETX for each link in that path. For the ease of mathematical presentation, we only consider use the forward delivery ratio to calculate these ETX values.

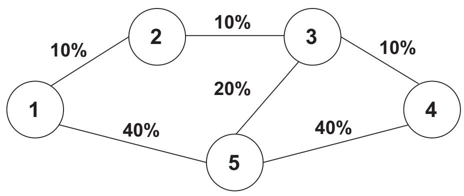


Fig. 2.1. Example of a WMN with link loss probabilities.

Consider the WMN in Fig. 2.1 as an example, where the link loss probability are shown along the edge. Consider the flow (1, 4). Under the shortest-path hop-count routing, there is only one shortest path (1, 5, 4). Under the shortest-path ETX routing, there are two shortest paths (1, 2, 3, 4) and (1, 5, 4).

When a flow (s, d) has multiple shortest paths, or $|\mathcal{R}_{sd}| > 1$, we use a traffic splitting method to deliver packets. Consider again the flow (1,4) in Fig. 2.1. Under the shortest-path ETX routing, there are two shortest paths r = (1, 2, 3, 4) and r' = (1, 5, 4). To simplify our presentation, we assume *uniform traffic splitting* in our analysis, or $A^r_{14}(t) = A_{14}(t)/|\mathcal{R}_{14}| = A_{14}(t)/2$. Again, we like to stress that our mathematical framework can accommodate more general splitting rule.

- *Incentive scheme*: Sprite [5] is a well-known incentive scheme to encourage nodes to collaborate. It has an elegant *cheat-proof* property and does not require any tamper-proof hardware. Briefly speaking, Sprite uses a centralize credit clearance service (CCS), to collect receipts from each forwarding node. Charges and rewards are based on these receipts, which provides incentive for nodes to cooperate and report actions honestly. Formally, Sprite can be described as follows:
  - A sender selects a path to delivery a message to the destination. Denote this path as $r = (s, n_1, \ldots, n_e, \ldots, n_{l_r})$, where $n_{l_r} = d$ (destination node), hop count of r is l_r.
  - The sender will be charged for transmitting the message.
  - CCS believes that a node along r has forwarded the message if and only if there is a successor of that node on the path reporting a valid receipt of that message.
  - CCS charges C^r from node s, and pays $P^r_{n_k}$ to node n_k:

$$C^r = (l_r - 1)\alpha + \beta - (l_r - e)\gamma\beta, \tag{2.1}$$

$$P^r_{n_k} = \begin{cases} \alpha, & \text{if } k < e = l_r, \\ \beta, & \text{if } k = e = l_r, \\ \gamma\alpha, & \text{if } k < e < l_r, \\ \gamma\beta, & \text{if } k = e < l_r, \end{cases} \tag{2.2}$$

with $\gamma < 1$ and $\beta < \alpha$. In here n_e is the last node on path r that submits a valid receipt.

Let us illustrate this with the network in Fig. 2.1. Assume that node 1 is to send a message to node 4 along the path r = (1, 2, 3, 4), where $l_r = 3$. When node 3 is the last node on path r that submits a valid receipt, then $e = 2, C^r = 2\alpha + \beta - \gamma\beta$, $P^r_2 = \gamma\alpha$, $P^r_3 = \gamma\beta$ and $P^r_4 = 0$. When node 4 is the last node on path r that submits a valid receipt, *i.e.* the message is send successfully, then $e = 3, C^r = 2\alpha + \beta$, $P^r_2 = \alpha, P^r_3 = \alpha$ and $P^r_4 = \beta$.

Let δ be the cost of forwarding a receipt from one node to another. Then a colluding node (if any) incurs a cost of δ and n_k must compensate the colluding node with δ. We state some definitions and property of the Sprite system [5] here.

Definition 1. For a player, an optimal strategy is a strategy that brings the maximum expected welfare to it, regardless of the strategies of all the other nodes.

Definition 2. A game is collusion-resistant, if any group of colluding players cannot increase the expected sum of their welfare by using any strategy profile other than that in which everybody tells the truth.

Definition 3. A game is cheat-proof if truth-telling is an optimal strategy for all nodes and the game is collusion-resistant.

Theorem 1. *For the receipt-submission game of Sprite, if* $\delta \geqslant \gamma\beta$ *and* $\delta \geqslant (l_r - 1)\gamma\alpha$*, then it is cheat-proof.*

Proof. please refer to [5] for complete derivation. □

- *Credit inequality metrics:* in economics, income is the sum of all forms of earnings received in a given period of time, and demand is also known as personal consumption or expenditure. In this work, we regard message forwarding as a service a node provides to others, so the traffic model and the routing scheme determine the income and demand of each node, where *income* means the payments received from CCS for providing forwarding service for other nodes, while *demand* means the payment to CCS for sending messages to others nodes. The *credit balance* of a node is the difference between income and demand of that node. Hence, a node can have a positive or negative credit balance, and this is influenced by the underlying routing policy, the traffic workload and network topology. In this paper, we use the *expected credit variation* of a node in each time slot to quantity its credit inequality. Here, the expected credit variation is the average credit variation of a node during a time slot. If it is positive, then that node will have surplus in credit; if it is negative, then that node will have deficit in credit. If each node's expected credit variation is zero, this means each node (and also the whole network) is *budget balanced* so each node can provide service for others in equilibrium.
- *Back-pressure algorithm:* resource allocation in wireless networks is complicated due to the shared nature of wireless medium. One particular allocation algorithm is called the *back-pressure algorithm* which encompasses several layers of the protocol stack from MAC to routing. It was proposed by Tassiulas and Ephremides in their seminal paper [8]. The back-pressure algorithm was shown to be *throughput-optimal*, *i.e.*, it can support any arrival rate vector which is supportable by any other resource allocation algorithm.

3. Mathematical model for credit evolution

In here, we present the mathematical model to capture the *credit evolution*, *i.e.*, the dynamic change of credit, for

Table 1
Summary of main notation.

Symbol	Meaning
t	Time index
$\mathcal{N}, N$	Node set and number of nodes
ϵ_{ij}	Loss probability of link (i, j)
$\mathcal{R}_{sd}$	Routes for flow (s, d)
$\mathcal{R}_{sd}^n$	Routes for flow (s, d) crossing node n
$A_{sd}(t)$	Traffic of flow (s, d) at time t
$C_n(t)$	Credit of node n at time t
Δc_n	Expected credit variation of node n
f_{mn}	Expected traffic of node m forwarded by node n
$\{P_{ij}(t)\}_{t=1}^{\infty}$	Transmission process of link (i, j)
$Q_n^r(t)$	Queue length of flow r at node n at time t
$\alpha, \beta, \gamma, \delta$	Sprite parameters
$\alpha_n, \underline{\alpha}, \bar{\alpha}$	Heterogeneous pricing parameters

each node in a WMN. Main notations are summarized in Table 1. We model the evolution of nodes' credit as a sequence of random variables, and for some network topologies, we are able to obtain the closed form expression for the expected credit variation. We consider two scenarios:

(1) *persistent transmission mode*: when a collision or packet lost occurs, a node will retransmit a packet until the packet is successfully received by its neighbor.
(2) *single transmission mode*: when a collision or packet lost occurs, a node will *not* retransmit. The source node is responsible to retransmit the packet until the destination node receives the packet. Since source nodes need to perform retransmission, this can be regarded as the upper bound (or the worst case) payment for source nodes under Sprite [5].

For simplicity of presentation, we make the following assumptions: (a) there is no cheating and no collusion for message-forwarding and receipt-submission (as guarantee by Sprite), and (b) the link capacity is sufficient to satisfy all demands.

3.1. General mathematical model

Consider a node, say n. We use $\mathcal{R}_{sd}^n = \{r \in \mathcal{R}_{sd} : n \in r\}$ to denote the route set (or path set) that node n is an intermediate node of some route r for flow (s, d). Let C^r be the cost of source node s and P_n^r be the payment to node $n \in r$ for successfully transmitting a packet along the path r to d. Let $C_n(0)$ be the initial credit of node n at time $t = 0$. At time t, node n has credit

$$C_n(t) = C_n(t-1) + \Delta C_n(t), \quad t \geqslant 1, \; n \in \mathcal{N}, \tag{3.1}$$

where

$$\Delta C_n(t) = \sum_{s \in \mathcal{S}, r \in \mathcal{R}_{sd_s}^n} A_{sd_s}^r(t) P_n^r + \sum_{s \in \mathcal{S}} h_{sn} A_s(t) \beta - \sum_{r \in \mathcal{R}_{nd_n}} A_{nd_n}^r(t) C^r \tag{3.2}$$

is the credit variation of node n in time slot t. In (3.2), the first and the second terms represent the payments from CCS to node n for forwarding messages and for reporting receipts for the message respectively. The third term represents the payment from node n to CCS for sending its messages to other nodes.

3.1.1. Persistent transmission mode

Under the *persistent transmission mode*, each intermediate node in the path would forward and/or retransmit an arbitrary number of times until a successful packet transmission occurs. In this case, we have $P_n^r \equiv \alpha$ for all $n \in r$ and $C^r = (l_r - 1)\alpha + \beta$ for each route r. Substituting them into Eq. (3.2), we have

$$\Delta C_n(t) = B_n(t)\beta + \sum_{m \neq n} F_{mn}(t)\alpha - \sum_{m \neq n} F_{nm}(t)\alpha, \tag{3.3}$$

where $B_n(t)$ is the difference between the traffic ending at node n and the one starting from node n, $F_{nm}(t)$ is the traffic that node n forwards for node m at time slot t, *i.e.*,

$$B_n(t) = \sum_{s \in \mathcal{S}} h_{sn} A_s(t) - A_n(t), \tag{3.4a}$$

$$F_{mn}(t) = \sum_{r \in \mathcal{R}_{md_m}^n} A_{md_m}^r(t). \tag{3.4b}$$

In (3.3), $B_n(t)$ describes the instantaneous increase or decrease in credit, due to sending message to or receiving message from other nodes, while the terms involving the (asymmetric) matrix $F_{mn}(t)$ describe the amount of credit that node n spends in acquiring the forwarding service of node m (and vice versa). Under the persistent transmission mode, the credit of CCS remains *unchanged*, *i.e.*, $C_0(t) \equiv C_0(0)$, $t \geqslant 0$, which means that we have *credit conservation* in CCS, *i.e.*, $\sum_{n \in \mathcal{N}} \Delta C_n(t) = 0$ for $t \geqslant 1$.

Given the assumptions we made about the traffic model, $\{B_n(t)\}_{t=1}^{\infty}$ and $\{F_{nm}(t)\}_{t=1}^{\infty}$ are i.i.d. sequences of random variables for all $n, m \in \mathcal{N}$ and $n \neq m$. By (3.3), the *expected credit variation* of node n is

$$\Delta c_n := \mathrm{E}[\Delta C_n(t)] = b_n\beta + \sum_{m \neq n} f_{mn}\alpha - \sum_{m \neq n} f_{nm}\alpha, \tag{3.5}$$

where $b_n = \mathrm{E}[B_n(t)]$, $f_{mn} = \mathrm{E}[F_{mn}(t)]$. Furthermore, by Eq. (3.1), the expected credit balance of node n is

$$\mathrm{E}[C_n(t)] = C_n(0) + t\Delta c_n, \quad t \geqslant 1, \; n \in \mathcal{N}.$$

3.1.2. Single transmission mode

Under this mode, intermediate nodes along the path will forward the packet *only once*. In this case, P_n^r and C^r in Eq. (3.2) are random variables.

Let $r = (s, n_1, \ldots, n_l)$ be a path for flow (s, d), where $n_l = d$ and we omit the subscript r from l_r. Let A_i denote the outcome that the message arrives at node n_i but fails to arrive at node n_{i+1}, $i = 1, \ldots, l$. Denote the probability that event A_i happens by p_i. The corresponding cost of node s and payment to intermediate node n_k for outcome A_i are denoted by c_i and $p_i^{n_k}$ for k respectively, $i = 1, \ldots, l$. Note that if the packet is not sent to node n_1, the CCS will not make an effective record. So one can assume the source node s always makes a successful transmission when we calculate the cost and payments. We have the following result.

Theorem 2. *It holds that*

(*i*) *the expected cost that node s successfully sends a message along path r to node d is* $\bar{c}/p_l$, *where* $\bar{c} = c_1p_1 + \cdots + c_lp_l$, *and*
(*ii*) *the expected payment to node* n_k *that node s successfully sends a message along path r to node d is* $\bar{p}_{n_k}/p_l$, $k = 1, \ldots, l$, *where* $\bar{p}_{n_k} = p_1^{n_k}p_1 + \cdots + p_l^{n_k}p_l$.

Proof. For the payoff scheme of Sprite (2.1) and (2.2), we regard node s sending a message along path r to node d as a trial. Then each trial results in exactly one of $A_1,\ldots,A_l$ of possible outcomes with probability p_i, $i = 1,\ldots,l$. We repeatedly perform independent but identical trials until the first outcome A_l was observed, *i.e.* the message is successfully received by node d. Denote the number of trials to get an outcome A_l by Y, where $Y = 1,2,\ldots$ By the basic property of conditional expectation [9], we have

$$\mathrm{E}[C^r] = \sum_{y=1}^{\infty}\Pr(Y = y)\mathrm{E}[C^r|Y = y]. \tag{3.6}$$

It is a Bernoulli trial that node s sends out a message to d along path r, where A_l indicates a "successful transmission" and everything else indicates a "failure". Then Y follows a geometric distribution with parameter p_l and

$$\Pr(Y = y) = (1 - p_l)^{y-1}p_l, \quad y = 1, 2, \ldots. \tag{3.7}$$

If $y = 1$, then $C^r = c_l$. Given $y = 2,3,\ldots$, let the random variable X_i be the number of times that outcome A_i was observed over the first $y - 1$ trials. The vector $\mathbf{X} = (X_1,\ldots,X_{l-1})$ follows a multinomial distribution with parameters $y - 1$ and $\mathbf{p}$, where $\mathbf{p} = \left(\frac{p_1}{p_1+\cdots+p_{l-1}}, \ldots, \frac{p_{l-1}}{p_1+\cdots+p_{l-1}}\right)$. It can be verified that

$$C^r = c_1X_1 + \cdots + c_{l-1}X_{l-1} + c_l.$$

So $\mathrm{E}[C^r|Y = y] = c_1\mathrm{E}[X_1] + \cdots + c_{l-1}\mathrm{E}[X_{l-1}] + c_l$. Since $\mathrm{E}[X_i] = \frac{(y-1)p_i}{p_1+\cdots+p_{l-1}}$, $i = 1, \ldots, l-1$ [23] and $p_1 + \cdots + p_l = 1$, we have

$$\mathrm{E}[C^r|Y = y] = \frac{y-1}{1-p_l}(c_1p_1 + \cdots + c_{l-1}p_{l-1}) + c_l. \tag{3.8}$$

Substituting Eqs. (3.7) and (3.8) into (3.6), we obtain $\mathrm{E}[C^r] = (c_1p_1 + \cdots + p_{l-1}c_{l-1})/p_l + c_l$. So item (i) holds. Similarly we can get the results in item (ii). □

Here we can further provide a closed-form expressions for $\mathrm{E}[C^r]$ and $\mathrm{E}\left[P^r_{n_k}\right]$, $k = 1, \ldots, l$. By the payoff scheme of Sprite (2.1) and (2.2), we have $c_i = (l - 1)\alpha + \beta - (l - i)\gamma\beta$ and

$$p_i^{n_k} = \begin{cases} \gamma\alpha, & \text{if } k < i, \\ \gamma\beta, & \text{if } k = i, \\ 0, & \text{if } k > i. \end{cases}$$

Substituting them into the results in Theorem 2, we have

$$\mathrm{E}[C^r] = \left(c_l - \gamma\beta\sum_{i=1}^{l-1}(l-i)p_i\right)\Big/p_l,$$
$$\mathrm{E}\left[P^r_d\right] = \beta,$$
$$\mathrm{E}\left[P^r_{n_k}\right] = \alpha + \gamma\left(p_k\beta + \sum_{i=k+1}^{l-1}p_i\alpha\right)\Big/p_l, \quad k = 1, \ldots, l-1,$$

where $p_i = \prod_{j=1}^{i-1}(1 - \epsilon_{n_jn_{j+1}})\epsilon_{n_in_{i+1}}$ for $i = 1,\ldots,l-1$, and $p_l = \prod_{j=1}^{l-1}(1 - \epsilon_{n_jn_{j+1}})$.

Remark. Let us provide a physical interpretation of Theorem 2. $1/p_l$ is the expected number of transmissions that node s makes for a message it sends along path r to d, $\bar{c}$ is the expected cost that s sends a message to d and $\bar{p}_{n_k}$ is the expected payments to node n_k, $k = 1,\ldots,l$. Therefore, the expected cost for s to successfully send a message to d is the product of the expected number of transmissions and the expected cost according to item (i) in Theorem 2. The similar results also hold for the payments to nodes $n_1, \ldots, n_l$.

To illustrate, consider the flow (1, 4) in Fig. 2.1, where node 1 sends a message along path $r = (1, 2, 3, 4)$ to node 4. We have $\mathrm{E}[C^r] = (100(2\alpha + \beta) - 29\gamma\beta)/81$, $\mathrm{E}\left[P^r_2\right] = (81\alpha + 9\gamma\alpha + 10\gamma\beta)/81$, $\mathrm{E}\left[P^r_3\right] = (9\alpha + \gamma\beta)/9$ and $\mathrm{E}\left[P^r_4\right] = \beta$.

3.2. Expected credit variation

Now consider the persistent transmission mode. By (3.4a), the expected traffic starting from n is λ_n and the one ending at n is $\sum_{s\in\mathcal{S}}h_{sn}\lambda_s$. So we have

$$b_n = \sum_{s\in\mathcal{S}}\lambda_sh_{sn} - \lambda_n. \tag{3.9}$$

The key problem is to determine f_{sn}, which is the expected traffic that node n forwards for s, $s \neq n$. By (3.4b) and the uniform traffic splitting rule, when flow (s, d) has multiple shortest paths, we have

$$F_{sn}(t) = \sum_{r\in\mathcal{R}^n_{sd_s}}A^r_{sd_s}(t) = \frac{|\mathcal{R}^n_{sd_s}|}{|\mathcal{R}_{sd_s}|}A_s(t),$$

where d_s is the random variable with $\Pr(d_s = d) = h_{sd}$ for all $d \in \mathcal{D}_s$. By the basic property of conditional expectation [9], we have

$$f_{sn} = \mathrm{E}[\mathrm{E}[F_{sn}(t)|d_s]] = \sum_{d\in\mathcal{D}_s}\lambda_sh_{sd}\frac{|\mathcal{R}^n_{sd}|}{|\mathcal{R}_{sd}|}. \tag{3.10}$$

To illustrate our analytical framework, we consider several topologies with the shortest hop-count routing, and derive the "*closed form expression*" on the expected credit variation.

3.2.1. Linear topology

Let us consider the linear network in Fig. 3.1. There is only *one* shortest path for each flow (s, d). So $|\mathcal{R}_{sd}| = 1$ for all $s \in \mathcal{S}$ and $d \in \mathcal{D}_s$. Both $|\mathcal{R}^1_{sd}|$ and $|\mathcal{R}^N_{sd}|$ are zero since node 1 and N do not lie in the routes of any flow.

For $1 < n < N$, node n only forwards traffic for the flows across it, *i.e.*, the flow's source and destination lie in the two sides of n. Then we have

$$|\mathcal{R}^n_{sd}| = \begin{cases} 0, & \text{if } \max(s,d) < n \text{ or } \min(s,d) > n, \\ 1, & \text{otherwise}. \end{cases}$$

Here $\max(s, d) < n$ or $\min(s, d) > n$ means the flow's source and destination locating at the same side of node n. Substituting the said facts into Eq. (3.10), we have

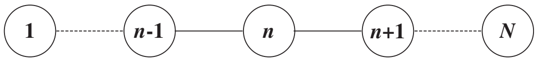


Fig. 3.1. Linear topology with N nodes.

$$f_{sn} = \begin{cases} \sum_{d \in \mathcal{D}_s, d > n} h_{sd}\lambda_s, & \text{if } s < n, \\ \sum_{d \in \mathcal{D}_s, d < n} h_{sd}\lambda_s, & \text{if } s > n. \end{cases} \tag{3.11}$$

3.2.2. Ring topology

Consider the ring WMN in Fig. 3.2. For $n \in \mathcal{N}$, we let $s = \text{mod}\ (n \mp k,\ N)$, $k = 1, 2, \ldots, \lfloor N/2 \rfloor$, where mod is the modulo operation and $\lfloor N/2 \rfloor$ means the largest integer not greater than $N/2$. In the sequel, we assume mod $(0, N) = \text{mod}\ (N, N) = N$ for convenience of formulation.

For all $n \in \mathcal{N}$, node n lies in the route of flow (s, d) only if the hop count between s and d is not greater than $\lfloor N/2 \rfloor$. Then we have

$$|\mathcal{R}_{sd}^n| = \begin{cases} 1, & \text{if } d = \text{mod}(n \pm l, N), \\ & \quad l = 1, \ldots, \lfloor N/2 \rfloor - k, \\ 0, & \text{otherwise.} \end{cases} \tag{3.12}$$

In the case that N is odd, each flow has only *one* shortest path. So $|\mathcal{R}_{sd}| = 1$ for all $s \in \mathcal{S}$ and $d \in \mathcal{D}_s$. Substituting this fact and (3.12) into (3.10), we have

$$f_{sn} = \sum_{l=1}^{\lfloor N/2 \rfloor - k} h_{s\ \text{mod}(n \pm l, N)} \lambda_s.$$

In the case that N is even, the flow $(s, \text{mod}(s + N/2, N))$ has *two* shortest paths for all s. Substituting this fact and (3.12) into (3.10), we have

$$f_{sn} = \left(\sum_{l=1}^{\frac{N}{2}-k-1} h_{s\ \text{mod}(n \pm l, N)} + \frac{h_{s\ \text{mod}\left(s + \frac{N}{2}, N\right)}}{2} \right) \lambda_s.$$

3.2.3. Grid topology

Consider the grid topology in Fig. 3.3, there are $N = HP$ nodes. To simplify notations, let $n = pH + h$, where $p = 0, 1, \ldots, P-1$, $h = 1, 2, \ldots, H$. For a given flow (s, d), let $s = p_s H + h_s$ and $d = p_d H + h_d$. To obtain the credit variation of node n, we need to know how much traffic it forwards for a given flow (s, d), which means the number of shortest path for flow (s, d).

Lemma 1. *The fact holds that*

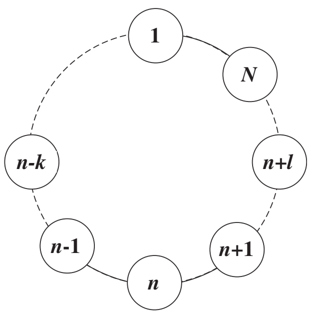


Fig. 3.2. Ring topology with N nodes.

$$|\mathcal{R}_{sd}| = \binom{|p_s - p_d| + |h_s - h_d|}{|p_s - p_d|}$$

for flow (s, d), where $\binom{n}{k}$ *is the binomial coefficient.*

Proof. The vertical and horizontal hop counts between node s and d are $|p_s - p_d|$ and $|h_s - h_d|$, respectively. Each shortest path for flow (s, d) maps to a way that $|p_s - p_d|$ perpendicular hops can be chosen from all $|p_s - p_d| + |h_s - h_d|$ hops. □

In the following derivation, let us simplify the notation and we let $k_1 = \binom{|p_s - p| + |h_s - h|}{|p_s - p|}$, $k_2 = \binom{|p - p_d| + |h - d_d|}{|p - p_d|}$, $k = \binom{|p_s - p_d| + |h_s - h_d|}{|p_s - p_d|}$ and use the notation $\binom{0}{0}$ to deNote 0.

Lemma 2. *If node n forwards traffic for flow (s, d), i.e. $\mathcal{R}_{sd}^n \neq \emptyset$, then $|\mathcal{R}_{sd}^n| = k_1 k_2$.*

Proof. We consider flows (s, n) and (n, d). By Lemma 1, there are k_1 and k_2 shortest paths for flows (s, n) and (n, d), respectively. Flow (s, d) has $k_1 k_2$ shortest paths through node n by *the multiplication principle.* □

To derive the closed-form of the expected credit variation of node n, we partition the other nodes into eight blocks with the dash line shown in Fig. 3.3. We have three cases to consider according to the relative locations of node s and n, respectively.

Case A: $p_s < p$. If $h_s < h$, *i.e.*, node s lies in the block I, we have $\mathcal{R}_{sd}^n \neq 0$ only when node d lies in blocks V, VII, or VIII. Substituting the results in Lemmas 1 and 2 into Eq. (3.10), we have

$$f_{sn} = k_1 \sum_{p_d = p}^{P-1} \sum_{h_d = h}^{H} h_{sd} \frac{k_2}{k} \lambda_s. \tag{3.13}$$

If $h_s = h$, *i.e.*, node s lies in the block II, we have $k_1 = 1$ and $\mathcal{R}_{sd}^n \neq 0$ only when node d does not lie in blocks I, II and

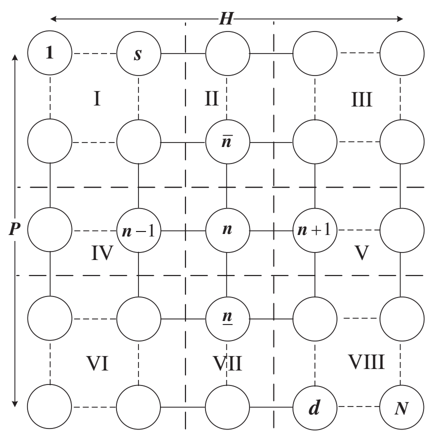


Fig. 3.3. Grid topology with $N = HP$ nodes.

VIII. So similar with Eq. (3.13), we have $f_{sn} = \sum_{p_d=p}^{P-1}\sum_{h_d=1}^{H} h_{sd}\frac{k_2}{k}\lambda_s$. If $h_s > h$, *i.e.*, node s lies in the block III, we have $f_{sn} = k_1\sum_{p_d=p}^{P-1}\sum_{h_d=1}^{h} h_{sd}\frac{k_2}{k}\lambda_s$.

Case B: $p_s = p$. Then $k_1 = 1$ and we have

$$f_{sn} = \begin{cases} \sum_{p_d=0}^{P-1}\sum_{h_d=h}^{H} h_{sd}\frac{k_2}{k}\lambda_s, & h_s < h, \\ \sum_{p_d=0}^{P-1}\sum_{h_d=1}^{h} h_{sd}\frac{k_2}{k}\lambda_s, & h_s > h. \end{cases}$$

Case C: $p_s > p$. Then we have

$$f_{sn} = \begin{cases} k_1\sum_{p_d=0}^{p}\sum_{h_d=h}^{H} h_{sd}\frac{k_2}{k}\lambda_s, & h_s < h, \\ \sum_{p_d=0}^{p}\sum_{h_d=1}^{H} h_{sd}\frac{k_2}{k}\lambda_s, & h_s = h, \\ k_1\sum_{p_d=0}^{p}\sum_{h_d=1}^{h} h_{sd}\frac{k_2}{k}\lambda_s, & h_s > h. \end{cases}$$

Substituting f_{sn} into Eq. (3.5), we obtain the desired result.

4. Balancing credit via heterogeneous pricing

Note that if a node has a lot of credit, this node has no incentive to forward packets for other nodes. On the other hand, when a node does not possess sufficient credit, it cannot transmit packet to its target receiver, so it needs to wait until accumulating sufficient credits. These are *undesirable* since they reduce collaboration and lower system throughput. We address this problem here by determining the *optimal incentive price* for each node so to ensure a *fair distribution of credits*.

4.1. Tradeoff between incentive and balancing

We introduce heterogeneous pricing by allowing different node to charge differently in forwarding a message. Under the Sprite mechanism, all nodes use the same price and we called this the *homogeneous pricing* (HomoPricing). We consider the *heterogeneous pricing* (HeteroPricing). Assume that node s sends a message to d along the path $r = (s, n_1, \ldots, n_{l_r})$, where $n_{l_r} = d$ and l_r is the hop count of path r. In HeteroPricing, the CCS charges C^r from node s, and pays $P^r_{n_k}$ to node n_k, where

$$C^r = \sum_{k=1}^{l_r-1}\alpha_{n_k} + \beta - (l_r - e)\gamma\beta,$$

$$P^r_{n_k} = \begin{cases} \alpha_{n_k}, & \text{if } k < e = l_r, \\ \beta, & \text{if } k = e = l_r, \\ \gamma\alpha_{n_k}, & \text{if } k < e < l_r, \\ \gamma\beta, & \text{if } k = e < l_r, \end{cases}$$

where n_e is the last node on path r that submits a valid receipt, $\gamma < 1$ and $\beta < \alpha_n$ for all n. HeteroPricing has the following property:

Theorem 3. *The receipt-submission game of HeteroPricing, if $\delta \geqslant \gamma\,\beta$ and $\delta \geqslant \gamma\sum_{k=1}^{l_r}\alpha_{n_k}$, is cheat-proof.*

Proof. The proof is similar to the derivation in [5]. □

Comparing the expected gain of credit from forwarding a message with that of not forwarding the message, an intermediate node m can expect a net gain of

$$p_2(1-\gamma)\alpha_m + p_1\gamma(\alpha_m - \beta), \tag{4.1}$$

where p_1 and p_2 are the probabilities that the message arrives at the next node and destination respectively. For the persistent transmission mode, similarly to (3.5) for HomoPricing, the expected credit variation of node n for HeteroPricing is

$$\Delta c_n = w_n + f_{nn}\alpha_n - \sum_{m\neq n} f_{nm}\alpha_m, \tag{4.2}$$

where $w_n = b_n\beta$ is independent of other nodes and

$$f_{nn} = \sum_{m\neq n} f_{mn}. \tag{4.3}$$

One can further decompose the expected credit variation into the *expected income* $w_n + f_{nn}\alpha_n$, and the *expected demand* $\sum_{m\neq n} f_{nm}\alpha_m$ in (4.2). The former depends on the forwarding traffic for others nodes and the price of node n, the later depends on the traffic sent by node n and the price of other nodes.

One needs to satisfy two desirable requirements: (1) to keep the expected credit variation (4.2) for each node as zero, *i.e.* $\Delta\mathbf{c} = (\Delta c_1, \ldots, \Delta c_N) = \mathbf{0}$, (2) to maximize the incentive to each node, *i.e.* maximize $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_N)$ according to (4.1). Therefore, a trade-off must be made between the two requirements. Taking the norm of $\Delta\mathbf{c}$ as the metric of the expected credit variation vector $\Delta\mathbf{c}$, we minimize $\|\Delta\mathbf{c}\|$ over the price vector $\boldsymbol{\alpha}$ which are subject to the box constraints, *i.e.*

$$\text{minimize} \quad \|\Delta\mathbf{c}\|, \tag{4.4a}$$

$$\text{subject to} \quad \underline{\alpha} \leqslant \alpha_n \leqslant \bar{\alpha}, \quad n = 1, \ldots, N, \tag{4.4b}$$

where $\underline{\alpha}$ and $\bar{\alpha}$ is the lower and upper bound of the price, respectively. Note that we need $\underline{\alpha} > \beta$. Furthermore, it should be chosen such that the expected net gain of node m in (4.1) be greater than the cost for forwarding a message. The upper bound $\bar{\alpha}$ should be less than $\delta/(h_{\max}\gamma)$, where $h_{\max}$ is the maximum hop count for possible routing. Then the price vector satisfies the established condition in Theorem 3.

To see the effect of the credit balancing method, we have to answer a more fundamental question: given an optimal price vector resulting from (4.4), what is the underlying physical meaning and how does it depend on the traffic workload of the system? We answer the problem via the *shadow prices* associated with the price constraints (4.4b) [10].

In constrained optimization, the *shadow price* is the change in the objective value of the optimal solution of an optimization problem obtained by relaxing the constraint by one unit: it is the marginal utility of relaxing the constraint, or equivalently the marginal cost of strengthening the constraint. Each constraint in an optimization problem has a shadow price or dual variable. The value of the shadow price can provide decision makers powerful insight into problem. In the sequel, we consider the shadow price for constraints (4.4b).

4.2. Taxicab norm pricing approach

We take taxicab norm in (4.4a) and call it as taxicab norm pricing scheme. Let x_n be the non-negative variable. We translate the non-smooth optimization problem (4.4) to a linear programming problem

$$\begin{aligned} \text{minimize} \quad & \sum_{n=1}^{N} x_n, \\ \text{subject to} \quad & w_n + f_{nn}\alpha_n - \sum_{m\neq n} f_{nm}\alpha_m \geqslant -x_n, \\ & w_n + f_{nn}\alpha_n - \sum_{m\neq n} f_{nm}\alpha_m \leqslant x_n, \\ & \underline{\alpha} \leqslant \alpha_n \leqslant \bar{\alpha}, \quad n = 1,\ldots,N. \end{aligned} \tag{4.5}$$

The dual of the optimization problem (4.5) is

$$\text{maximize} \quad \sum_{n=1}^{N} (w_n(z_n - y_n) + \underline{\alpha}\mu_n - \bar{\alpha}\nu_n), \tag{4.6a}$$

$$\text{subject to} \quad \sum_{m\neq n} f_{mn}(z_m - y_m) - f_{nn}(z_n - y_n) + \mu_n - \nu_n = 0, \tag{4.6b}$$

$$y_n + z_n = 1, \tag{4.6c}$$

$$y_n, z_n, \mu_n, \ \nu_n \geqslant 0, \quad n = 1,\ldots,N. \tag{4.6d}$$

Let $(\boldsymbol{\alpha}^*, \mathbf{x}^*)$ be the solution to (4.5). Then the expected credit variation of node n is $\Delta c_n^* = w_n + f_{nn}\alpha_n^* - \sum_{m:m\neq n} f_{nm}\alpha_m^*$. It can be easily checked that $x_n^* = |\Delta c_n^*|$ for all $n \in \mathcal{N}$.

Let $\mathcal{R} = \{n \in \mathcal{N} : \Delta c_n^* > 0\}$ be the set of rich node, *i.e.*, whose expected credit variation is positive, and $\mathcal{P} = \{n \in \mathcal{N} : \Delta c_n^* < 0\}$ is the set of poor node, *i.e.*, whose expected credit variation is negative. Define sets of node with the lowest price $\mathcal{L} = \{n \in \mathcal{N} : \alpha_n^* = \underline{\alpha}\}$, with the middle price $\mathcal{M} = \{n \in \mathcal{N} : \underline{\alpha} < \alpha_n^* < \bar{\alpha}\}$, and with the highest price $\mathcal{H} = \{n \in \mathcal{N} : \alpha_n^* = \bar{\alpha}\}$. The following theorem shows how the optimal price vector depends on the traffic workload.

Theorem 4. *Let α^* be the optimal price vector of Taxicab norm pricing approach, and μ_n^* and ν_n^* be the shadow prices for the lower and upper bound constrains, respectively. We have*

(i) $\forall n \in \mathcal{R} \cap (\mathcal{H} \cup \mathcal{M})$, *it holds that* $\mu_n^* = \nu_n^* = 0$; *In addition,* $f_{mn} = 0, \forall m \in \mathcal{P}, m \neq n$.
(ii) $\forall n \in \mathcal{R} \cap \mathcal{L}$, *it holds that* $\mu_n^* = 2\sum_{m\in\mathcal{P},m\neq n} f_{mn}$ *and* $\nu_n^* = 0$.
(iii) $\forall n \in \mathcal{P} \cap \mathcal{H}$, *it holds that* $\nu_n^* = 2\sum_{m\in\mathcal{R},m\neq n} f_{mn}$ *and* $\mu_n^* = 0$.
(iv) $\forall n \in \mathcal{P} \cap (\mathcal{M} \cup \mathcal{L})$, *it holds that* $\mu_n^* = \nu_n^* = 0$; *In addition,* $f_{mn} = 0, \forall m \in \mathcal{R}, m \neq n$.

Proof. Let $(\mathbf{y}^*, \mathbf{z}^*, \mu^*, \nu^*)$ be a solution of (4.6). It can be easily checked that $\Delta c_n^* = x_n^*$ for all $n \in \mathcal{R}$ and $\Delta c_n^* = -x_n^*$ for all $n \in \mathcal{P}$. By the well known complementary theorem in linear programming [10], we have that $y_n^* = 0$ for all $n \in \mathcal{R}, z_n^* = 0$ for all $n \in \mathcal{P}, \nu_n^* = 0$ for all $n \in \mathcal{L}, \mu_n^* = 0$ for all $n \in \mathcal{H}$, and $\mu_n^* = \nu_n^* = 0$ for all $n \in \mathcal{M}$.

Furthermore, we have $z_n^* = 1$ for all $n \in \mathcal{R}$ and $y_n^* = 1$ for all $n \in \mathcal{P}$ by the second dual feasibility condition (4.6c). Now the first dual feasibility (4.6b) reduces to

$$\sum_{m\in\mathcal{R},m\neq n} f_{mn} - \sum_{m\in\mathcal{P},m\neq n} f_{mn} - f_{nn}(z_n^* - y_n^*) + \mu_n^* - \nu_n^* = 0. \tag{4.7}$$

Case A: $n \in \mathcal{R}$. It holds that $y_n^* = 0, z_n^* = 1$. Eq. (4.7) reduces to

$$\sum_{m\in\mathcal{R},m\neq n} f_{mn} - \sum_{m\in\mathcal{P},m\neq n} f_{mn} - f_{nn} + \mu_n^* - \nu_n^* = 0.$$

Substituting Eq. (4.3) into it, we have that

$$-2\sum_{m\in\mathcal{P},m\neq n} f_{mn} + \mu_n^* - \nu_n^* = 0. \tag{4.8}$$

If $n \in \mathcal{H}$, we have $\mu_n^* = 0$. By Eq. (4.8) and ν_n^* non-negative, we have $\nu_n^* = 0$ and $f_{mn} = 0$ for all $m \in \mathcal{P}$. If $n \in \mathcal{M}$, we have $\mu_n^* = \nu_n^* = 0$. By Eq. (4.8), we have $f_{mn} = 0$ for all $m \in \mathcal{P}$. So item (i) holds. If $n \in \mathcal{L}$, we have $\nu_n^* = 0$. By Eq. (4.8), we have $\mu_n^* = 2\sum_{m\in\mathcal{P},m\neq n} f_{mn}$. So item (ii) holds.

Case B: $n \in \mathcal{P}$. It holds that $y_n^* = 1, z_n^* = 0$. Eq. (4.7) reduces to

$$\sum_{m\in\mathcal{R},m\neq n} f_{mn} - \sum_{m\in\mathcal{P},m\neq n} f_{mn} + f_{nn} + \mu_n^* - \nu_n^* = 0.$$

Substituting Eq. (4.3) into it, we have that

$$2\sum_{m\in\mathcal{R},m\neq n} f_{mn} + \mu_n^* - \nu_n^* = 0. \tag{4.9}$$

If $n \in \mathcal{H}$, we have $\mu_n^* = 0$. By Eq. (4.9), we have $\nu_n^* = 2\sum_{m\in\mathcal{R},m\neq n} f_{mn}$. So item (iii) holds.

If $n \in \mathcal{M}$, we have $\mu_n^* = \nu_n^* = 0$. By (4.9), we have $f_{mn} = 0$ for all $m \in \mathcal{R}$. If $n \in \mathcal{L}$, we have $\nu_n^* = 0$. By Eq. (4.9), we have $f_{mn} = 0$ for all $m \in \mathcal{R}$. So item (iv) holds. □

Let us provide some economic interpretations for the optimal price vector. Suppose node n has surplus credits. The shadow price of the lower bound for n's price is zero. If node n does not take the lowest price, *i.e.*, $n \in \mathcal{R} \cap (\mathcal{H} \cup \mathcal{M})$, the shadow price of the upper bound is zero and node n will not forward any traffic for any poor node. If node n takes the lowest price, *i.e.*, $n \in \mathcal{R} \cap \mathcal{L}$, the shadow price of the lower bound for n's price is twice of the sum of the traffic that n forwards for poor nodes. Suppose node n has a deficit in credits. The shadow price of the upper bound for n's price is zero. If node n takes the highest price, *i.e.*, $n \in \mathcal{P} \cap \mathcal{H}$, the shadow price of the lower bound for n's price is twice of the sum of the traffic that n forwards for rich nodes. If node n does not take the highest price, *i.e.*, $n \in \mathcal{P} \cap (\mathcal{L} \cup \mathcal{M})$, the shadow price of the lower bound for n's price is zero. In addition, node n will not forward any traffic for any rich node.

Remark. Note that we can carry out similar analysis of the shadow price for the pricing method with *infinity norm* or even *Euclidean norm* in (4.4a). In view of the complexity, both infinity norm and taxicab norm pricing approaches need to solve a linear programming problem. For the Euclidean norm pricing approach, one needs to solve a linear least-squares problem with box constraints, which is also solvable in polynomial time.

5. Credit evolution with back-pressure algorithms

In this section, we model the credit evolution based on the queuing status of a node and examine how the shared nature of wireless medium may effect the credit distribution and the performance of the network. Here, the *back-pressure algorithm* is responsible for the scheduling. Besides maintaining queues for flow r at each of its crossing nodes, each source node also needs a *packet admission strategy* to decide whether to accept a packet that was generated but during the packet generation, the source node does not have sufficient credit to pay the CCS.

5.1. Credit evolution based on the queuing status

We assume that the packet length is constant and the system is slotted with the slot length equal to the packet length. The transmissions are synchronized at the beginning of a slot. The discrete time t corresponds to the slot $(t-1, t]$.

The transmission process $\{P_{(i,j)}(t)\}_{t=1}^{\infty}$ of a link (i, j) is modeled as a Bernoulli process, *i.e.*, $\Pr(P_{(i,j)}(t) = 1) = p_{ij}$ and $\Pr(P_{(i,j)}(t) = 0) = 1 - p_{ij}$. We select the transmitting links at each slot such that collisions are avoided, the possible packet losses are mainly due to channel errors. Links can be active or inactive. Each active link can transmit one packet in its queue. We use Π to denote the set of all possible feasible schedules according to some *interference constraints set*, $\pi \in \Pi$ is a vector with element of 0 or 1, and

$$\pi_{(i,j)} = \begin{cases} 1, & \text{if link } (i,j) \text{ is activated,} \\ 0, & \text{otherwise.} \end{cases}$$

Remark. To simplify the accounting computation, each source node n pays $C^r = \sum_{i \in r} \alpha_r + (\beta - \alpha_{r_d})$ to CCS for a packet before this packet enters the network. Then the CCS pays α_n to node n when it successfully transmits a packet.

It is well known that the traditional back-pressure algorithm utilizes all possible paths between source–destination pairs in packet or message delivery, thus achieving load balancing. We refer this as the *adaptive-routing*, in which the back-pressure algorithm is used to adaptively select a route for each packet. While this might be needed in a heavily loaded network but it is not appropriate in a light or moderately load regime. Exploring all paths can be detrimental – it may lead to packets traversing excessively long paths between source and destinations leading to large end-to-end packet delays. Here, we consider a *fixed-routing* policy, in which a route is chosen by the smallest hop count or ETX as routing metrics but packet scheduling is based on the back-pressure framework.

We regard each flow r as a class of customer and denote the queue length of flow r at node n at the end of slot t (or the beginning of slot $t+1$) by $Q_n^r(t)$. Note that data packets of any flow are delivered to the higher layer upon reaching the destination node, so $Q_{r_d}^r(t) \equiv 0$. We use n_n^r and p_n^r to denote the next hop and the previous hop of node n along the flow r. Specifically, at time slot t, the queue length dynamics and credits evolution are as follows:

Step a. Link (i, j) finds the differential backlogs $D_{(i,j)}^r(t) = \left(Q_i^r(t) - Q_j^r(t)\right)_+$ for each flow r through it. Let $w_{(i,j)}(t) = p_{ij} \max_{r:(i,j) \in r} \{D_{(i,j)}^r(t)\}$ be the weight of link (i, j) and $\mathbf{w}(t) = (w_{(i,j)}(t) : (i,j) \in \mathcal{E})$ be the link weight vector at slot t.

Step b. A maximum weighted activation vector $\hat{\boldsymbol{\pi}}$ is selected from Π

$$\hat{\boldsymbol{\pi}} = \arg\max_{\boldsymbol{\pi} \in \Pi} \mathbf{w}(t)^T \boldsymbol{\pi}.$$

We denote the scheduling vector for flow r by $\hat{\boldsymbol{\pi}}_r$.

Step c. Node n checks its credits and gets the packet admission strategy $\mu_n^r(t)$ for each flow starting from n, where $\mu_n^r(t)$ takes either of the values 0 or 1 only. The credit dynamics for node n is

$$\begin{aligned} C_n(t+1) = C_n(t) &- \sum\nolimits_{r_s = n} \mu_n^r(t) A_{nr_d}(t) C^r \\ &+ \sum\nolimits_{(n,k) \in \mathcal{E}} P_{(n,k)}(t) \hat{\pi}_{(n,k)} \alpha_n \\ &+ \sum\nolimits_{r_d = n} P_{(m,n)}(t) \hat{\pi}_{(m,n)}^r \beta, \end{aligned}$$

where the second term is the spending of node n on message sending, the third is the income of node n for message forwarding, and the forth term is the income of node n for submitting a receipt for message ending at node n.

Step d. The source of flow r updates the queue length, *i.e.*,

$$Q_n^r(t+1) = \left(Q_n^r(t) + \mu_n^r(t) A_{nr_d}(t) - P_{(n,n_n^r)}(t) \hat{\pi}_{(n,n_n^r)}^r\right)_+,$$

where $(z)_+ = \max(0, z)$ and $n = r_s$. The queue length dynamics for all $n \in r$ and $n \neq r_s$ are

$$Q_n^r(t+1) = \left(Q_n^r(t) + P_{(p_n^r,n)}(t) \hat{\pi}_{(p_n^r,n)}^r - P_{(n,n_n^r)}(t) \hat{\pi}_{(n,n_n^r)}^r\right)_+.$$

Note that in Step b, the maximum weighted schedule depends on the interference constraints Π. In this paper, we assume the *node-exclusive spectrum sharing model* of primary interference in wireless networks, which implies that any node communicates with at most one other node in any time slot. In this case, the activated set is any set of links such that no two links of the set share a common node, which is a *matching* in $\mathcal{G}$. The scheduling algorithm boils down to finding maximum weighted matchings.

In fact, the node-exclusive interference model can be viewed as a generalization of the bipartite graph model for modeling high-speed packet switches [12]. It has been used in [13,14] to model wireless networks. While this is a somewhat simplified model, the main results can be readily generalized to other more complex interference models, e.g., the two-hop interference model. Note also that the latter model is very close to the model of IEEE 802.11 DCF [15].

5.2. Packet admission strategy

In a credit-based incentive scheme, a source node needs to determine whether to admit the packet into the network when it lacks in credits. We consider two packet admission strategies, *i.e., credit sensitive* (CS) admission control and *credit insensitive* (CI) admission control.

Under CS, each source node admits a packet into the system only when its credit balance is sufficient to cover the charge for sending the packet; otherwise the source node drops the packet. In this case, the source node's credit remains non-negative, but some packets generated will be lost. In the following section, we will explore the probability of a generated packet successfully delivered in HomoPricing and HeteroPricing under CS. Under CI, a source node always admit a packet generated into the network even if it has insufficient credit. In this case, the source node's credit may become negative, but all generated packets are sent. In the next section, we will explore the credit inequality in HomoPricing and HeteroPricing under CI.

6. Performance evaluation

In this section, we use simulations to study the performance of the proposed credit evolution model, focusing on the credit distribution among nodes under various scenarios and the effectiveness of the differentiated pricing to achieve credit equality.

6.1. Simulation setup

We consider various network topologies of WMNs. For the linear and ring topologies, we set $N = 10$. For the grid topology, set $H = 5$ and $P = 5$, or $N = 25$. We also consider a topology consisted of 36 nodes that are randomly distributed in an area of 1000 by 1000 m as shown in Fig. 6.1 where we use the lognormal shadowing propagation model [11]. Let d_{ij} and p_{ij} be the distance and the delivery probability for the link from node i to node j, respectively. Based on [11], p_{ij} can be approximated as a function of d_{ij}:

$$p_{ij} = \begin{cases} 1 - \left(\frac{d_{ij}}{D}\right)^{2\rho} \Big/ 2, & \text{if } d_{ij} \leqslant D, \\ \left(\frac{2D - d_{ij}}{D}\right)^{2\rho} \Big/ 2, & \text{if } D < d_{ij} \leqslant 2D, \\ 0, & \text{otherwise}, \end{cases}$$

where ρ is the power attenuation factor ranging from 2 to 6, and D is defined as the distance such that $p_{ij}(D) = 0.5$. In our simulation, we set $\rho = 5$ and $D = 200$ m. We assume j is not in the transmission range of node i if $p_{ij} < 0.6$. The average degree of a vertex is 3.44.

All simulations are done with the Mathwork's matlab on an Intel Core i5 computer operating at 2.0 GHz. We simulate the credit evolution based on the queuing status with discrete event dynamic system where the queue limit of each flow is set to 300. The max-weight scheduling in Step (b) is model as the maximum weight match and is formulated as a 0–1 integer linear programming problem. We solve it with the function *binprog.m* in the optimization toolbox of Matlab.

We use uniform traffic workload, or $\lambda_s = \lambda$ and $h_{sd} = \frac{1}{N-1}$ for all $s, d \in \mathcal{N}$ as the benchmark. A skewed one is considered in Section 6.1. The other traffic models are used respectively in system's input and output performance sections.

To study in detail the credit inequality, we use the unit traffic of each source as the benchmark, *i.e.*, $\lambda_s = 1$ for all s under the uniform traffic model. In HomoPricing, we set the parameters $\alpha = 1$, $\beta = 0.1$ and $\gamma = 0.001$. In the pricing model (4.4) for HeteroPricing, we set $\underline{\alpha} = 0.5$ and $\bar{\alpha} = 10$. In the queue-based model, we assume $p_{ij} = 1$ for all $(i, j) \in \mathcal{E}$.

6.2. Credit inequality measures

In this set of experiments, we focus on the factors that lead to the credit inequality. In addition to the expected credits variation, we also consider the expected incomes and demands defined in Section 4 in order to understand the impact of transmission modes, routing metrics and traffic patterns.

6.2.1. Persistent transmission mode vs. single transmission mode

We consider the random topology as depicted in Fig. 6.1 with the shortest-path ETX routing. The expected credits variation, CDF of the expected demands and incomes, and scatter plot of (demand, income) for the persistent transmission mode (with legend Persistent-TM) and the single transmission mode (with legend Single-TM) are shown in Fig. 6.2.

From Fig. 6.2(a), the expected credits variation of nodes under the single transmission mode is larger than that under the persistent transmission mode. The difference between the expected credit variation of those nodes which are on the edge of the network, e.g. node 1, 2, 3, 35 and 36, under these two transmission modes, are *much larger* than those in the center of the network. The CCS has null credit under the persistent-TM. While under the single-TM, the CCS has 55 credits.

Let us explore the reasons for the above results. From the CDF of the expected demands and incomes in Fig. 6.2(b), we observe that the expected incomes under the two transmission modes are *almost consistent*, while the expected demand under the persistent-TM is more concentrate than that under the single-TM. This can be also observed from Fig. 6.2(c), where the scattering points of the persistent-TM are almost in the lower left of that of the Single-TM. Based on these observations, we conclude that the credit inequality under the two transmission modes result from the inequality of the expected demands. Furthermore, the inequality of demands under the persistent-TM is less than the one under the single-TM. So the expected credits variation under the persistent-TM is less than the one under the single-TM.

Observation 1. The transmission modes have little impact on the expected income but it has an impact on the expected demand. Hence, we can see that the single transmission mode will have a higher credit inequality than the persistent transmission mode.

In the following experiments, we focus on HomoPricing under the persistent transmission mode. The expected credits variation and the distribution of the expected

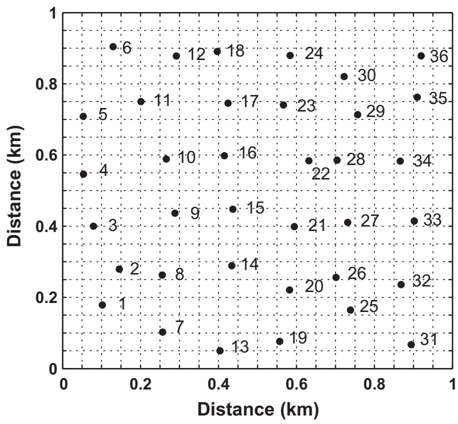


Fig. 6.1. Specific topology randomly generated.

income and demand for the linear, ring and grid topologies under different routing metrics and different traffic models are considered.

6.2.2. Comparison between different routing metrics

Fig. 6.3 shows the expected credits variation, incomes and demands of each node for the linear and ring topologies under the hop count metric (with legend HC) and the ETX metric (with legend ETX), where the link loss probabilities are uniformly generated over [0, 0.4] for ETX metric.

For the linear topology, each flow has the *same routing* under both of the hop-count metric and the ETX metric. The expected credits variation under the two routing matrices are the same as shown in Fig. 6.3(a), where nodes 1 and 10, which are on the edge of the network, have the smallest expected credits variation, while nodes 5 and 6, which are in the center of the network, have the highest expected credits variation. Based on the demands and incomes shown in Fig. 6.3(b) and (c), we can see that the large credit inequality is due to the mis-match between expected income and demand, e.g., node 5 and 6 have large expected income but small expected demand, while node 1 and 10 are exactly opposite. This leads this network to the large credit inequality.

For the ring topology, each node plays the *same role* and maintains credit equality under hop count metric. From Fig. 6.3, one can observe that the ETX metric has little effect on the expected credits variation. Specially, the ETX metric has no effect on the expected demands and has slight effect on the expected incomes.

To test the effect of ETX metric on the credit inequality in the grid topology, we consider two scenarios. One is the ETX-Edge scenario, where the links out from those nodes on the edges of the network have zero loss probability and others are set to 0.3. The other is the ETX-Cross scenario, where the links out from those nodes in the middle of the network have zero loss probability while others are set to 0.3. The results are shown in Fig. 6.4. Compared with the shortest-path hop-count routing, the ETX-Edge case makes the credit more equal because there are more opportunities for nodes on the edge of the network to forward message for others. On the contrary, the ETX-Cross case has a higher credit inequality since there are more opportunities for nodes in the center of the network to earn more credits. We can see these from Fig. 6.4(b) and (c), where the expected demands are almost consistent and routing is the only cause of large credit inequality among the expected incomes under three cases.

Observation 2. The routing algorithms (*i.e.*, hop count and ETX) have little impact on the expected demands but have a high impact on the expected income.

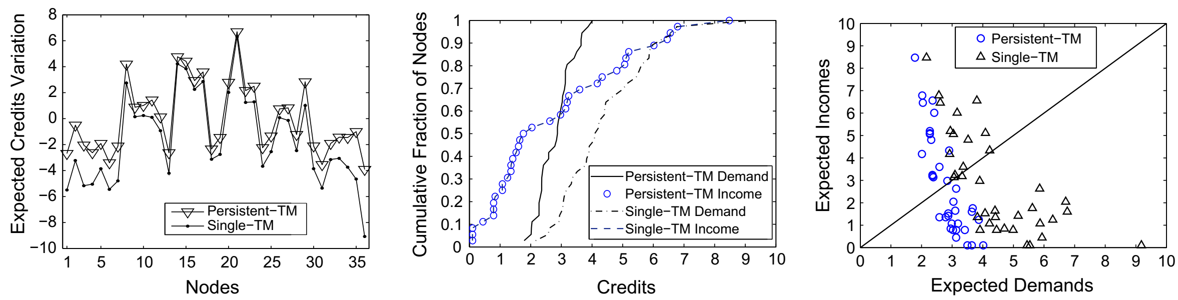
6.2.3. Comparison between different traffic patterns

Besides the uniform traffic model (with legend UTM), we consider a *skewed* one (with legend STM), *i.e.*, $\lambda_N = \lfloor N/2 \rfloor \lambda$ and $\lambda_s = \lambda$ for all $s \neq N$. In addition, $h_{Nd} = \frac{1}{N-1}$ for all $d \neq N$, and for all $s \neq N$, we have

$$h_{sd} = \begin{cases} \frac{1}{2(N-2)} & d \neq s, N, \\ \frac{1}{2} & d = N. \end{cases}$$

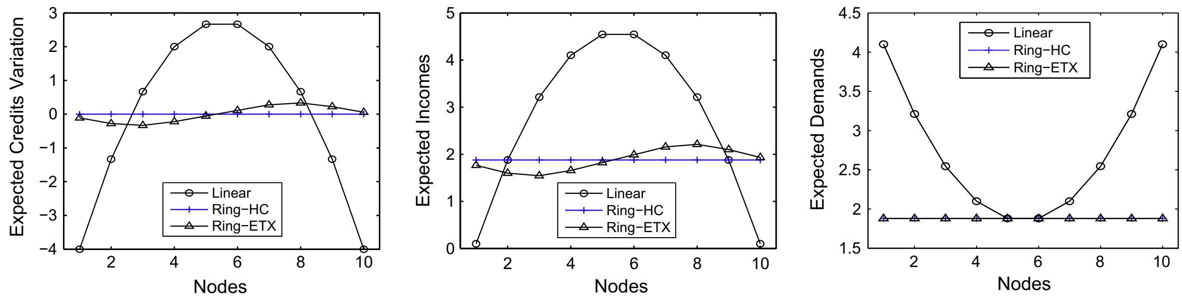
This traffic model can represent a WMN with N being the access point.

Fig. 6.5 shows the results for the linear and ring topologies with the shortest-path hop-count routing. The skewed traffic in both topologies makes those nodes near the access point have large expected incomes and small ex-



(a) Expected credits variation (b) CDF of the expected demands and incomes (c) Scatter plot of (demand, income)

Fig. 6.2. The comparison of persistent transmission mode against single transmission mode for the specific topology in Fig. 6.1 with the uniform traffic model and the shortest-path ETX routing.



(a) Expected credits variation (b) Expected incomes (c) Expected demands

Fig. 6.3. Hop count metric against ETX metric for linear and ring topologies.

pected demands, as shown in Fig. 6.5(b) and (c). All facts increase the credit inequality among nodes. The difference between the linear and ring topologies is that the credit inequality is asymmetric in the former and symmetric in the later around the access point.

Fig. 6.6(a) illustrates the expected credit variation for a grid topology. Since a grid topology is composed of *multiple* linear networks (five linear networks in our consideration), the expected credit variation will have a *local behavior* (*i.e.*, nodes 1–5, nodes 6–10, etc.) which is similar to a linear network in Fig. 6.5(a). For the nodes *far away* from the access point *N*, the expected income is relatively stable even with larger expected demands. While the nodes *near* the access point *N*, the expected demands become smaller with huge expected incomes. All these contribute to the credit inequality.

Observation 3. The traffic patterns can affect both the expected incomes and demands. Generally, the skewed traffic can cause *higher* credit inequality.

6.3. System performance

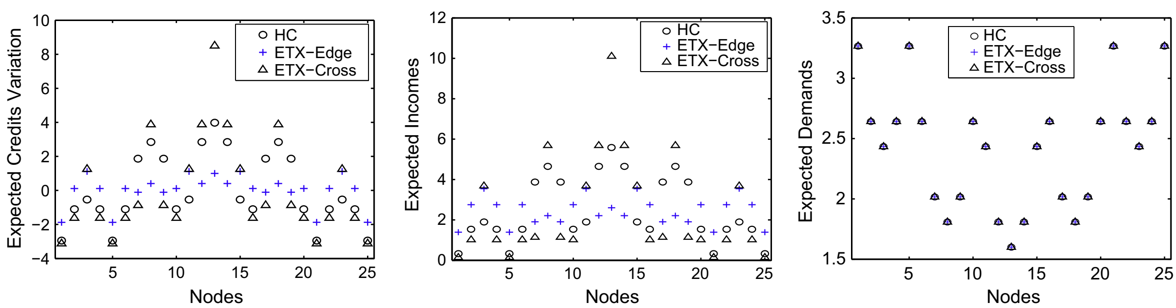
Here, we study the overall system performance. We use the stationary traffic model for HeteroPricing as described in Section 4. We evaluate the performance of HomoPricing and HeteroPricing with the topology in Fig. 6.1 with the shortest-path ETX routing. Furthermore, we use the *back-pressure algorithm* as the underlying link scheduling.

First, we consider the credit inequality measures for HomoPricing and HeteroPricing in the case that the packet admission strategy is credit insensitive. Fig. 6.7 illustrates this result. From Fig. 6.7(a), it can be seen that independent of the pricing scheme, nodes 6, 31 and 36 have no chance to provide forwarding service, therefore their expected credit variation is negative, and pricing cannot resolve the credit inequality problem. However, all other nodes can provide forwarding service, therefore, heterogeneous pricing can help in reducing the credit inequality of these nodes.

Another way to validate that heterogeneous pricing is effective in reducing the credit inequality among nodes is from Fig. 6.7(b). One can observe that the CDFs for income and demand under the heterogeneous pricing are very *similar* and they have a smaller variance than the CDFs of homogeneous pricing. This justifies why heterogeneous pricing is effective in reducing credit inequality. One can validate this claim again from Fig. 6.7(c), in which there are more points along the 45 degree line under the heterogeneous pricing. Hence, it is effective to achieve credit equality.

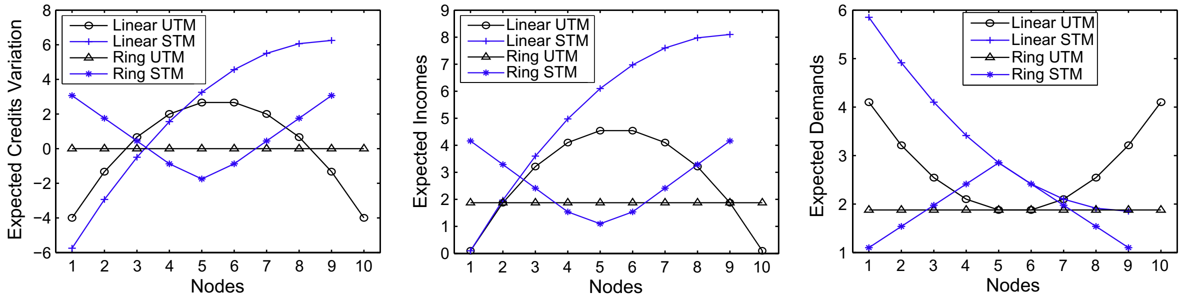
Next, we evaluate the system performance with the following metrics:

- *message success rate*: the percentage of generated packets actually sent by the source nodes. Note that this depends both on the packet admission strategy and the pricing schemes,
- *end-to-end throughput*: the given packets divided by the total time required to transfer the packets.



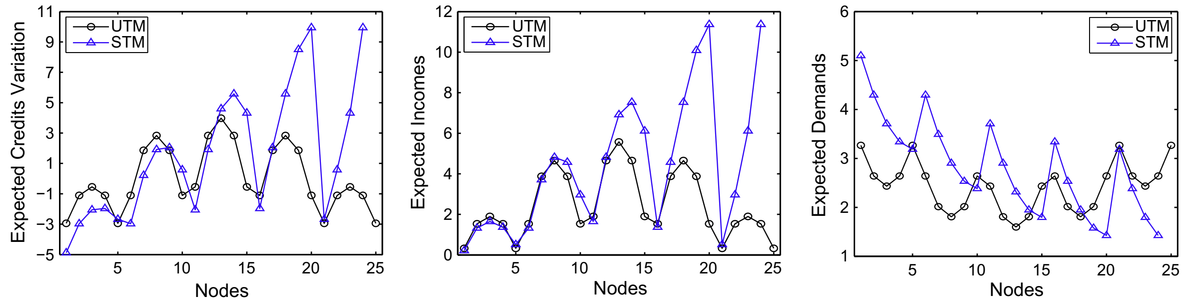
(a) Expected credits variation (b) Expected incomes (c) Expected demands

Fig. 6.4. Hop count metric against ETX metric for grid topology.



(a) Expected credits variation (b) Expected incomes (c) Expected demands

Fig. 6.5. Uniform traffic against skewed traffic for linear and ring topologies.



(a) Expected credits variation (b) Expected incomes (c) Expected demands

Fig. 6.6. Uniform traffic against skewed traffic for grid topology.

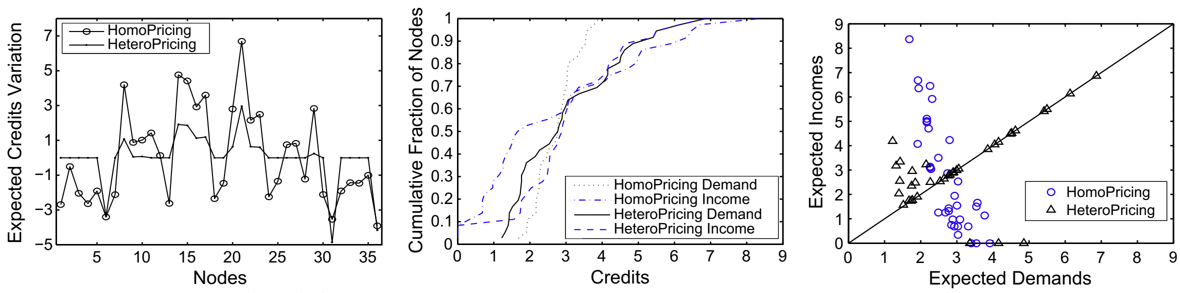
- *end-to-end delay*: the average delay over all successfully delivered packets of each flow. Note that this depends on the link scheduling and the traffic workloads in the network.

It is important to point out that the message success rate is *always* 1 under CI, but for CS, the message success rate depends on the *total initial credits* among nodes. We quantify this as the ratio of total initial credits in the network over the total expected demands for each node. We call this the *credit injection ratio* and denote it as r_c. Unless we state otherwise, the total initial credits are distributed evenly among all nodes in our simulation.

6.3.1. System's output performance

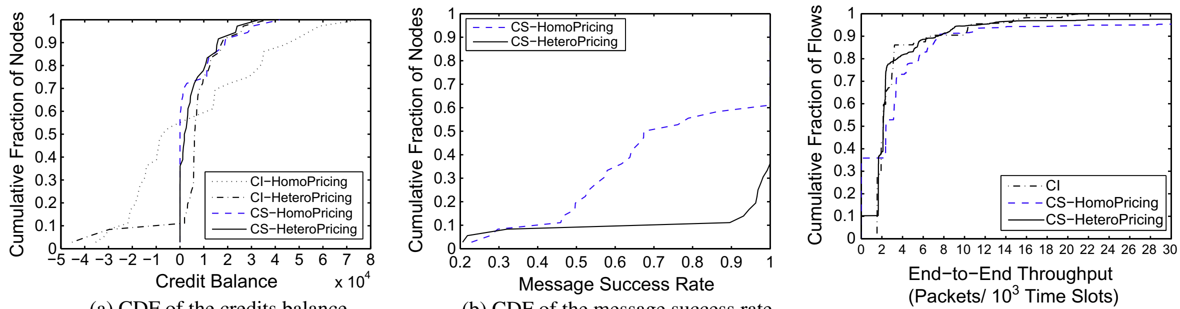
We consider a network wherein each source node has 10^4 packets to deliver and there are $36 \times 35 = 1260$ flows with $r_c = 0.2$. Fig. 6.8 illustrates this result.

Fig. 6.8(a) considers the DCF of credit balance. Under the credit insensitive (CI) admission control, over 50% nodes in CI-HomoPricing and less than 10% in CI-HeteroPricing have a deficit in credits, which means these nodes need to acquire more credit if they want to have a non-negative credit balance or to have a message success rate of 1. However, heterogeneous pricing is more effective in reducing the number of nodes which are deficit in credit. The difference between the CDFs of credit balance of CS-



(a) Expected credits variation (b) CDF of the expected demands and incomes (c) Scatter plot of (demand, income)

Fig. 6.7. The credit inequality comparison between HomoPricing and HeteroPricing under credit insensitive admission control.



(a) CDF of the credits balance (b) CDF of the message success rate (c) CDF of the end-to-end throughput

Fig. 6.8. The system's output performance comparison between HomoPricing and HeteroPricing, where each source node has 10,000 packets and $r_c = 0.2$.

HeteroPricing and CI-HeteroPricing, as well as CS-HomoPricing and CI-HomoPricing are quite small, this indicates that the pricing based on the stationary traffic is feasible and effective in controlling credit equality.

Fig. 6.8(b) illustrates the message success rate in CS-HeteroPricing vs. the CS-HomoPricing. It shows that nearly 50% of the flows improve their success rate to 1 and only 10% flows's success rate is within the interval of [0.4, 0.6] under the CS-HeteroPricing. This indicates that CS-HeteroPricing admits more packets into the network as compared with the CS-HomoPricing.

From Fig. 6.8(c), we observe that under CI, over half of the flows have the throughput of 2 packets per time unit, nearly 30% flows have the throughput of 5 packets per time unit, and less than 10% flows have the throughput between 10 and 20 packets per time unit. Nearly 40% flows in CS-HomoPricing and 10% in CS-HeteroPricing have zero throughput. A flow with zero throughput means that the source does not have enough credit for packet sending. Note that less than 5% flows under CS, especially in CS-HomoPricing, can achieve very high throughput, but this is only because heterogeneous pricing carries a *higher workload* (as indicated in Fig. 6.8(b)) than homogeneous pricing scheme. The average end-to-end throughput is 3.3694, 4.6269, and 5.3141 for CI, CS-HeteroPricing and CS-HomoPricing respectively. Since 4.6269 is comparable to 5.3141, one can conclude that heterogeneous pricing can carry higher workload than homogeneous pricing, while achieve comparable end-to-end throughput.

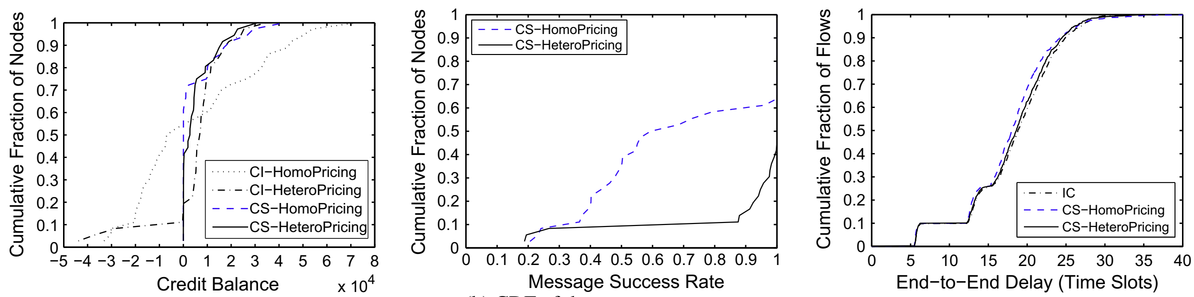
6.3.2. System's input performance

We use the similar simulation setup in [2,6] to generate traffic randomly. The start of a session (namely a source–destination pair) at a node (in which the node is the source) is a Poisson arrival. The expected time interval between two sessions from the same node is 10^3 time slots. The destination of each session is selected uniformly from all nodes (excluding the source). The number of packets in each session is a constant of 10. Fig. 6.9 shows a typical result wherein the system runs 10^6 time slots with $r_c = 0.2$.

From Figs. 6.8 and 6.9, we can see that the system bears the same input and output performance on credit balance and message success rate, *i.e.*, CS-HeteroPricing admits more packets into the network as compared with the CS-HomoPricing. From Fig. 6.9(c), we see that nearly the same end-to-end delay can be achieved under the three cases. The average end-to-end delay is 18.23, 17.94, and 17.61 for CI, CS-HeteroPricing and CS-HomoPricing respectively. Since 17.94 is comparable to 17.61, one can conclude that heterogeneous pricing can carry higher workload than homogeneous pricing, but achieve comparable end-to-end delay.

6.3.3. System performance vs. the total initial credits

We investigate the impact of the total initial credit on the system performance. Fig. 6.10 shows the average message success rate and average end-to-end delay as a function of the credit injection ratio under the same scenarios as system's input performance experiments.



(a) CDF of the credits balance (b) CDF of the message success rate (c) CDF of the end-to-end delay

Fig. 6.9. The system's input performance comparison between HomoPricing and HeteroPricing, where the system runs 10^6 time slots and $r_c = 0.2$.

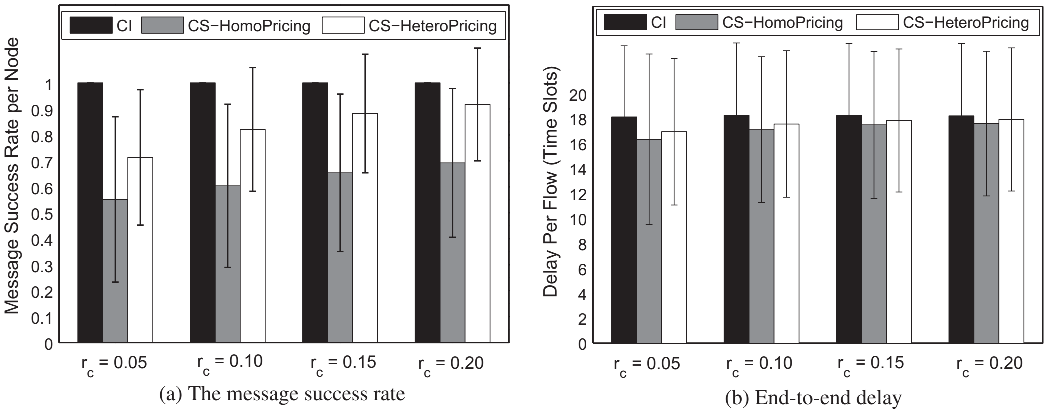


Fig. 6.10. Dynamics of system performance vs. the injected credit ratio. The bars in (a) show the average per node message success rate over 10^6 runs. The bars in (b) show the average per flow delay over 10^6 runs. Standard deviations are shown using lines.

In Fig. 6.10(a), it shows that the message success rate per-node increases with increased total initial credits since nodes have more opportunity to exchange the forwarding service. But the rate of increase reduces gradually.

Fig. 6.10(b) shows no obvious increase in delay per-flow happens with the increasing of initial credits.

Based on Fig. 6.10, increase in the total initial credits can make the system bear more workload with no degradation in the end-to-end delay performance. But it may cause wealth condensation [19]. Therefore, how to choose a proper total credits injecting into the system is an interesting and important problem.

7. Related work

Let us present some relevant work on credit-based incentive network protocols. Santhanam et al. [25] analyze selfishness of MRs in a multi-operator WMN and explore its overall negative impact on network performance. To stimulate cooperation among selfish nodes in mobile ad hoc networks, incentives have been proposed. When incentives are introduced, abuse and forgery must be prevented. For example, with the assumption of tamper-proof hardware on board, each relaying node earns some virtual credit that is protected by the tamper-proof hardware. Buttyan and Hubaux [3] proposes two models to reward forwarding nodes, the packet purse model (PPM) and packet trade model (PTM). A serious disadvantage of the Packet Trade Model is that it is possible to overload the network since the sources do not have to pay. Under the PPM, it is difficult to estimate the number of nuglets that the source should put in the packet initially. Buttyan and Hubaux [4] proposes a mechanism to overcome the estimation problem in PPM, because the packets do not need to carry nuglets. At the same time, the property of refraining users from overloading the network is retained. SPRITE [5] ensures the authenticity of currency by employing a centralized authority called the credit clearance service (CCS). Every node keeps a receipt of the packet it receives and submits it to the CCS. The CCS then determines the charge and credit for every node in the transmission path from a game-theoretic perspective so that other nodes are motivated to report correctly, even when selfish nodes collude and submit false receipts. Zhong et al. [6] present Corsac, a cooperation-optimal protocol which consists of a routing protocol and a forwarding protocol. The routing protocol of Corsac integrates VCG with a novel cryptographic technique to address the challenge in wireless ad hoc networks that a link's cost is determined by two nodes together. Corsac also applies efficient cryptographic techniques to design a forwarding protocol to enforce the routing decision, such that following the routing decision is the optimal action for each to maximize its utility.

Incentive systems implement micropayment in the network so to stimulate the selfish nodes to cooperate. However, micropayment schemes were originally proposed for Web-based applications so one has to make them efficient for wireless mesh networks. Authors in [26,27] provide a practical incentive system which considers the differences between Web-based applications and cooperation stimulation. They also propose a novel incentive mechanism where fairness can be achieved by using credits to reward the cooperative nodes.

Other "secure" incentive approaches make use of reputation-based schemes to detect and isolate uncooperative nodes. Marti et al. [1] proposes a monitoring agent called a watchdog at every node that overhears the transmission of its neighbors to detect non-forwarding misbehavior. Buchegger and Boudec [28] proposes the CONFIDANT protocol that assigns a rating for every node based on watchdog and second-rating information gathered from other nodes. The second-rating information prevents spurious rating and detects inconsistencies in the two observations. Accordingly, a path manager selects the best path by avoiding selfish nodes. To avoid a retaliation situation after a node has been falsely perceived as selfish, Jaramillo and Srikant [2] proposes DARWIN so cooperation can be restored quickly. Last but not least, Felegyhazi et al. investigate non-cooperative communication scenarios within a game theory framework [29,30].

Using network coding, wireless mesh networks can significantly improve their performance. In [31–33] authors study how to stimulate selfish nodes to cooperate in wireless mesh networks using network coding. Chen and Zhong [31] proposes a simple but practical reputation system that rewards cooperative behavior in routing and packet forwarding, and penalizes non-cooperative behavior. Chen and Zhong [31] uses a combination of game theoretic and cryptographic techniques to solve the incentive compatibility. Xia et al. [33] proposes a stimulus scheme under a multi-path inter-session network coding setting.

For routing protocols in WMNs, *Srcr* [16] is a state-of-the-art path routing protocol, where link weights are assigned based on the ETX metric [7]. Back-pressure algorithm possesses the throughput-optimal property. There are several technical challenges in the back-pressure algorithm. One is its complexity issue and the other is how to realize it in a decentralized setting [12,14]. Also, one needs to consider how to reduce the inefficiency in terms of end-to-end delay [17]. Although there are abundant work on network incentive protocols, but none of them addresses the *credit* and/or *reputation* distribution since they ultimately affect the operability and sustainability of the underlying wireless mesh network.

Simple models have been proposed to capture the distribution of money [18] in economics. There also exist models for study of the condensation of materials [19] in physics. Friedman et al. [20] study the credit-based P2P system and conclude that it is possible for system to collapse (*i.e.*, no node will have incentive to cooperate) when there are too much internal credits in the network. Zhao et al. [21,22] proposed a general analytical framework to analyze and design a large family of incentive protocols for P2P networks. The main difference of incentive scheme in WMNs and P2P networks is that the peers is not directly interacting with each other and we need to consider the interactions between the incentive mechanisms and the routing protocols.

This work is an extension of our previous conference paper [24]. Our extension includes deducing the general calculation on the expected traffic that a node forwards for other nodes under the persistent transmission mode as well as proofs on the theoretical properties. We provide generalizations in Sections 3 and 4 and economic interpretations on the optimal price vector. We also extend our work to consider credit evolution with back-pressure algorithms, and perform new experiments on these credit evolutions under more general network settings. We also enhance this work by considering system performance under HomoPricing and HeteroPricing.

8. Conclusion

In this paper, we present a mathematical framework to analyze the interaction of the credit-based incentive scheme (e.g., such as Sprite) and the underlying routing protocols (e.g., shortest-path, ETX or back-pressure routing). We showed that under some traffic workload, the WMNs can have large *credit-inequality*, which can cause some nodes not able to transmit any packet. To redeem this problem, we propose a differentiated pricing mechanism so as to evenly distribute credit among nodes such that the norm of the expected credit variation is closed to zero (or achieving the credit equality). Like a complex economic system, except balancing credit via pricing, we can examine the progressive tax on the credit-based incentive scheme for help the nodes with low credits. The mathematical methodology we propose in this work opens doors to investigate the sustainability of wireless networks that employ different incentive mechanisms and/or routing protocols.

Acknowledgement

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References

[1] S. Marti, T. Giuli, K. Lai, M. Baker, Mitigating routing misbehavior in mobile ad hoc networks, in: Proceedings of MobiCom, Boston, MA, August 2000, pp. 255–265.
[2] J. Jaramillo, R. Srikant, DARWIN: distributed and adaptive reputation mechanism for wireless ad-hoc networks, in: Proceedings of MobiCom, Montréal, Québec, Canada, September 2007, pp. 87–97.
[3] L. Buttyan, J.P. Hubaux, Enforcing service availability in mobile ad-hoc WANs, in: Proceedings of MobiHOC, Boston, MA, August 2000, pp. 87–96.
[4] L. Buttyan, J.P. Hubaux, Stimulating cooperation in self-organizing mobile ad hoc networks, ACM Journal for Mobile Networks and Applications 8 (2003) 579–592.
[5] S. Zhong, J. Chen, Y.R. Yang, Sprite, a simple, cheat-proof, credit-based system for mobile ad-hoc networks, in: Proceedings of IEEE INFOCOM, San Francisco, California, April 2003, pp. 1987–1997.
[6] S. Zhong, L. Li, Y. Liu, Y.R. Yang, On designing incentive-compatible routing and forwarding protocols in wireless ad-hoc networks – an integrated approach using game theoretical and cryptographic techniques, in: Proceedings of MobiCom, Cologne, Germany, September 2005, pp. 117–131.
[7] D.S.J. De Couto, D. Aguayo, J. Bicket, R. Morris, A high-throughput path metric for multi-hop wireless routing, in: Proceedings of MobiCom, San Diego, September 2003, pp. 134–146.
[8] L. Tassiulas, A. Ephremides, Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks, IEEE Transactions on Automatic Control 37 (12) (1992) 1936–1948.
[9] Geoffrey R. Grimmett, David R. Stirzaker, Probability and Random Process, third ed., Oxford University Press, 2001.
[10] George B. Dantzig, Linear Programming and Extensions, Princeton University Press, 1963.
[11] The Network Simulator – ns-2. <http://www.isi.edu/nsnam/ns/>.
[12] J.G. Dai, B. Prabhakar, The throughput of data switches with and without speedup, in: Proceedings of IEEE INFOCOM, Tel Aviv, Israel, March 2000, pp. 556–564.
[13] B. Hajek, G. Sasaki, Link scheduling in polynomial time, IEEE Transactions on Information Theory 34 (5) (1988) 910–917.
[14] X. Lin, N.B. Shroff, The impact of imperfect scheduling on cross-layer congestion control in wireless networks, IEEE/ACM Transactions on Networking 14 (2) (2006) 302–315.
[15] P. Chaporkar, K. Kar, S. Sarkar, Throughput guarantees through maximal scheduling in wireless networks, in: Proceedings of 43rd Annual Allerton Conference on Communication, Control and Computing, Monticello, IL, September 2005.
[16] J. Bicket, D. Aguayo, S. Biswas, Morris, Architecture and evaluation of an unplanned 802.11b mesh network, in: Proceedings of MobiCom, Cologne, Germany, September 2005, pp. 31–42.
[17] L. Ying, S. Shakkottai, A. Reddy, S. Liu, On combining shortest-path and back-pressure routing over multihop wireless networks, IEEE/ACM Transactions on Networking 19 (3) (2011) 841–854.
[18] R. Steckel, C. Moehling, Rising inequality: trends in the distribution of wealth in industrializing New England, The Journal of Economic History 61 (1) (2001) 160–183.
[19] J.-P. Bouchaud, M. Mézard, Wealth condensation in a simple model of economy, Physica A 282 (3) (2000) 536–545.
[20] Eric J. Friedman, Joseph Y. Halpern, Ian Kash, Efficiency and equilibria in a scrip system for P2P networks, in: Proceeding of the

7th ACM Conference on Electronic Commerce (EC'06), Ann Arbor, USA, June 2006.
[21] B.Q. Zhao, John C.S. Lui, D.M. Chiu, Analysis of adaptive incentive protocols for P2P networks, in: Proceedings of IEEE INFOCOM, Rio de Janeiro, Brazil, April 2009, pp. 325–333.
[22] B.Q. Zhao, John C.S. Lui, D.M. Chiu, Mathematical framework for analyzing adaptive incentive protocols in P2P networks, IEEE/ACM Transactions on Networking 20 (2) (2012) 367–380.
[23] Evans, Merran, Nicholas Hastings and Brian Peacock, Statistical Distributions, third ed., Wiley, 2000.
[24] Patrick P.C. Lee, Hongying Liu, John C.S. Lui, Analyzing credit evolution for credit-based incentive schemes in wireless mesh networks, in: Proceedings of 8th Int. Conference on Sensor, Mesh and Ad-Hoc Communications and Networks, Salt Lake City, Utah, June 2011, pp. 512–520.
[25] L. Santhanam, B. Xie, D. Agrawal, Selfishness in mesh networks: wired multihop MANETs, IEEE Wireless Communications 15 (4) (2008) 16–23.
[26] M.E. Mahmoud, X. Shen, PIS: a practical incentive system for multihop wireless networks, IEEE Transactions on Vehicular Technology 59 (8) (2010) 4012–4015.
[27] M.E. Mahmoud, X. Shen, Stimulating cooperation in multi-hop wireless networks using cheating detection System, in: Proceedings of IEEE INFOCOM, San Diego, CA, March 2010.
[28] S. Buchegger, J.-Y.L. Boudec, Performance analysis of the CONFIDANT protocol: cooperation of nodes-fairness in dynamic ad-hoc networks, in: Proceedings of MobiHOC, EPFL Lausanne, Switzerland, June 2002, pp. 226–236.
[29] M. Felegyhazi, J.-P. Hubaux, L. Buttyan, Nash equilibria of packet forwarding strategies in wireless ad hoc networks, IEEE Transactions on Mobile Computing 5 (5) (2006) 463–476.
[30] D.E. Charilas, A.D. Panagopoulos, A survey on game theory applications in wireless networks, Computer Networks 54 (18) (2010) 3421–3430.
[31] T. Chen, S. Zhong, INPAC: an enforceable incentive scheme for wireless networks using network coding, in: Proceedings of IEEE INFOCOM, San Diego, CA, March 2010.
[32] T. Chen, A. Bansal, S. Zhong, A reputation system for wireless mesh networks using network coding, Journal of Network and Computer Applications 34 (2) (2011) 535–541.
[33] Zhuoqun Xia, Zhigang Chen, Xiaoheng Deng, Ming Zhao, An enforceable incentive scheme in wireless multi-path inter-session network coding game, Journal of Networks 7 (2) (2012) 351–356.

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