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Modeling eBay-like reputation systems: Analysis, characterization and insurance mechanism design

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ABSTRACT

E-commerce systems like eBay are becoming increasingly popular. Having an effective reputation system is critical because it can assist buyers to evaluate the trustworthiness of sellers, and improve the revenue for reputable sellers and E-commerce operators. We formulate a stochastic model to analyze an eBay-like reputation system and propose four measures to quantify its effectiveness: (1) new seller ramp up time, (2) new seller drop out probability, (3) long term profit gains for sellers, and (4) average per seller transaction gains for E-commerce operators. By analyzing a dataset from eBay, we discover that eBay suffers a long ramp up time, low long term profit gains and low average per seller transaction gains. We design a novel insurance mechanism consisting of an insurance protocol and a transaction mechanism to improve the above four measures. We formulate an optimization framework to select appropriate parameters for our insurance mechanism. We conduct experiments on an eBay's dataset and show that our insurance mechanism reduces ramp up time by 91%, improves both the long term profit gains and the average per seller transaction gains by 26.66%. It also guarantees that new sellers drop out with a small probability (close to 0).

1. Introduction

E-commerce systems are becoming increasingly popular and typical examples include Amazon [1], eBay [2], and Taobao [3] of Alibaba (the largest E-commerce system in China), etc. Through an E-commerce system, buyers and sellers can transact online. Sellers advertise products in their online stores (which reside in the E-commerce's web site), buyers can purchase products from any online stores, and the E-commerce system can charge a transaction fee from sellers for each completed transaction. Note that in an E-commerce system, it is possible to buy products from a seller whom the buyer has never transacted with, and this seller may not even be trustworthy [4]. This situation results in a high risk of buying low quality products. To overcome such problems, E-commerce systems usually deploy reputation systems [4].

Usually, E-commerce systems maintain and operate a reputation mechanism to reflect the *trustworthiness* of sellers [2,3]. A high reputation seller can attract more transactions leading to higher revenue [4]. The eBay-like reputation system is the most widely deployed reputation policy, which is used in eBay and Taobao, etc. This type of reputation system is a credit-based system. More precisely, a seller needs to collect enough credits from buyers in order to improve his reputation. These credits are obtained in form of feedback ratings, which are expressed by buyers after transactions are completed. Feedback ratings in eBay and Taobao are of three levels: positive (−1), neutral (0), and negative (−1). A *reputation score*, which is a cumulative sum of all the past feedback ratings of a seller, can be used to reflect the *trustworthiness* of a seller. Furthermore,

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the reputation scores and feedback ratings of all buyers are public information and they are accessible by all buyers and sellers in such an E-commerce system.

Consider this eBay-like reputation system, a "*new*" seller needs to spend a long time, or what we call a long *ramp up time*, so to collect enough credits to be considered reputable. This is because new sellers are initialized with a reputation score of zero, and buyers are less willing to buy products from a seller with low reputation scores. The ramp up time is critical to an E-commerce system since a long ramp up time discourages new sellers to join the E-commerce system. Furthermore, a new user usually starts an online store with certain budgets, and maintaining such online stores involves cost. If a new seller uses up his entire budget and has not yet ramped up his reputation, he may discontinue his online business (i.e., or drop out) due to low revenue. Therefore, a long ramp up time increases the risk that a new seller drops out and discourages potential new sellers to join. Finally, a long ramp up time also results in a low profit gain for a seller. Because before ramping up, a seller can only attract few transactions due to his low reputation score. To an E-commerce operator, this also results in a reduction in revenue.

Reducing ramp up time is challenging and to the best of our knowledge, this is the "*first work*" which reveals the importance of the ramp up time in eBay-like reputation systems. This paper aims to explore the following two fundamental questions: (1) *How to identify key factors which influence the ramp up time*? (2) *How to take advantage of these factors to reduce the ramp up time*? Our contributions are:

- We reveal the ramp up time problem in eBay-like reputation systems. We propose four performance measures to explore this problem: (1) new seller ramp up time, (2) new seller drop out probability, (3) long term profit gains for sellers, and (4) average per seller transaction gains for an E-commerce operator.
- We develop a stochastic model to identify key factors which influence these four performance measures. We apply our model to analyze a real-life dataset from eBay. We discover that the eBay system suffers a long ramp up time, a high new seller drop out probability, low long term profit gains and low average per seller transaction gains.
- We design a novel insurance mechanism to improve these four performance measures. Our insurance mechanism consists of an *insurance protocol* and a *transaction mechanism*. We formulate an optimization problem to select appropriate parameters for our insurance mechanism, which aims to maximize sellers' incentive in subscribing our insurance. We present an efficient approach to locate the optimal insurance parameters as well.
- We conduct experiments using a dataset from eBay. We infer model parameters from the data and show that our insurance mechanism reduces the ramp up time by 91%, and it improves both the long term profit gains and the average per seller transaction gains by 26.66%. It also guarantees that new sellers drop out with a small probability (very close to 0) and reduces the risk that buyers transact with untrustworthy sellers.

This paper is organized as follows. In Section 2, we present the system model for E-commerce systems. In Section 3 we formulate four measures to explore the ramp up time problem. In Section 4, we derive analytical expressions for these four measures. In Section 5, we present the design of our insurance mechanism. In Section 6, we present tradeoffs in selecting insurance parameters. In Section 7, we present experimental results using an eBay's dataset. Related work is given in Section 8 and we conclude in Section 9.

2. E-commerce system model

An E-commerce system consists of *users*, *products* and a *reputation system*. A user can be a *seller* or a *buyer* or both. Sellers advertise products in their online stores and set a price for each product. Buyers, on the other hand, purchase products through online stores and provide feedbacks to indicate whether a seller advertises products honestly or not. A reputation system is maintained by E-commerce operators to reflect the *trustworthiness* of sellers. A high reputation seller can attract more transactions leading to a high revenue. The reputation system aggregates all the feedbacks, and computes a reputation score for each seller. The reputation scores and feedbacks are public information which are accessible by all buyers and sellers.

Products are categorized into different types. For example, eBay categorizes products into "Fashion", "Electronics", "Collectibles & Art", etc. [2]. We consider $L \geq 1$ types of product. Consider a type $\ell \in \{1, \dots, L\}$ product. A seller sets a price $p_\ell \in [0, 1]$ and the E-commerce operator charges a transaction fee of $T \triangleq \alpha p_\ell$, where $\alpha \in (0, 1)$, after the product is sold.[1] There is a per unit manufacturing cost of $c_\ell \in [0, 1]$. A seller earns a profit of u_ℓ by selling one product, or

$$u_\ell = (1 - \alpha)p_\ell - c_\ell. \tag{1}$$

For the ease of presentation, our analysis focuses on one product type. Note that our analysis can be easily generalized to multiple product types, but for clarity of presentation, we omit the subscript unless we state otherwise.

[1] We can also consider a fixed transaction fee model and our analysis is still applicable. But for brevity, let us consider a transaction fee which is proportional to the selling price.

Table 1
Notation list.

p, c	Price and manufacturing cost of a product
T, u	Transaction fee, unit profit of selling a product
Q_a, Q_i, Q_e, Q_p	Advertised, intrinsic, estimated, perceived product quality
d, C_S	Shipment delay, shipment cost
γ	Critical factor in expressing feedback ratings
$\mathcal{F}, r$	Reputation profile, reputation score for a seller
r_h, θ	Reputation threshold, consistency threshold
β	Discounting factor in estimating product quality
$\mathcal{P}(Q_e, p)$	Probability that a buyer buys a product having estimated quality Q_e and price p
P_{ba}, P_{br}	Probability that a buyer buys a product from a seller labeled as average, reputable
λ_1, λ_2	Buyer's arrival rate before, after a seller ramps up
T_w	The maximum time that a seller is willing to wait to get ramped up
T_r, P_d	Ramp up time, new seller drop out probability
$G_s,\ G_e$	Long term expected profit gains for a seller, average per seller transaction gains for the E-commerce operator
δ	Discounting factor in computing long term expected profit gains G_s
$\lambda_T(\tau)$	Transaction's arrival rate at time slot τ
C_I, D_I, T_d, T_c	Insurance price, deposit, duration time and clearing time
$C_I^*, D_I^*, T_d^*, T_c^*$	Optimal insurance price, deposit, duration time and clearing time
$\widetilde{D}_I$	Insurance deposit threshold to revoke an insurance certificate
λ_I	Transaction's arrival rate to an insured seller
T_r^I, P_r^I	Ramp up time, new seller drop out probability when subscribing an insurance
G_s^I, G_e^I	Long term profit gains, average per seller transaction gains when subscribing an insurance
$\triangle G^I$	Marginal profit gain improvement

2.1. Transaction model

Sellers advertise the product quality in their online stores. Let $Q_a \in [0, 1]$ be the *advertised quality*. The larger the value of Q_a implies the higher the advertised quality. Buyers refer to the advertised quality Q_a in their product adoption. Each product also has an intrinsic quality which we denote as $Q_i \in [0, 1]$ (i.e., the ground truth of the product's quality). The larger the value of Q_i implies the higher the intrinsic quality. Since sellers aim to promote their products, we have $Q_a \geq Q_i$. We emphasize that the intrinsic quality Q_i is a private information, e.g., it is only known to the seller. On the other hand, the advertised quality Q_a is public information which is accessible by all buyers and sellers.

Buyers estimate the product quality by referring to the advertised quality Q_a (we will present the estimating model later). Let $Q_e \in [0, 1]$ be the *estimated quality*. The larger the value of Q_e implies the higher the estimated quality. To purchase a product, a buyer must submit a payment p to the E-commerce system, which will be given to the corresponding seller when the buyer receives the product. There is usually a shipment time (or delay) in any E-commerce systems. We denote the delay as d. Upon receiving a product, a buyer can evaluate its quality and at that moment, he has the *perceived quality*, which we denote as $Q_p \in [0, 1]$. The larger the value of Q_p implies the higher the perceived quality. We assume that buyers can perceive the *intrinsic quality*, i.e., $Q_p = Q_i$. Buyers are satisfied (disappointed) if they find out that the product is at least as good as (less than) it is advertised, or $Q_p \geq Q_a$ ($Q_p < Q_a$).

To attract buyers, an E-commerce system needs to incentivize sellers to advertise honestly, i.e., $Q_a = Q_i$. Many E-commerce systems achieve this by deploying a reputation system. We next introduce a popular reputation system used by many E-commerce systems such as eBay [2] or Taobao [3]. Table 1 summarizes key notations in this paper.

2.2. Baseline reputation system

The eBay-like E-commerce system maintains a reputation system to reflect the trustworthiness of sellers. It consists of a *feedback rating system* and *a rating aggregation policy*. We first consider a baseline reputation mechanism.

Buyers express feedback ratings to indicate whether a seller advertises honestly or not. The eBay-like reputation system adopts a feedback rating system consisting of three rating points,[2] i.e., $\{-1, 0, 1\}$. A positive rating (rating 1) indicates that a product is at least as good as it is advertised, i.e., $Q_p \geq Q_a$. A neutral rating (rating 0) indicates that a buyer is indifferent about the product that he purchased. This happens when the perceived quality is slightly lower than it is advertised, i.e., $Q_p \in [Q_a - \gamma, Q_a)$, where $\gamma \in [0, 1]$ denotes the critical factor. The smaller the value of γ implies that buyers are more critical in expressing ratings, e.g., $\gamma = 0$ means that buyers have zero tolerance on seller overstating the product quality. A negative rating (rating -1) represents that the perceived quality is far smaller than the advertised quality, i.e., $Q_p < Q_a - \gamma$. We have

$$\text{feedback rating} = \begin{cases} 1, & \text{if } Q_p \geq Q_a, \\ 0, & \text{if } Q_a - \gamma \leq Q_p < Q_a, \\ -1, & \text{if } Q_p < Q_a - \gamma. \end{cases}$$

[2] Note that we can easily generalize the model to consider more rating points.

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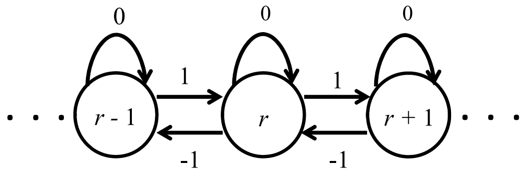

Fig. 1. Transition diagram of a seller's reputation score r.

All the historical ratings are known to all buyers and sellers.

For the rating aggregation policy, each seller is associated with a reputation score, which is the summation of all his feedback ratings. We denote it by $r \in \mathbb{Z}$. A new seller who enters the E-commerce system is initialized with zero reputation score, or $r = 0$. A positive feedback rating increases r by one, a negative feedback rating decreases r by one, and a neutral feedback rating 0 does not change r. Fig. 1 depicts the transition diagram of a seller's reputation score. Note that the reputation score r is a public information accessible by all buyers and sellers.

To assist buyers to evaluate the trustworthiness of a seller, E-commerce systems not only announce the seller's reputation score r, but also his reputation profile. Let $\mathcal{F} \triangleq (r, n^+, n^0, n^-)$ be the reputation profile, where n^+, n^0, n^- represent the *cumulative number of feedback ratings* equal to $1, 0, -1$ respectively. Note that this form of reputation is commonly deployed, say in eBay [2] and Taobao [3].

Shipment delay in real-world E-commerce systems usually results in certain delay in the reputation update. To characterize the dynamics of a reputation updating process, we consider a discrete time system and divide the time into slots, i.e., $[0, d), [d, 2d), \ldots$, where d is the shipment delay. We refer to a time slot $\tau \in \mathbb{N}$ as $[\tau d, (\tau + 1)d)$. Let $N(\tau)$ be the number of products sold by a seller in the time slot τ. Suppose $N^+(\tau), N^0(\tau), N^-(\tau)$ of these transactions result in positive, neutral and negative feedbacks respectively, where $N^+(\tau)+N^0(\tau)+N^-(\tau) = N(\tau)$. Let $\mathcal{F}(\tau) \triangleq (r(\tau), n^+(\tau), n^0(\tau), n^-(\tau))$ be the reputation profile at time slot τ. Initially, the reputation profile of this seller is $\mathcal{F}(0) = (0, 0, 0, 0)$. The reputation profile $\mathcal{F}(\tau)$ is updated as:

$$\begin{cases} \left(n^+(\tau + 1), n^0(\tau + 1), n^-(\tau + 1)\right) = \left(n^+(\tau) + N^+(\tau), n^0(\tau) + N^0(\tau), n^-(\tau) + N^-(\tau)\right), \\ r(\tau + 1) = r(\tau) + N^+(\tau) - N^-(\tau). \end{cases} \tag{2}$$

For simplicity, we drop the time stamp τ in the reputation profile when there is no confusion. We next present a probabilistic model to characterize the impact of sellers' reputation profiles on buyers' product adoption behavior. This model serves as an important building block for us to explore the effectiveness of this baseline reputation system.

2.3. Model for product adoption behavior

A reputation system forges trust among sellers and buyers. This trust plays a critical role in product adoption. More precisely, buyers evaluate the trustworthiness of sellers from sellers' reputation profiles. Buyers seek to minimize their risk in product purchase and they prefer to buy from reputable sellers.

Based on the reputation profile $\mathcal{F}$, our model classifies sellers into two types: "*reputable*" and "*average*". To be labeled as reputable, a seller's reputation profile must satisfy two conditions. The first one is that a seller needs to collect enough credits, i.e., positive feedbacks from buyers. More precisely, his reputation score must be at least greater than or equal to some positive reputation threshold r_h, i.e., $r \geq r_h$. A new seller is initialized with zero reputation score, i.e., $r = 0$. To accumulate a reputation score of at least r_h, a seller needs to accomplish sufficient number of honest transactions. The second condition is that a seller should be *consistently honest*. More concretely, the fraction of positive feedbacks should be larger than or equal to a consistency threshold $\theta \in (0, 1]$, i.e., $n^+/(n^+ + n^- + n^0) \geq \theta$. The larger the value of θ implies that an E-Commerce operator is more critical about the honest consistency. We formally define a reputable seller and an average seller as follows.

Definition 2.1. A seller is labeled as reputable if and only if the following two conditions are met

C1: $r \geq r_h$ and,
C2: $n^+/(n^+ + n^- + n^0) \geq \theta$.

Otherwise, a seller is labeled as an average seller.

Remark. The reputation threshold r_h and consistency threshold θ quantify how difficult it is for a seller to earn a reputable label. The larger the r_h and θ, the more difficult it is to earn a reputable label. An E-commerce operator can control r_h and θ. Some E-commerce systems may not set r_h and θ explicitly, where our model is still applicable because we can infer r_h and θ from historical transaction data.

A buyer estimates the product quality by referring to the advertised quality Q_a and the reputation profile of a seller. If a seller's reputation profile indicates that this seller is reputable, then a buyer believes that this seller advertises honestly.

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This buyer therefore estimates the product quality as the advertised quality, i.e., $Q_e = Q_a$. On the contrary, if the reputation profile indicates that a seller is average, a buyer believes that this seller is likely to overstate the product quality. Hence the estimated quality is lower than the advertised quality, i.e., $Q_e = \beta Q_a$, where $\beta \in [0, 1]$ denotes the discounting factor. The smaller the value of β implies that buyers are less willing to trust an average seller. We have

$$Q_e = \begin{cases} Q_a, & \text{if } r \geq r_h \text{ and } n^+/(n^+ + n^- + n^0) \geq \theta, \\ \beta Q_a, & \text{otherwise.} \end{cases}$$

A buyer makes the purchasing decision based on the estimated quality Q_e and the product price p. More concretely, the probability that a buyer buys a product increases in Q_e and decreases in p. Formally, we have

$$\Pr[\text{adopts a product}] \triangleq \mathcal{P}(Q_e, p), \tag{3}$$

where $\mathcal{P}$ can be any function as long as it increases in Q_e and decreases in p.

3. Problems formulation

We use four performance measures to quantify the effectiveness of the baseline reputation system mentioned in Section 2. These measures are: (1) ramp up time T_r, (2) new seller drop out probability P_d, (3) long term expected profit gains for a seller G_s, and (4) average per seller transaction gains for the E-commerce system operator G_e. We present our problem formulations and our objective is to identify key factors which can influence these measures. Lastly, we explore an interesting question of whether there are other mechanisms which can reduce the ramp up time and the new seller drop out probability, and improve the long term expected profit gains and average per seller transaction gains.

3.1. *Ramp up time*

Sellers and E-commerce system operators are interested in the minimum time that a new seller needs to collect enough credits, i.e., positive feedbacks from buyers, so that the seller can be classified as reputable. For one thing, a reputable seller can attract more buyers which may result in more transactions, and higher transaction volume implies higher transaction gains to the E-commerce operator. We next formally define the ramp up process and the ramp up condition.

Definition 3.1. A new seller's reputation is initialized as $r = 0$. He needs to collect enough credits, i.e., positive feedbacks from buyers, so that his reputation r can increase to at least r_h. The process of increasing his reputation to r_h is called the ramp up process. Furthermore, when $r \geq r_h$, then we say that the ramp up condition is satisfied.

Recall that $r(\tau)$ denotes the reputation score of a seller at time slot τ. We now formally define the ramp up time.

Definition 3.2. Ramp up time is the minimum time that a seller must spend to accumulate a reputation score of r_h. Let T_r denote the ramp up time, we have

$$T_r \triangleq d \cdot \arg\min_{\tau}\{r(\tau) \geq r_h\}. \tag{4}$$

The ramp up time quantifies how long it will take to collect enough credits from buyers. It is critical to the seller's profit gains. To see this, we next quantify how the ramp up time can affect the transaction's arrival rate.

A seller can attract more buyers when he satisfies the ramp up condition because his online store will receive higher click rate by buyers, therefore increasing his profit gains. Let λ_1 (λ_2) be the buyer's arrival rate before (after) a seller satisfies the ramp up condition. We assume that the buyer's arrival process, both before and after a seller satisfies the ramp up condition, follows a Poisson counting process with parameter λ_1 (before ramping up) and λ_2 (after ramping up) respectively, where $\lambda_1 < \lambda_2$ to indicate that a ramped up seller can attract more buyers. Recall that in Eq. (3) we express the probability that a buyer purchases a product as $\mathcal{P}(Q_e, p)$. If a buyer purchases a product, we say a seller obtains a transaction. Based on the Poisson property, it is easy to see that the transaction's arrival process is also a Poisson counting process. Let $\lambda_T(\tau)$ be the transaction's arrival rate at time slot τ. Let $\mathcal{P}(Q_e(\tau), p)$ be the probability that a buyer adopts a product at time slot τ, where $Q_e(\tau)$ denotes the estimated quality at time slot τ. We can express the transaction's arrival rate as

$$\lambda_T(\tau) \triangleq \begin{cases} \lambda_1 \mathcal{P}(Q_e(\tau), p), & \text{if } r(\tau) < r_h, \\ \lambda_2 \mathcal{P}(Q_e(\tau), p), & \text{if } r(\tau) \geq r_h. \end{cases} \tag{5}$$

Eq. (5) serves as an important building block for us to explore the key factors which influence the ramp up time T_r. Let us formulate our first problem.

Problem 1. Identify key factors which influence the ramp up time T_r, and design a mechanism which can take advantage of these factors to reduce T_r.

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3.2. New seller drop out probability

In real-world E-commerce systems, a new seller may drop out, or move to another E-commerce system, if he does not collect enough credits (i.e., ramp up) within certain time because he cannot obtain enough transactions. For example, a new seller in eBay may drop out if he does not ramp up in one year. This is because a new seller starts an online store with certain budgets, and there are costs associated with maintaining this online business. To model this scenario, let $T_w > 0$ denote the maximum time that a new seller is willing to wait to get ramped up. In other words, if the ramp up time is longer than T_w, a new seller will drop out from that E-commerce system. We assume $T_w/d \in \mathbb{N}$ to accommodate the delay (d) in reputation update.

Definition 3.3. A new seller drops out, if and only if $T_r > T_w$. Let P_d denote the probability that a new seller drops out. We formally have

$$P_d \triangleq \Pr[T_r > T_w]. \tag{6}$$

Sellers and the E-commerce operator are interested to know this new seller drop out probability. The smaller the value of P_d implies that sellers are more likely to continue his online business in the E-commerce system. This is an important measure since a small P_d can attract more new sellers to join the E-commerce system, which will result in higher transaction gains for the E-commerce operator. On the other hand, a large P_d discourages new sellers to participate and can lead to some transaction gain losses for the E-commerce system operator. We therefore consider the second problem.

Problem 2. Identify key factors which influence the new seller drop out probability P_d, and design a mechanism which can take advantage of these factors to reduce P_d.

3.3. Long term profit gains and transaction gains

The profit gain and transaction gain are critical to sellers and E-commerce system operators respectively. We focus on the scenario that sellers are long lived and they aim to maximize their long term profit gains. Recall that u, the unit profit of selling one product, is expressed in Eq. (1). Also recall that $N(\tau)$ denotes the number of products sold in the time slot τ. We emphasize that $N(\tau)$ is a random variable and it follows a Poisson distribution with parameter $\lambda_T(\tau)d$, where $\lambda_T(\tau)$ is derived in Eq. (5). A seller earns a profit of $uN(\tau)$ in the time slot τ. We consider a discounted long term profit gain with a discounting factor of $\delta \in (0, 1]$.

Definition 3.4. Let G_s denote the long term expected profit gains for a seller, which can be expressed as

$$G_s \triangleq E\left[\sum_{\tau=0}^{\infty} \delta^\tau uN(\tau)\right]. \tag{7}$$

Note that when a seller earns a profit u, he also contributes a transaction fee $T = \alpha p$ to the E-commerce operator.

Definition 3.5. Let G_e denote the average per seller transaction gains that a seller pays to the E-commerce operator. We can express it as

$$G_e \triangleq E\left[\sum_{\tau=0}^{\infty} \delta^\tau TN(\tau)\right] = E\left[\sum_{\tau=0}^{\infty} \delta^\tau \alpha pN(\tau)\right] = \frac{\alpha p}{u} G_s. \tag{8}$$

Note that G_s is important to a seller while G_e is important to the E-commerce operator.

Problem 3. Identify key factors which influence the profit gains G_s and the average per seller transaction gains G_e, and design a mechanism to use these factors to improve G_e and G_s.

We next derive $E[T_r]$, P_d, G_s, and G_e. Through this analysis, we identify key factors which influence the above mentioned performance measures. These insights will serve as important building blocks for us to design a mechanism.

4. Analyzing the baseline reputation system

Here, we derive analytical expressions for the expected ramp up time ($E[T_r]$), the new seller drop out probability (P_d), the long term expected profit gains (G_e) and the average per seller transaction gains (G_e). Through this we identify that the reputation threshold (r_h), as well as the probability that a buyer buys a product from an "average labeled" seller (P_{ab}) are two critical factors which influence T_r, P_d, G_s and G_e. Our results indicate that the baseline reputation mechanism described in Section 2, suffers a long ramp up time, a high new seller drop out probability, and small long term profit gains or transaction gains. These insights show that one needs to have a new mechanism to reduce T_r, P_d, and to improve G_s and G_e. We will present this new mechanism in Section 5.

Table 2
Expected ramp up time $E[T_r]$ in days ($P_{ba} = 0.02$, $d = 3$).

λ_1	5	10	15	20	25
$E[T_r]$ ($r_h = 200$)	2001.5	1001.5	668.2	501.5	401.5
$E[T_r]$ ($r_h = 150$)	1501.5	751.5	501.5	376.5	301.5
$E[T_r]$ ($r_h = 100$)	1001.5	501.5	334.8	251.5	201.5

4.1. Deriving the expected ramp up time $E[T_r]$

Let us derive the analytical expression for the expected ramp up time $E[T_r]$. This measure quantifies on average, how long it will take to ramp up a new seller under the baseline reputation mechanism mentioned in Section 2. We consider the scenario that buyers advertise the product quality honestly, i.e., $Q_a = Q_i$. As to how an eBay-like reputation mechanism can guarantee rational sellers to advertise honestly, one can refer to [5]. We like to point out that new sellers can achieve the *lowest ramp up time* by advertising honestly ($Q_a = Q_i$). This is because overstating the product quality, i.e., $Q_a > Q_i$, leads to neutral or negative ratings. Understating the product quality, i.e., $Q_a < Q_i$, results in a decrease in transaction's arrival rate. Hence, the assumption that $Q_a = Q_i$ can be viewed as deriving the best case of T_r for the baseline reputation system. We define the following notations to simplify our analysis.

Definition 4.1. Let $P_{ba} \triangleq \mathcal{P}(\beta Q_i, p)$ and $P_{br} \triangleq \mathcal{P}(Q_i, p)$ denote the probability that a buyer buys a product from an "average labeled" seller and a "reputable" seller respectively.

Theorem 4.1. *The expected ramp up time is*

$$E[T_r] = d \sum_{\tau=1}^{\infty} \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba}(\tau-1)d} \frac{(\lambda_1 P_{ba}(\tau-1)d)^k}{k!}. \tag{9}$$

Furthermore, $E[T_r]$ increases in the reputation threshold r_h, and decreases in the transaction's arrival rate $\lambda_1 P_{ba}$.

Proof. Please refer to Appendix for derivation. □

Remark. A new seller is more difficult to get ramped up if the E-Commerce operator sets a high reputation threshold r_h, or the transaction's arrival rate to an "average labeled" seller ($\lambda_1 P_{ba}$) is low. The computational complexity in evaluating $E[T_r]$ derived in Eq. (9) is $\Theta(\sum_{\tau=1}^{\infty} r_h) = \Theta(\infty)$. We next state a Theorem to approximate $E[T_r]$.

Theorem 4.2. *Let $\widehat{E}[T_r] = d \sum_{\tau=1}^{\widetilde{\tau}} \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba}(\tau-1)d} \frac{(\lambda_1 P_{ba}(\tau-1)d)^k}{k!}$ denote an estimation of $E[T_r]$ derived in Eq. (9). If $\widetilde{\tau} > \max\left\{\left(\ln(1-e^{-0.8\lambda_1 P_{ba} d}) + \ln\epsilon\right)/(0.8\lambda_1 P_{ba} d),\ 125\frac{r_h-1}{\lambda_1 P_{ba} d}\right\}$, then $|\widehat{E}[T_r] - E[T_r]| \le \epsilon$.*

Proof. Please refer to Appendix for derivation. □

Table 2 presents numerical examples on $E[T_r]$, where we fix $P_{ba} = 0.02$, i.e., buyers purchase products from an "average labeled" seller with probability 0.02, and fix $d = 3$, i.e., the reputation updating delay is three days. We vary the buyer's arrival rate λ_1 from 5 to 25, i.e., on average each day an "average labeled" seller attracts 5 to 25 buyers to visit his online store. We vary the reputation threshold (r_h) from 100 to 200. Applying Theorem 4.2 we set $\widetilde{\tau} = \max\left\{\left(\ln(1-e^{-0.048\lambda_1}) + \ln 0.01\right)/(0.048\lambda_1),\ 125\frac{r_h-1}{\lambda_1 0.06}\right\}$ to guarantee $|\widehat{E}[T_r] - E[T_r]| \le 0.01$. When $r_h = 200$, as λ_1 increases from 5 to 25, the expected ramp up time ($E[T_r]$) drops from 2001.5 to 401.5 days, a deduction ratio of 80%. When the buyer's arrival rate is low, say $\lambda_1 = 5$, as the reputation threshold r_h drops from 200 to 100, the expected ramp up time $E[T_r]$ drops from 2001.5 to 1001.5 days, a reduction ratio of 50%. These results show that the expected ramp up time ($E[T_r]$) is large in general. Namely, it is difficult for new sellers to quickly get ramped up under the baseline reputation system. We next explore the new seller drop out probability.

4.2. Deriving the new seller drop out probability P_d

We now derive the analytical expression for P_d. This probability quantifies how difficult it is for a new seller to *survive* in an E-commerce system. Note that P_d is also crucial for new sellers because a potential new seller can use it to decide whether or not to open an online store in that E-commerce system. Therefore, a low drop out probability P_d is attractive to new sellers, while a high P_d discourages new sellers to join.

Theorem 4.3. *The new seller drop out probability is $P_d = \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba} T_w} \frac{(\lambda_1 P_{ba} T_w)^k}{k!}$. The P_d decreases in $\lambda_1 P_{ba}$, T_w and increases in r_h.*

Proof. Please refer to Appendix for derivation. □

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Table 3
New seller drop out probability P_d ($\lambda_1 = 20, T_w = 180, d = 3$).

P_{ba}	0.01	0.02	0.03	0.04	0.05
P_d ($r_h = 200$)	1.00000	1.00000	1.00000	0.99999	0.92514
P_d ($r_h = 150$)	1.00000	1.00000	0.99992	0.68056	0.00991
P_d ($r_h = 100$)	1.00000	0.99897	0.20819	0.00005	0.00000

Table 4
Long term expected profit gains G_s and average per seller transaction gains G_e ($\lambda_1 = 20, \lambda_2 = 50, u = 1, T = 0.1, \delta = 0.99, T_w = 180, P_{br} = 0.1, d = 3$).

P_{ba}	0.01	0.02	0.03	0.04	0.05
G_s ($r_h = 200$)	27.171	54.341	81.512	108.687	198.027
G_s ($r_h = 150$)	27.171	54.341	81.575	376.868	1014.980
G_s ($r_h = 100$)	27.171	55.200	767.515	1065.440	1154.580
G_e ($r_h = 200$)	2.7171	5.4341	8.1512	10.8687	19.8027
G_e ($r_h = 150$)	2.7171	5.4341	8.1575	37.6868	101.4980
G_e ($r_h = 100$)	2.7171	5.5200	76.7515	106.5440	115.4580

Remark. Theorem 4.3 states that a new seller can reduce the drop out probability by extending his ramp up deadline line (T_w), and a new seller is more likely to drop out if the reputation threshold (r_h) increases or the transaction's arrival rate to an "average labeled" seller ($\lambda_1 P_{ba}$) decreases.

Table 3 presents numerical examples on the new seller drop out probability P_d, where we set $\lambda_1 = 20$, i.e., on average, each day an "average labeled" seller attracts 20 buyers to visit his store, $d = 3$, and $T_w = 180$, i.e., sellers drop out if they do not ramp up in 180 days. We vary P_{ba}, the probability that a buyer buys products from an "average labeled" seller, from 0.01 to 0.05, and vary the reputation threshold r_h from 100 to 200. Consider $r_h = 200$. As P_{ba} increases from 0.01 to 0.05, the new seller drop out probability P_d decreases from 1 to 0.92514. This implies a very high drop out probability. Consider $P_{ba} = 0.03$. As the reputation threshold r_h drops from 200 to 100, we see that P_d drops from 1 to 0.20819, a reduction ratio of around 80%. It is interesting to observe that when the P_{ba} is small, the new seller drop out probability is quite high. In fact when $P_{ba} = 0.01$, P_d is very close to 1. In other words, if buyers are less willing to buy from "average labeled" sellers, new sellers will be more likely to drop out. We next explore key factors which influence long term expected profit gains and average per seller transaction gains.

4.3. Deriving the long term profit gains G_s and G_e

Let us now derive analytical expressions for the long term expected profit gains G_s and the average per seller transaction gains G_e respectively. They are important measures because a large G_s is attractive to new sellers and a small G_s discourages new sellers to join the E-commerce system, while the average per seller transaction gains G_e is crucial to the E-commerce system operator.

Theorem 4.4. *The long term expected profit gains for a new seller can be expressed as*

$$G_s = \frac{\lambda_2 P_{br} d}{1-\delta} - \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba} T_w} \frac{(\lambda_1 P_{ba} T_w)^k}{k!} \frac{\lambda_2 P_{br} d \delta^{T_w/d}}{1-\delta} + \sum_{\tau=0}^{T_w/d-1} \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba} \tau d} \frac{(\lambda_1 P_{ba} \tau d)^k}{k!} (\lambda_1 P_{ba} - \lambda_2 P_{br}) d \delta^{\tau}. \quad (10)$$

Furthermore, $G_e = \frac{\alpha p}{u} G_s$.

Proof. Please refer to Appendix for derivation. □

Remark. Theorem 4.4 quantifies the impact of various factors on the long term expected profit gains (G_s) and average per seller transaction gains (G_e), e.g., the reputation threshold r_h, the buyer's arrival rate λ_1, λ_2, etc. We next present some numerical examples to illustrate their impact.

Table 4 presents numerical examples on the long term expected profit gains (G_s) and the average per seller transaction gains (G_e), where we set $\lambda_1 = 20, T = 0.1, \lambda_2 = 50, u = 1, T_w = 180, d = 3, P_{br} = 0.1$ (i.e., buyers buy products from a reputable seller with probability 0.1). We vary P_{ba}, the probability that a buyer purchases products from an "average labeled" seller, and the ramp up threshold r_h respectively to examine their impact on G_s and G_e. Consider $r_h = 200$. As P_{ba} increases from 0.01 to 0.05, G_s improves from 27.171 to 198.027, an improvement ratio of 7.29 times. Similarly, the average per seller transaction gains G_e is also improved by 7.29 times. This implies that P_{ba} is critical to both sellers' profit gains and the E-commerce system operator's transaction gains. Consider $P_{ba} = 0.05$. As r_h drops from 200 to 100, G_s improves from 198.027 to 1154.580, an improvement ratio of 5.83. This improvement ratio also holds for the average per seller transaction grains G_e. It is interesting to observe that when P_{ba} is small, both G_s and G_e are quite small. In fact when $P_{ba} = 0.01$, the G_s

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is around 27.171 and G_e is around 2.7171. Namely, if buyers are less willing to buy from "average labeled" sellers, sellers (E-commerce operators) will have low long term profit gains (average per seller transaction gains).

Summary: The reputation threshold r_h and P_{ba} are critical to the ramp up time, the new seller drop out probability, the long term profit gains and the average per seller transaction gains. The baseline (or eBay-like) reputation system presented in Section 2 suffers a long ramp up time, a high new seller drop out probability, low long term profit gains and low average per seller transaction gains. Hence, it is important to ask whether we can design a new mechanism that an E-commerce system can use to improve all the performance measures $E[T_r]$, P_d, G_s and G_e. We next explore this interesting question.

5. Insurance mechanism design

The objective of our insurance mechanism is to help new sellers ramp up quickly. Our insurance mechanism consists of an *insurance protocol* and a *transaction mechanism*.

We first describe the *insurance protocol*. The E-commerce system operator provides an insurance service to new sellers. Each insurance has a price of $C_I > 0$, a duration time of $T_d > 0$, and a clearing time of $T_c > 0$. Without loss of generality, we assume $\frac{T_d}{d} \in \mathbb{N}$ and $\frac{T_c}{d} \in \mathbb{N}$ in order to accommodate the delay (d) in reputation update. The insurance clearing time takes effect when an insurance expires. To buy an insurance, a seller must provide the E-commerce operator an insurance deposit of D_I. Hence, the total payment by the new seller to the E-commerce system operator is $C_I + D_I$. We refer to this insurance as the (C_I, T_d, T_c, D_I)-insurance. Only new sellers can subscribe to this insurance. If a seller subscribes an insurance, the E-commerce system operator issues an insurance certificate to him, and this certificate is known to the public (i.e., all buyers and sellers). This certificate only takes effect within the insurance duration time T_d. The E-commerce system operator treats a seller with an insurance certificate as trustworthy. To guarantee that such sellers will advertise their product quality honestly, the E-commerce system operator requires such sellers obey the following rules based on our *transaction mechanism*.

We now describe the *transaction mechanism*. Only sellers with an insurance certificate have to obey these transaction rules. Let us focus on a seller with an insurance certificate. When ordering a product from this seller, a buyer sends his payment p to the E-commerce system operator. After receiving the product, if this buyer provides a positive feedback, then this transaction completes, i.e., the E-commerce operator forwards the payment $(1 - \alpha)p$ to the seller and charges a transaction fee of αp. This transaction also completes if this buyer expresses a neutral feedback. A neutral feedback means that a seller slightly overstated his product quality, i.e. $Q_i < Q_a < Q_i + \gamma$. To avoid such overstating, the E-commerce company revokes a seller's insurance certificate once the fraction of positive feedbacks falls below the consistency factor (θ), i.e., $n^+/(n^+ + n^0 + n^-) < \theta$. A negative feedback results in the transaction being revoked. More concretely, the E-commerce operator gives the payment p back to the buyer and does not charge any transaction fee from the seller (provided that it is within the duration time T_d or the clearing time T_c). The buyer needs to ship the product back to the seller but the buyer does not need to pay for the shipment cost C_S, because it will be deducted from a seller's insurance deposit D_I. If the insurance deposit is not enough to cover C_S, the E-commerce operator makes a supplemental payment. To avoid this undesirable outcome, the E-commerce company revokes a seller's insurance certificate, once a seller's deposit falls below a threshold $\widehat{D}_I < D_I$. The insurance clearing time takes effect when an insurance is invoked. At the end of the clearing time, the E-commerce company returns the remaining deposit (if it is not deducted to zero) to the seller.

Remark. Note that sellers may collude with buyers to inflate their reputation by fake transactions [6]. One way to avoid such collusion is by increasing the transaction fee such as [7]. The shipment cost may exceed D_I due to a large number of products to be returned. This can be avoided with high probability by setting a large $\widehat{D}_I$ (refer to Theorem 5.2). We also derive the minimum clearing time (T_c) to guarantee that a seller with an insurance certificate needs to obey the transaction mechanism (refer to Theorem 5.2).

5.1. Analyzing the insurance mechanism

We first show that buyers treat a seller having an insurance certificate as trustworthy. We derive the transaction rate for a seller who has an insurance certificate. We then derive the improved $E[T_r]$, P_d, G_s and G_e.

Buyers treat sellers having an insurance certificate as trustworthy. This is an important property of our insurance mechanism because it influences the probability that a buyer adopts a product from a seller. Suppose in time slot τ, a seller has an insurance certificate. If this seller advertises honestly $Q_a = Q_i$, then the buyer who buys a product from this seller will be satisfied (express positive feedback rating). In this case, the payment from the buyer will be forwarded to the seller. Hence this seller earns a profit of u. If this seller overstates his product quality beyond the lenient factor (γ), i.e., $Q_a > Q_i + \gamma$, then according to our insurance mechanism, the payment by the buyer will be returned to the buyer. The seller needs to pay a shipment cost of C_S to ship back the product and C_S will be deducted from his insurance deposit D_I. Hence, if a seller overstates the product quality beyond the lenient factor, he will lose a total shipment cost of at least $\min\{\widehat{D}_I, C_S N(\tau)\}$ in time slot τ, where $N(\tau)$ denotes the number of product selling. A seller with an insurance certificate must obey the same consistency factor (θ) as reputable sellers in being honest, i.e., $n^+/(n^+ + n^0 + n^-) \geq \theta$, because if not his insurance certificate will be revoked by the E-commerce operator. Given these properties, buyers trust a seller with an insurance certificate.

Table 5
Impact of our insurance on $E[T_r]$, P_d, G_s and G_e.

	$r_h=100$				$r_h=150$				$r_h=200$			
	$E[T_r]$	P_d	G_s	G_e	$E[T_r]$	P_d	G_s	G_e	$E[T_r]$	P_d	G_s	G_e
Baseline	168	0.2	768	76.8	252	1	81.6	8.16	335	1	81.5	8.15
Insurance	21.5	0	1500	150	31.5	0	1500	150	41.5	0	1500	150
Improvement	87%	100%	95%	95%	88%	100%	1738%	1738%	88%	100%	1740%	1740%

Recall that the E-commerce operator also trusts a seller with an insurance certificate. Therefore, an insured seller can attract transactions with an arrival rate being equivalent to those reputable sellers. Let λ_T^I denote the transaction's arrival rate to a seller with an insurance certificate. We have

$$\lambda_T^I = \lambda_2 P_{br}. \tag{11}$$

We now quantify the impact of our insurance mechanism on the four performance measures. Let $T_r^I, P_d^I, G_s^I, G_e^I$ denote the ramp up time, the new seller drop out probability, the long term profit gains and the average per seller transaction gains respectively, when a new seller subscribes our insurance.

Theorem 5.1. *Suppose a new seller subscribes to our proposed insurance mechanism, the expected ramp up time and the new seller drop out probability are*

$$E[T_r^I] = d\sum_{\tau=1}^{\infty}\sum_{k=0}^{r_h-1} e^{-\sum_{\ell=0}^{\tau-2}\widetilde{\lambda}_T(\ell)} \frac{\left(\sum_{\ell=0}^{\tau-2}\widetilde{\lambda}_T(\ell)\right)^k}{k!}, \qquad P_d^I = \sum_{k=0}^{r_h-1} e^{-\sum_{\ell=0}^{T_w/d-1}\widetilde{\lambda}_T(\ell)} \frac{\left(\sum_{\ell=0}^{T_w/d-1}\widetilde{\lambda}_T(\ell)\right)^k}{k!},$$

where $\widetilde{\lambda}_T(\ell) = \lambda_2 P_{br} d$ for all $\ell = 0, 1, \ldots, T_d/d - 1$, and $\widetilde{\lambda}_T(\ell) = \lambda_1 P_{ba} d$ for all $\ell = T_d/d, \ldots, \infty$. The long term expected profit gains for a new seller who subscribes an insurance is:

$$\begin{aligned} G_s^I = \mathbf{I}_{\{T_d<T_w\}} \Bigg(& \frac{\lambda_2 P_{br} d}{1-\delta} - \sum_{k=0}^{r_h-1} e^{-\lambda_2 P_{br} T_d}\frac{(\lambda_2 P_{br} T_d)^k}{k!} \sum_{k'=0}^{r_h-1-k} e^{-\lambda_1 P_{ba}(T_w-T_d)} \frac{(\lambda_1 P_{ba}(T_w-T_d))^{k'}}{k'!}\frac{\lambda_2 P_{br} d\delta^{T_w/d}}{1-\delta} \\ & + \sum_{\tau=T_d/d}^{T_w/d-1}\sum_{k=0}^{r_h-1} e^{-\lambda_2 P_{br} T_d}\frac{(\lambda_2 P_{br} T_d)^k}{k!} \sum_{k'=0}^{r_h-1-k} e^{-\lambda_1 P_{ba}(\tau d - T_d)}\frac{(\lambda_1 P_{ba}(\tau d-T_d))^{k'}}{k'!}(\lambda_1 P_{ba} - \lambda_2 P_{br}) d\delta^{\tau}\Bigg) \\ & + \mathbf{I}_{\{T_d\ge T_w\}}\left(\frac{\lambda_2 P_{br} d}{1-\delta} - \sum_{k=0}^{r_h-1} e^{-\lambda_2 P_{br} T_w}\frac{(\lambda_2 P_{br} T_w)^k}{k!}\frac{\lambda_2 P_{br} d\delta^{T_w/d}}{1-\delta}\right). \end{aligned} \tag{12}$$

G_s^I increases in T_d. Furthermore, $G_e^I = \frac{\alpha p}{u} G_s^I$.

Proof. Please refer to Appendix for derivation. □

Remark. Theorem 5.1 quantifies the impact of our insurance mechanism on four important performance measures. Before we discuss more about how to select insurance parameters, i.e., C_I, D_I, T_d, T_c, let us illustrate the effectiveness of our insurance mechanism using some numerical examples. The computational complexity in evaluating $E[T_r^I]$ is $\Theta(\sum_{\tau=1}^{\infty} r_h) = \Theta(\infty)$. Theorem 4.2 can be easily extended to approximate $E[T_r^I]$.

Table 5 presents numerical examples on $E[T_r], P_d, G_s$ and G_e under the baseline reputation setting and the insurance setting, where we refer to improvement as deducting ratio for $E[T_r]$ and P_d, and as improving ratio for G_s and G_e. We use the following setting: $\lambda_1 = 20, \lambda_2 = 50, u = 1, T_w = 180, d = 3, P_{br} = 0.1, P_{ba} = 0.03, C_I = 100, D_I = 100, \widehat{D}_I = 50, C_S = 0.5, T = 0.1, T_d = 99, T_c = 3, \delta = 0.99$. When $r_h = 100$, we have $E[T_r] = 168$ and $E[T_r^I] = 21.5$. In other words, our insurance mechanism reduces the expected ramp up time from 168 days to only 21.5 days, or over 87% reduction. It is interesting to observe that our incentive mechanism reduces the new seller drop out probability from $P_d = 0.2$ to $P_d^I = 0$. Namely, our insurance mechanism can guarantee that new sellers ramp up before the deadline line T_w with a high probability (very close to 1.0). In addition, our insurance mechanism improves long term expected profit gains from $G_s = 761$ to $G_s^I = 1485$, a 95% improvement. This improvement ratio also holds for average per seller transaction gains. As r_h increases from 100 to 200, the improvement on the $E[T_r], P_d, G_s, G_e$, becomes more significant. We next state the appropriate values for $C_I, D_I, \widehat{D}_I$ and T_c in the following theorem.

Theorem 5.2. *An upper bound for the insurance price C_I is $C_I < G_s^I - G_s$. If D_I and $\widehat{D}_I$ satisfy $D_I \ge \widehat{D}_I \ge C_S \max\left\{\ln\epsilon^{-1} - \lambda_2 P_{br} T_d, e^2\lambda_2 P_{br} T_d\right\}$, then $\Pr[\text{shipment cost exceeds} D_I] \le \epsilon$. If $T_c \ge d$, then all products sold by a seller with an insurance certificate can be guaranteed to obey the insurance mechanism.*

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Proof. Please refer to Appendix for derivation. □

Remark. Note that the insurance price should be lower than $G_s^I - G_s$, otherwise sellers have no incentive to subscribe an insurance. The clearing time should be larger or equal to d in order to guarantee all products sold by a seller with an insurance certificate obey the insurance mechanism. To guarantee that the insurance deposit covers the shipment cost for returning products with high probability, D_I and $\widehat{D}_I$ need to be no less than $C_S \max\left\{\ln \epsilon^{-1} - \lambda_2 P_{br} T_d, e^2 \lambda_2 P_{br} T_d\right\}$. We next formally show how to select C_I, D_I, T_d, T_c subject to different tradeoffs.

6. Tradeoffs in insurance mechanism

Here, we formulate a metric, the marginal profit gain improvement ΔG^I, to quantify sellers' incentive in subscribing our insurance. We formulate an optimization framework to select appropriate parameters for our insurance mechanism, i.e., C_I, D_I, T_d, T_c, which aims to maximize ΔG^I. We present an efficient method to locate the optimal C_I, D_I, T_d, T_c.

6.1. Metrics

An E-commerce system operator wants to incentivize new sellers to subscribe to our insurance. To quantify sellers' incentive in subscribing an insurance, we define marginal profit gain improvement by an (C_I, T_d, T_c, D_I)-insurance as

$$\Delta G^I \triangleq G_s^I - G_s - C_I - D_I + D_I \delta^{(T_d+T_c)/d}. \tag{13}$$

The physical meaning is that when subscribing an insurance, a seller pays $C_I + D_I$ in total, and a remaining deposit of D_I (because sellers advertise honestly, hence no deposit is deducted) will be returned to a seller after $(T_d + T_c)/d$ time slots. The larger the value of ΔG^I, the higher the incentive that new sellers will subscribe to our insurance.

6.2. Insurance design to maximize marginal profit gains improvement

Our objective is to select C_I, D_I, T_d, T_c so as to maximize ΔG^I. The optimization formulation is:

$$\max_{C_I, D_I, T_d, T_c} \ \Delta G^I \triangleq G_s^I - G_s - C_I - D_I + D_I \delta^{(T_d+T_c)/d}.$$

$$s.t. \quad \text{Eqs. (10) and (12) hold, } C_I \geq 0, \quad T_d/d \in \mathbb{N},$$

$$T_c \geq d, \quad T_c/d \in \mathbb{N}, \tag{14}$$

$$D_I \geq C_S \max\{\ln \epsilon^{-1} - \lambda_2 P_{br} T_d, e^2 \lambda_2 P_{br} T_d\}, \tag{15}$$

where Inequality (15) specifies the minimum deposit, which is derived in Theorem 5.2, and Inequality (14) guarantees that all products sold by a seller with an insurance certificate can be guaranteed to obey the insurance mechanism.

Theorem 6.1. *The optimal insurance price, insurance clearing time and insurance deposit satisfy* $C_I^* = 0$, $T_c^* = d$ *and* $D_I^* = C_S \max\left\{\ln \epsilon^{-1} - \lambda_2 P_{br} T_d, e^2 \lambda_2 P_{br} T_d\right\}$ *respectively.*

Proof. Please refer to Appendix for derivation. □

Remark. The above theorem implies that to maximize ΔG^I (or maximize sellers' incentive to subscribe an insurance), an E-commerce operator should set the insurance price to be zero, set the clearing time to be d, and set the insurance deposit to be $\max\{\ln \epsilon^{-1} - \lambda_2 P_{br} T_d, e^2 \lambda_2 P_{br} T_d\}$. Theorem 6.1 simplifies our optimization formulation as follows:

$$\max_{T_d} \Delta G^I \triangleq G_s^I - G_s - (1 - \delta^{T_d/d+1}) C_S \max\left\{\ln \epsilon^{-1} - \lambda_2 P_{br} T_d, e^2 \lambda_2 P_{br} T_d\right\}.$$

$$s.t. \quad \text{Eqs. (10) and (12) hold, and } T_d/d \in \mathbb{N}.$$

The remaining issue is to locate the optimal insurance duration time for the above simplified optimization problem. In the following theorem, we derive an upper bound for the optimal duration time.

Theorem 6.2. *The optimal insurance duration time* T_d^* *satisfies* $T_d^* \leq d \max\left\{\left\lceil \frac{\ln 0.5}{\ln \delta} \right\rceil - 1, \left\lceil 2 \frac{u\lambda_2 P_{br}(1-\delta)^{-1} - G_s}{d C_S e^2 \lambda_2 P_{br}} \right\rceil\right\}$.

Proof. Please refer to Appendix for derivation. □

Remark. With the upper bound of T_d^* derived Theorem 6.2, and note that $T_d/d \in \mathbb{N}$, one can easily locate the optimal insurance duration time by exhaustive search. And the complexity is $\Theta(\max\{\lceil \frac{\ln 0.5}{\ln \delta} \rceil - 1, \lceil 2 \frac{u\lambda_2 P_{br}(1-\delta)^{-1} - G_s}{d C_S e^2 \lambda_2 P_{br}} \rceil\})$. Actually, this complexity is quite low, e.g., for the eBay setting in Section 7, the complexity is $\Theta(693)$. Hence our exhaustive search method is quite efficient and reasonable.

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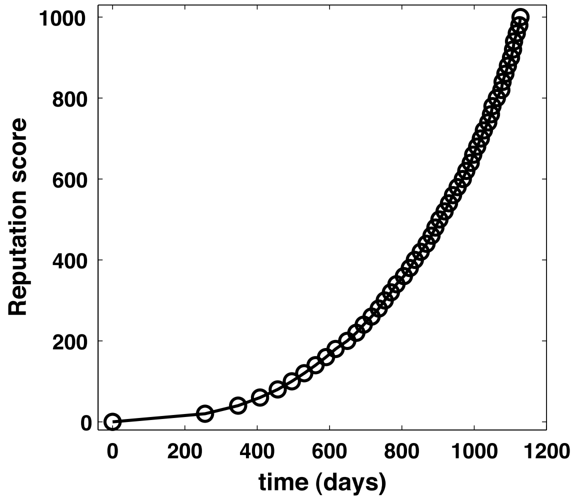

Fig. 2. Reputation score accumulating in the eBay system.

Table 6
Statistics for eBay feedback rating dataset.

Number of sellers	4362
Total number of ratings	18,533,913
Maximum/Minimum on number of ratings per seller	117,100/1
Mean/Median on number of ratings per seller	4190/1437
Rating metric	$\{-1, 0, 1\}$

7. Experiments on real-world data

We conduct experiments using a real-life dataset from eBay. We identify that the ramp up time in eBay is quite long (i.e., 777 days on average), new sellers drop out probability is quite high, and both expected long term profit gains and average transaction gains are low. We select the optimal parameters for our insurance mechanism and show that our insurance mechanism reduces the ramp up time by 91%, improves both expected long term profit gains and average per seller transaction gains by 26.66%. It also guarantees that new sellers ramp up before the deadline T_w with a high probability (very close to 1).

7.1. eBay dataset

We crawled and obtained historical feedback ratings from eBay, the overall statistics are shown in Table 6. This dataset was obtained in April 2013. Each seller's historical ratings in our dataset starts from the first day that a seller joined eBay to April 2013. Ebay is a popular E-commerce system that assists customers in online product purchasing, and it uses a reputation mechanism to assist buyers to assess the trustworthiness (or reputation) of sellers. Buyers express a rating $\in \{-1, 0, 1\}$ to the seller after purchasing a product. This rating indicates whether a seller is trustworthy or not. Ratings are public to all buyers and sellers.

7.2. Inferring model parameters

We first infer the reputation threshold r_h. Recall that r_h mainly affects the transaction's arrival rate. More precisely, a seller having a reputation score smaller than r_h attracts transactions with a rate significantly smaller than a seller having a reputation score higher than r_h. In our data, we find out that the fraction of transactions that result in positive scores is 99.43%. Namely, the transaction's arrival rate is roughly the same as the reputation score accumulation rate. To identify r_h, we plot the reputation score accumulation rate in Fig. 2, where the vertical axis shows the reputation score and the horizontal axis shows the average number of days to accumulate a given number of reputation score. One can observe that to accumulate a reputation score of 200, it takes around 600 days on average, a relatively long duration. In other words, sellers having reputation score smaller than 200 attract transactions with a small rate. To increase the reputation score from 200 to 1000, it only takes around 500 days. This implies that the transaction rate is significantly larger than that corresponds to a seller having a reputation score smaller than 200. We therefore set the reputation threshold as 200, or $r_h = 200$.

We now infer the transaction's arrival rate. We infer the transaction's arrival rate before a seller ramps up ($\lambda_1 P_{ba}$) as the total number of transactions by sellers having a reputation score smaller than r_h, divided by the total time to accumulate

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Table 7
Optimal insurance parameters for eBay.

	C_I^*	D_I^*	T_d^*	T_c^*
eBay	0	566.93	93 days	3 days

Table 8
Expected ramp up time in eBay ($C_I = 0, D_I = \widehat{D}_I = 566.93, T_d = 93$ days and $T_c = 3$ days).

	Baseline ($E[T_r]$)	Insurance ($E[T_r^I]$)	Improvement ($(E[T_r] - E[T_r^I])/E[T_r]$)
eBay	792 days	75 days	91%

Table 9
New seller drop out probability in eBay ($C_I = 0, D_I = \widehat{D}_I = 566.93, T_d = 93$ days and $T_c = 3$ days).

	Baseline (P_d)	Insurance (P_d^I)
$T_w = 1$ year	1	0
$T_w = 2$ years	0.7853	0
$T_w = 3$ years	1.03×10^{-7}	0

these transactions. From our data, we obtain the following:

$$\lambda_1 P_{ba} = \frac{\text{Number of transactions by sellers having } r < r_h}{\text{Total time to accumulate these transactions}} = 0.253.$$

Namely, on average, before ramping up a seller can attract 0.2578 transactions per day. Similarly, we infer the transaction's arrival rate to a reputable seller as the total number of transactions by reputable sellers (i.e., having a reputation score larger than r_h and the fraction of positive score is larger than 0.9), divided by the total time to accumulate these transactions. From our data, we obtain the following:

$$\lambda_2 P_{br} = \frac{\text{Number of transactions by sellers having } r > r_h}{\text{Total time to accumulate these transactions}} = 2.724.$$

Namely, on average, a reputable seller attracts 2.7496 transactions per day. We next present the experimental results.

7.3. Experimental results

We set a shipment delay of $d = 3$ days, a shipment cost of $C_S = 0.1$, a unit profit of $u = 1$, a transaction fee of $T = 0.1$, a discounting factor of $\delta = 0.999$. We also use the inferred parameters in Section 7.2.

We first select the optimal insurance parameter for eBay. We input the parameters inferred in Section 7.2 into our optimization framework mentioned in Section 6. Table 7 presents the optimal insurance parameters, where C_I^*, D_I^*, T_d^*, T_1^* represent the optimal insurance price, deposit, duration time and clearing time respectively. It is interesting to observe that the optimal insurance price, duration time, deposit and clearing time are $C_I^* = 0$, $D_I^* = 566.93$, $T_d^* = 93$ days and $T_c^* = 3$ days respectively. In the following experiments, we set these optimal insurance parameters as default to show the effectiveness of our insurance mechanism.

We explore the ramp up time ($E[T_r]$). We input the inferred parameters into our model. We obtain the expected ramp up time of eBay and the expected ramp up time when a new seller subscribes to our proposed insurance. Table 8 presents the expected ramp up time, where $E[T_r]$ denotes the ramp up time in eBay, $E[T_r^I]$ denotes the ramp up time when a new seller subscribes to an insurance, and $(E[T_r] - E[T_r^I])/E[T_r]$ denotes the reduction ratio. One can observe that the ramp up time in eBay is 792 days, a very long duration. Our insurance mechanism reduces the ramp up time to 75 days, with a reduction of 91%. This is a significant reduction.

We now examine the new seller drop out probability (P_d). We vary T_w (the maximum time that a seller is willing to wait to get ramped up) from one year to three years. Table 9 presents the new seller drop out probability when a seller subscribes to or declines an insurance. Consider $T_w = 1$ year, i.e., a seller is willing to wait one year to get ramped up. In the eBay setting, the probability that he will drop out is very close to 1. Sellers can reduce the drop out probability to 0.7853 if they are willing to wait $T_w = 2$ years. However, the drop out probability is still high. It can be reduced to 1.3×10^{-7}, if the seller is willing to wait $T_w = 3$ years. However a waiting time of three years is too long. This implies that it is difficult for new sellers to continue their business in eBay. It is interesting to observe that if a seller subscribes to an insurance, the drop out probability is very close to 0 for all $T_w = 1, 2, 3$ years.

We now explore the long term expected profit gains and the average per seller transaction gains (G_s and G_e). We input the inferred parameters into our model. We compute G_s and G_s^I. Table 10 presents numerical results on G_s and G_s^I, where $(G_s^I - G_s)/G_s$ and $(G_e^I - G_e)/G_e$ denote the improvement ratio. One can observe that a seller in eBay can earn a long term

Table 10
Long term profit gains and average per seller transaction gains in eBay ($C_I = 0$, $D_I = \widehat{D}_I = 566.93$, $T_d = 93$ days and $T_c = 3$ days, $T_w = 3$ years).

Baseline (G_s)	Insurance (G_s^I)	Improvement ($\frac{G_s^I - G_s}{G_s}$)	Baseline (G_e)	Insurance (G_e^I)	Improvement ($\frac{G_e^I - G_e}{G_e}$)
6452	8172	26.66%	645.2	817.2	26.66%

profit gain of $G_s = 6452$ on average. It can be improved to $G_s^I = 8172$ on average, if a new seller subscribes to our insurance. The long term profit gains is improved by $(G_s^I - G_s)/G_s = 26.66\%$. This improvement ratio also holds for average per seller transaction gains G_e.

8. Related work

Research on reputation systems [4] for internet services has been quite active. Many aspects of reputation systems have been studied, i.e., reputation metric formulation and calculation [8–10], attacks and defense techniques for reputation systems [11,6,12,13], and effectiveness of reputation systems [14]. A survey can be found in [15].

Reputation metric formulation and calculation have been studied extensively. Two most representative reputation calculating models are eBay-like reputation models [8] and transitive trust based models [11]. The eBay-like reputation model computes the reputation score by summarizing explicit human feedbacks (or ratings) [16,8,17,18]. The transitive trust based model [11,19,10,9,12] assumes that if user A trusts user B and user B trusts user C, then user A trusts user C. More precisely, each user is represented by a node in a graph, and the weighted directed link from A to B quantifies the degree that user A trusts user B. For this model, many algorithms were developed to compute an overall reputation score for each user [11,19,10,9,12]. These works provided theoretical foundations for reputation computing. Our work is different from them in that we bring out the ramp up time problem in eBay-like reputation systems, which has not been studied before. We use eBay data to show that the ramp up time is critical to the effectiveness of eBay-like E-commerce systems. Our results uncover many practical insights to enrich the theoretical research on reputation formulation and calculation.

Several works have explored attack and defense techniques for reputation systems. One type of potential attacks is that users may not give honest feedbacks. Peer-prediction method based mechanisms were proposed to elicit honest feedbacks [20–22]. Another type of potential attacks is reputation inflation, or self-promotion. Many works have been done to address this issue [11,6,12,13]. A survey on attack and defense techniques for reputation systems can be found in [6]. Note that these defense techniques may result in a long ramp up time or low profit gains for sellers. This may discourage new sellers to join an E-commerce system. Our work uncovers an important factor, i.e., ramp up time, in which many defense techniques need to be aware of. This will improve the practical usage of defense techniques.

The most closely related works are [7,14,5,23], which studied effectiveness of eBay-like reputation mechanisms. Authors in [7] derived the minimum transaction fee to avoid ballot stuffing (i.e., fake positive feedbacks). Author in [14] proposed to use buyer friendship relationship to filter out unfair ratings. In [14], the author explored the impact of buyers biases' (i.e., leniency or criticality) in express feedback ratings on sellers in advertising product quality. In [23], authors studied the impact of negative feedbacks by buyers. Our work differs from theirs in that we bring out a new problem of ramp up time, which is critical to the effectiveness of eBay-like reputation mechanisms. We identify key factors that influence the ramp up time and we propose an insurance mechanism to reduce the ramp up time.

9. Conclusion

This is the first paper which reveals the ramp up time problem in eBay-like reputation mechanisms. Via theoretical analysis and experiments using data from eBay, we showed that the ramp up time is critical to the effectiveness of eBay-like reputation systems. We formulated four performance measures to explore the ramp up time problem: (1) new seller ramp up time, (2) new seller drop out probability, (3) long term profit gains for sellers and (4) average per seller transaction gains for E-commerce operators. We developed a stochastic model to identify key factors which influence the above four performance measures. We applied our model to study a dataset from eBay. We discovered that the eBay system suffers a long ramp up time, a high new seller drop out probability, low profit gains and low average per seller transaction gains. We designed a novel insurance mechanism to improve the above four performance measures. We formulated an optimization framework to select appropriate parameters for our insurance mechanism so as to incentivize new sellers to subscribe the insurance. We conducted experiments on a dataset from eBay to show that our insurance mechanism reduces the ramp up time by 91%, improves both long term profit gains and average per seller transaction gains by 26.66%. It also guaranteed that new sellers drop out with a small probability (very close to 0). Furthermore, it can reduce the risk that a buyer purchases a product from an untrustworthy seller.

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Appendix

Proof of Theorem 4.1. Note that each new seller advertises product quality honestly. In this scenario, each transaction results in one positive feedback. Recall our reputation updating rule specified in Eq. (2), we have that the reputation score at time slot τ equals the number of transactions arriving within time slot 0 to time slot $\tau-1$. Recall the definition of T_r in Eq. (4), we have that $T_r/d \in \mathbb{N}$. With these observations and by some basic probability arguments, we have

$$\begin{aligned} E[T_r] &= \sum_{\tau=1}^{\infty} \tau d \Pr[T_r = \tau d] = d \sum_{\tau=1}^{\infty} \tau \Pr[T_r/d = \tau] \\ &= d \sum_{\tau=1}^{\infty} \Pr[T_r/d \geq \tau] = d \sum_{\tau=1}^{\infty} (1 - \Pr[T_r/d \leq (\tau-1)]) \\ &= d \sum_{\tau=1}^{\infty} (1 - \Pr[r(\tau-1) \geq r_h]) = d \sum_{\tau=1}^{\infty} \left(1 - \Pr\left[\sum_{\ell=0}^{\tau-2} N(\ell) \geq r_h\right]\right). \end{aligned}$$

Note that $\sum_{\ell=0}^{\tau-2} N(\ell)$ is a random variable which follows a Poisson distribution with parameter $\lambda_1 P_{ba}(\tau-1)d$. We have $E[T_r] = d\sum_{\tau=1}^{\infty}(1 - \sum_{k=r_h}^{\infty} e^{-\lambda_1 P_{ba}(\tau-1)d} \frac{(\lambda_1 P_{ba}(\tau-1)d)^k}{k!})$. Evaluating the first order derivative on $E[T_r]$ with respect to r_h and $\lambda_1 P_{ba}$ respectively, one can easily obtain the monotonous property of $E[T_r]$. □

Proof of Theorem 4.2. First we can easily have $|\widehat{E}[T_r] - E[T_r]| = d\sum_{\tau=\widetilde{\tau}+1}^{\infty}\sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba}(\tau-1)d}\frac{(\lambda_1 P_{ba}(\tau-1)d)^k}{k!}$. We next derive an upper bound for the probability $\sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba}(\tau-1)d}\frac{(\lambda_1 P_{ba}(\tau-1)d)^k}{k!}$. Actually, this probability is equivalent to $\Pr[N(\tau-1) \leq r_h - 1]$, where $N(\tau-1)$ follows a Poisson distribution with parameter $\lambda_1 P_{ba} d(\tau-1)$. Let us restrict our attention to the τ that satisfies $r_h - 1 < \lambda_1 P_{ba} d(\tau-1)$, which yields $\tau > \frac{r_h-1}{\lambda_1 P_{ba} d} + 1$. Using a Chernoff bound [24] argument, we have $\Pr[N(\tau-1) \leq r_h - 1] \leq e^{-\lambda_1 P_{ba} d(\tau-1)}(e\lambda_1 P_{ba} d(\tau-1))^{r_h-1}/(r_h-1)^{r_h-1}$. For simplicity, let $x = \frac{\lambda_1 P_{ba} d(\tau-1)}{r_h-1}$. Straightforwardly, $x > 1$. Then we have $\Pr[N(\tau-1) \leq r_h - 1] \leq e^{-(r_h-1)x}(ex)^{r_h-1} = e^{-(r_h-1)(x-\ln x-1)}$.

Now we show that for any $c \in [1-e^{-0.5}, 1]$, we have that $x - \ln x - 1 \geq cx$ holds for all $x \geq (1-c)^{-3}$. We aim to show that $(1-c)x - \ln x - 1 \geq 0$. For simplicity, let $y = (1-c)x$. Then we have $(1-c)x - \ln x - 1 = y - \ln\frac{y}{1-c} - 1 = y - \ln y + \ln(1-c) - 1 \geq 1 + \ln y + \frac{(\ln y)^2}{2} - \ln y + \ln(1-c) - 1 = \frac{(\ln y)^2}{2} + \ln(1-c)$. To make $\frac{(\ln y)^2}{2} + \ln(1-c) \geq 0$, we only need $\ln y \geq \sqrt{-2\ln(1-c)}$. Note that $c > 1 - e^{-0.5}$, hence $-2\ln(1-c) > 1$. Therefore it is sufficient to make $\ln y \geq -2\ln(1-c)$, which yields $x \geq (1-c)^{-3}$.

Hence, given $c \in [1-e^{-0.5}, 1]$, for all $x \geq (1-c)^{-3}$ we have $\Pr[N(\tau-1) \leq r_h - 1] \leq e^{-(r_h-1)cx}$. Then we have $|\widehat{E}[T_r] - E[T_r]| = d\sum_{\tau=\widetilde{\tau}+1}^{\infty}\Pr[N(\tau-1) \leq r_h - 1] \leq d\sum_{\tau=\widetilde{\tau}+1}^{\infty} e^{-(r_h-1)cx} = d\sum_{\tau=\widetilde{\tau}+1}^{\infty} e^{-(r_h-1)c\frac{\lambda_1 P_{ba} d(\tau-1)}{r_h-1}} = d\sum_{\tau=\widetilde{\tau}+1}^{\infty} e^{-c\lambda_1 P_{ba} d(\tau-1)} = e^{-c\lambda_1 P_{ba} d\widetilde{\tau}}/(1-e^{-c\lambda_1 P_{ba} d})$. To make $e^{-c\lambda_1 P_{ba} d\widetilde{\tau}}/(1-e^{-c\lambda_1 P_{ba} d}) \leq \epsilon$, we only need $\widetilde{\tau} \geq (\ln(1-e^{-c\lambda_1 P_{ba} d}) + \ln\epsilon)/(c\lambda_1 P_{ba} d)$. Note that $x \geq (1-c)^{-3}$ is equivalent to $\frac{\lambda_1 P_{ba} d(\tau-1)}{r_h-1} \geq (1-c)^{-3}$, which yields that $\tau \geq \frac{r_h-1}{\lambda_1 P_{ba} d}(1-c)^{-3} + 1$. Hence we need $\widetilde{\tau} \geq \frac{r_h-1}{\lambda_1 P_{ba} d}(1-c)^{-3}$. In summary, we need $\tau > \max\{(\ln(1-e^{-c\lambda_1 P_{ba} d}) + \ln\epsilon)/(c\lambda_1 P_{ba} d), \frac{r_h-1}{\lambda_1 P_{ba} d}(1-c)^{-3}\}$. Setting $c = 0.8$ we complete this proof. □

Proof of Theorem 4.3. Applying similar derivation as Theorem 4.1, we have that the reputation score at time slot τ equals the number of transactions arriving within time slot 0 to $\tau-1$. Note that $T_w/d \in \mathbb{N}$. Recall the definition in Eq. (4) we have that $T_r > T_w$ if and only if $r(T_w/d) < r_h$. Using some basic probability arguments, we have $P_d = \Pr[r(T_w/d) < r_h] = \Pr\left[\sum_{\tau=0}^{T_w/d-1} N(\tau) < r_h\right]$. Note that $\sum_{\tau=0}^{T_w/d-1} N(\tau)$ is a random variable which follows a Poisson distribution with parameter $\lambda_1 P_{ba} T_w$. We have

$$P_d = \sum_{k=0}^{r_h-1} \Pr\left[\sum_{\tau=0}^{T_w/d-1} N(\tau) = k\right] = \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba} T_w}\frac{(\lambda_1 P_{ba} T_w)^k}{k!}.$$

Evaluating the first order derivative on P_d with respect to r_h, T_w and $\lambda_1 P_{ba}$ respectively, one can easily obtain the monotonous property of P_d. □

Proof of Theorem 4.4. By the linearity of expectation we have $G_s = E\sum_{\tau=0}^{\infty}\delta^\tau u N(\tau) = \sum_{\tau=0}^{\infty}\delta^\tau u E[N(\tau)]$. We next derive $E[N(\tau)]$. Let $L(\tau) \in \{\text{reputable}, \text{average}\}$ denote the label of a seller in time slot τ. By the basic rule of conditional expectation we have

$$E[N(\tau)] = \Pr[L(\tau) = \text{reputable}]E[N(\tau)|L(\tau) = \text{reputable}] + \Pr[L(\tau) = \text{average}]E[N(\tau)|L(\tau) = \text{average}].$$

A reputable seller attracts transactions with rate $\lambda_2 P_{br}$. Note that the length of a time slot is d. We can then have $E[N(\tau)|L(\tau) = \text{reputable}] = \lambda_2 P_{br} d$ for all $\tau = 0, 1, \ldots, \infty$. An average seller attracts transactions with rate $\lambda_1 P_{ba}$. Hence we have $E[N(\tau)|L(\tau) = \text{average}] = \lambda_1 P_{ba} d$ for all $\tau = 0, 1, \ldots, T_w/d - 1$, and $E[N(\tau)|L(\tau) = \text{average}] = 0$, for all $\tau = T_w/d, \ldots, \infty$. The last statement follows the fact that a seller drops out if he does not earn a reputable label before the deadline T_w. Given a time slot τ, a seller is labeled as average at this time slot if and only if $r(\tau) < r_h$. Using some basic probability arguments, we have $\Pr[L(\tau) = \text{average}] = \Pr[r(\tau) < r_h] = \Pr\left[\sum_{\ell=0}^{\tau-1} N(\ell) < r_h\right]$. Note that $\sum_{\ell=0}^{\tau-1} N(\ell)$ is a random variable which follows a Poisson distribution with parameter $\lambda_1 P_{ba} \tau d$. We have

$$\Pr[L(\tau) = \text{average}] = \sum_{k=0}^{r_h-1} \Pr\left[\sum_{\ell=0}^{\tau-1} N(\ell) = k\right] = \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba}\tau d} \frac{(\lambda_1 P_{ba} \tau d)^k}{k!}.$$

Then it follows that $\Pr[L(\tau) = \text{reputable}] = 1 - \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba}\tau d} \frac{(\lambda_1 P_{ba}\tau d)^k}{k!}$, for all $\tau = 0, 1, \ldots, \infty$, and $\Pr[L(\tau) = \text{reputable}] = 1 - \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba} T_w} \frac{(\lambda_1 P_{ba} T_w)^k}{k!}$ for all $\tau = T_w/d, \ldots, \infty$. Combine them all we have

$$E[N(\tau)] = \mathbf{I}_{\{\tau < T_w/d\}} \left(1 - \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba}\tau d} \frac{(\lambda_1 P_{ba}\tau d)^k}{k!}\right) \lambda_2 P_{br} d + \mathbf{I}_{\{\tau \geq T_w/d\}} \left(1 - \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba} T_w} \frac{(\lambda_1 P_{ba} T_w)^k}{k!}\right) \lambda_2 P_{br} d$$
$$+ \mathbf{I}_{\{\tau < T_w/d\}} \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba}\tau d} \frac{(\lambda_1 P_{ba}\tau d)^k}{k!} \lambda_1 P_{ba} d.$$

Then with some basic probability arguments we have

$$\begin{aligned} G_s =& \sum_{\tau=0}^{T_w/d-1} \left(1 - \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba}\tau d} \frac{(\lambda_1 P_{ba}\tau d)^k}{k!}\right) \lambda_2 P_{br} d\delta^\tau + \sum_{\tau=T_w/d}^{\infty} \left(1 - \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba} T_w} \frac{(\lambda_1 P_{ba} T_w)^k}{k!}\right) \lambda_2 P_{br} d\delta^\tau \\ &+ \sum_{\tau=0}^{T_w/d-1} \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba}\tau d} \frac{(\lambda_1 P_{ba}\tau d)^k}{k!} \lambda_1 P_{ba} d\delta^\tau \\ =& \frac{\lambda_2 P_{br} d}{1-\delta} - \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba} T_w} \frac{(\lambda_1 P_{ba} T_w)^k}{k!} \frac{\lambda_2 P_{br} d\delta^{T_w/d}}{1-\delta} + \sum_{\tau=0}^{T_w/d-1} \sum_{k=0}^{r_h-1} e^{-\lambda_1 P_{ba}\tau d} \frac{(\lambda_1 P_{ba}\tau d)^k}{k!} (\lambda_1 P_{ba} - \lambda_2 P_{br}) d\delta^\tau. \end{aligned}$$

This proof is then complete. □

Proof of Theorem 5.1. We first derive $E[T_r^I]$ and P_d^I. Note that sellers advertise honestly. This means that all transactions result in positive feedbacks. This implies that the insurance certificate expires at the end of the duration time. Note that $N(\ell)$, the number of transactions at time slot $\ell = 0, 1, \ldots, \infty$ before a seller ramps up, follows a Poisson distribution, and we denote its parameter by $\widetilde{\lambda}_T(\ell)$. Applying Eq. (11), we have $\widetilde{\lambda}_T(\ell) = \lambda_2 P_{br} d$ for all $\ell = 0, 1, \ldots, T_d/d - 1$, and $\widetilde{\lambda}_T(\ell) = \lambda_1 P_{ba} d$ for all $\ell = T_d/d, \ldots, \infty$. Then with a similar derivation as Theorem 4.1 we obtain the expected ramp up time $E[T_r^I]$. Furthermore, with a similar derivation as Theorem 4.3 we obtain P_d^I.

Let us now derive long term profit gains (G_s^I) and average per seller transaction gains (G_e^I). We derive G_s first. Note that $E[N(\tau)] = \Pr[L(\tau) = \text{reputable}]E[N(\tau)|L(\tau) = \text{reputable}] + \Pr[L(\tau) = \text{average}]E[N(\tau)|L(\tau) = \text{average}]$. Let us consider the first case that $T_d \geq T_w$. Recall that an insured seller attracts transactions with rate $\lambda_2 P_{br}$. Hence we have $E[N(\tau)] = \lambda_2 P_{br} d$ for all $\tau = 0, 1, \ldots, T_w/d - 1$. Note that a seller drops out if he does not earn a reputable label before the deadline T_w. We can then have that for all $\tau = T_w/d, \ldots, \infty$ it holds that $E[N(\tau)] = \lambda_2 P_{br} d$ if $L(\tau) = \text{reputable}$, otherwise $E[N(\tau)] = 0$. Given any $\tau \in \{T_w/d, \ldots, \infty\}$, we have that

$$\begin{aligned} \Pr[L(\tau) = \text{reputable}] &= 1 - \Pr[L(\tau) = \text{average}] = 1 - \Pr[r(T_w/d) < r_h] \\ &= 1 - \Pr\left[\sum_{\ell=0}^{T_w/d-1} N(\ell) < r_h\right] = 1 - \sum_{k=0}^{r_h-1} e^{-\lambda_2 P_{br} T_w} \frac{(\lambda_2 P_{br} T_w)^k}{k!}. \end{aligned}$$

Hence $E[N(\tau)] = \mathbf{I}_{\{\tau < T_w/d\}} \lambda_2 P_{br} d + \mathbf{I}_{\{\tau \geq T_w/d\}} \left(1 - \sum_{k=0}^{r_h-1} e^{-\lambda_2 P_{br} T_w} \frac{(\lambda_2 P_{br} T_w)^k}{k!}\right) \lambda_2 P_{br} d$. Then it follows that

$$\begin{aligned} G_s^I &= \sum_{\tau=0}^{T_w/d-1} \lambda_2 P_{br} d\delta^\tau + \sum_{\tau=T_w/d}^{\infty} \left(1 - \sum_{k=0}^{r_h-1} e^{-\lambda_2 P_{br} T_w} \frac{(\lambda_2 P_{br} T_w)^k}{k!}\right) \lambda_2 P_{br} d\delta^\tau \\ &= \frac{\lambda_2 P_{br} d}{1-\delta} - \sum_{k=0}^{r_h-1} e^{-\lambda_2 P_{br} T_w} \frac{(\lambda_2 P_{br} T_w)^k}{k!} \frac{\lambda_2 P_{br} d\delta^{T_w/d}}{1-\delta}. \end{aligned}$$

Now let us consider the case that $T_d < T_w$. First, we can easily have that $E[N(\tau)] = \lambda_2 P_{br} d$ holds for all $\tau = 0, 1, \ldots, T_d/d - 1$. Now consider $\tau \in \{T_d/d, \ldots, T_w/d - 1\}$. We have $E[N(\tau)|L(\tau) = \text{reputable}] = \lambda_2 P_{br} d$ and

$E[N(\tau)|L(\tau)=\text{average}]=\lambda_1 P_{ba}d$. Furthermore,

$$\begin{aligned}
\Pr[L(\tau)=\text{average}] &= \Pr[r(\tau)<r_h]=\Pr\left[\sum_{\ell=0}^{\tau-1}N(\ell)<r_h\right]\\
&=\Pr\left[\sum_{\ell=0}^{T_d/d-1}N(\ell)+\sum_{\ell=T_d/d}^{\tau-1}N(\ell)<r_h\right]\\
&=\sum_{k=0}^{r_h-1}\Pr\left[\sum_{\ell=0}^{T_d/d-1}N(\ell)=k\right]\sum_{k'=0}^{r_h-1-k}\Pr\left[\sum_{\ell=T_d/d}^{\tau-1}N(\ell)=k'\right]\\
&=\sum_{k=0}^{r_h-1}e^{-\lambda_2 P_{br}T_d}\frac{(\lambda_2 P_{br}T_d)^k}{k!}\sum_{k'=0}^{r_h-1-k}e^{-\lambda_1 P_{ba}(\tau d-T_d)}\frac{(\lambda_1 P_{ba}(\tau d-T_d))^{k'}}{k'!}.
\end{aligned}$$

Now consider $\tau\in\{T_w/d,\ldots,\infty\}$. For this case we can derive $\Pr[L(\tau)=\text{average}]$ as

$$\begin{aligned}
\Pr[L(\tau)=\text{average}] &= \Pr[r(T_w/d)<r_h]\\
&=\sum_{k=0}^{r_h-1}e^{-\lambda_2 P_{br}T_d}\frac{(\lambda_2 P_{br}T_d)^k}{k!}\sum_{k'=0}^{r_h-1-k}e^{-\lambda_1 P_{ba}(T_w-T_d)}\frac{(\lambda_1 P_{ba}(T_w-T_d))^{k'}}{k'!}.
\end{aligned}$$

Combine them all we have

$$\begin{aligned}
G_s^I &= \sum_{\tau=0}^{T_d/d-1}\lambda_2 P_{br}d\delta^\tau+\sum_{\tau=T_d/d}^{T_w/d-1}\left(1-\sum_{k=0}^{r_h-1}e^{-\lambda_2 P_{br}T_d}\frac{(\lambda_2 P_{br}T_d)^k}{k!}\sum_{k'=0}^{r_h-1-k}e^{-\lambda_1 P_{ba}(\tau d-T_d)}\frac{(\lambda_1 P_{ba}(\tau d-T_d))^{k'}}{k'!}\right)\lambda_2 P_{br}d\delta^\tau\\
&\quad+\sum_{\tau=T_w/d}^{\infty}\left(1-\sum_{k=0}^{r_h-1}e^{-\lambda_2 P_{br}T_d}\frac{(\lambda_2 P_{br}T_d)^k}{k!}\sum_{k'=0}^{r_h-1-k}e^{-\lambda_1 P_{ba}(T_w-T_d)}\frac{(\lambda_1 P_{ba}(T_w-T_d))^{k'}}{k'!}\right)\lambda_2 P_{br}d\delta^\tau\\
&\quad+\sum_{\tau=T_d/d}^{T_w/d-1}\sum_{k=0}^{r_h-1}e^{-\lambda_2 P_{br}T_d}\frac{(\lambda_2 P_{br}T_d)^k}{k!}\sum_{k'=0}^{r_h-1-k}e^{-\lambda_1 P_{ba}(\tau d-T_d)}\frac{(\lambda_1 P_{ba}(\tau d-T_d))^{k'}}{k'!}\lambda_1 P_{ba}d\delta^\tau\\
&=\frac{\lambda_2 P_{br}d}{1-\delta}-\sum_{k=0}^{r_h-1}e^{-\lambda_2 P_{br}T_d}\frac{(\lambda_2 P_{br}T_d)^k}{k!}\sum_{k'=0}^{r_h-1-k}e^{-\lambda_1 P_{ba}(T_w-T_d)}\frac{(\lambda_1 P_{ba}(T_w-T_d))^{k'}}{k'!}\frac{\lambda_2 P_{br}d\delta^{T_w/d}}{1-\delta}\\
&\quad+\sum_{\tau=T_d/d}^{T_w/d-1}\sum_{k=0}^{r_h-1}e^{-\lambda_2 P_{br}T_d}\frac{(\lambda_2 P_{br}T_d)^k}{k!}\sum_{k'=0}^{r_h-1-k}e^{-\lambda_1 P_{ba}(\tau d-T_d)}\frac{(\lambda_1 P_{ba}(\tau d-T_d))^{k'}}{k'!}(\lambda_1 P_{ba}-\lambda_2 P_{br})d\delta^\tau.
\end{aligned}$$

This proof is then complete. □

Proof of Theorem 5.2. We want to derive the reasonable price that an E-Commerce operator can charge for the insurance. The marginal long term profit gain of an insured seller is $G_s^I-C_I$. Note that the marginal long term profit gain without insurance is G_s. Thus sellers have the incentive to buy an insurance if the marginal profit gain corresponds to buying an insurance is larger than the marginal profit gain without insurance, i.e., $G_s^I-C_I>G_s$, which yields $C_I<G_s^I-G_s$.

Note that sellers advertise honestly. Let $N'(T_d)$ denote the total number of products sold in insurance duration time. It is easy to see that $N'(T_d)$ follows a Poisson distribution with parameter $\lambda_2 P_{br}T_d$. The worst case is that all buyers hold the product till the last minute of the clearing time T_c and then return it. Using a Chernoff bound [24] argument, one can easily bound the shipment cost (at the worst case) as

$$\Pr[N'(T_d)C_S\ge N'C_S]\le e^{-\lambda_2 P_{br}T_d}(e\lambda_2 P_{br}T_d)^{N'}/N'^{N'}.$$

Setting $N'=\max\{\ln\epsilon^{-1}-\lambda_2 P_{br}T_d, e^2\lambda_2 P_{br}T_d\}$ we have $\Pr[N'(T_d)C_S\ge N'C_S]\le\epsilon$. The clearing time follows the shipment delay d. □

Proof of Theorem 6.1. Observe that C_I, D_I and T_c are not parameters in determining G_s^I derived in Eq. (12). Hence, examining Eq. (13), one can observe that ΔG^I decreases in C_I, D_I and T_c respectively. This proof is then complete. □

Proof of Theorem 6.2. Let T_d^* denote the optimal value of T_d. Observe that for each given T_d, it is possible to be optimal if the value of ΔG^I is no less than zero. In other words, T_d^* should satisfy

$$G_s^I-G_s-(1-\delta^{T_d^*/d+1})C_S\max\left\{\ln\epsilon^{-1}-\lambda_2 P_{br}T_d^*, e^2\lambda_2 P_{br}T_d^*\right\}\ge 0.$$

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Note that $\max\{\ln \epsilon^{-1} - \lambda_2 P_{br} T_d^*, e^2\lambda_2 P_{br} T_d^*\} \ge e^2 \lambda_2 P_{br} T_d^*$. Then it follows that

$$G_s^l - G_s - (1-\delta^{T_d^*/d+1}) C_S e^2 \lambda_2 P_{br} T_d^* \ge 0 \Leftrightarrow G_s^l - G_s \ge (1-\delta^{T_d^*/d+1}) C_S e^2 \lambda_2 P_{br} T_d^*.$$

Observe that $G_s^l \le \frac{u\lambda_2 P_{br}}{1-\delta}$, which yields

$$\frac{u\lambda_2 P_{br}}{1-\delta} - G_s \ge (1-\delta^{T_d^*+1}) C_S e^2 \lambda_2 P_{br} T_d^* \Leftrightarrow \frac{\frac{u\lambda_2 P_{br}}{1-\delta} - G_s}{C_S e^2 \lambda_2 P_{br}} \ge (1-\delta^{T_d^*/d+1}) T_d^*.$$

Observe that to make $1-\delta^{T_d^*/d+1} \ge \frac{1}{2}$, we only need $T_d^*/d + 1 \ge \frac{\ln 0.5}{\ln \delta}$, which yields that $T_d^* \ge d\frac{\ln 0.5}{\ln \delta} - d$. Then it follows that

$$T_d^* \le d \left\lceil \max\left\{ \frac{\ln 0.5}{\ln \delta} - 1, 2\frac{\frac{u\lambda_2 P_{br}}{1-\delta} - G_s}{d C_S e^2 \lambda_2 P_{br}} \right\} \right\rceil = d \max\left\{ \left\lceil \frac{\ln 0.5}{\ln \delta} \right\rceil - 1, \left\lceil 2\frac{\frac{u\lambda_2 P_{br}}{1-\delta} - G_s}{d C_S e^2 \lambda_2 P_{br}} \right\rceil \right\}.$$

This proof is then complete. □

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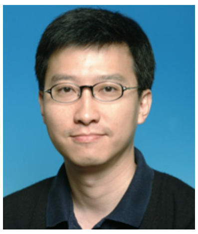

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