CENG 4480 Midterm (Fall 2018)

Name: __________
ID: __________

Solutions

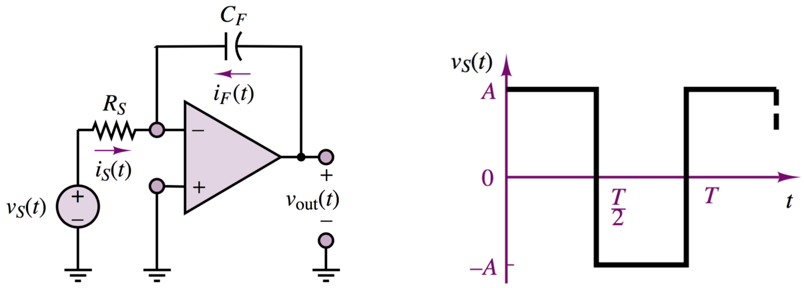
Q1 (40%) Check or fill the correct answer:

1. A circuit where the input signal power is less than the output signal power is called **amplifier/attenuator**.
2. The effect of the **positive/negative** feedback connection from output to inverting input is to force V^+ to be equal to V^-. A Schmitt Trigger based on inverting comparator has a **positive/negative** feedback.
3. Bandwidth is defined as the frequency range over which the voltage gain of the amplifier is above ___% or -3dB of its maximum output value.
4. Given a differentiator circuit and a sine wave as input signal, the waveform of the output is **cosine/square**.
5. The sampling rate of the ADC used depends on the maximum frequency f_{max} of input signal. According to Nyquist sampling theorem, the sampling frequency $f_{sampling}$ should be **greater/lesser** or equal to **two/three** times f_{max}.
6. When measuring angles by IMU, the measurement from **gyroscopes/accelerometers** has the tendency to drift.
7. **high-pass filter/sample-and-hold circuit** is used to reduce the glitch.
8. Given the same corner frequency, which low-pass filter has a better filtering performance? **one-pole low-pass filter /two-pole low-pass filter**
9. Give a reason why Arduino is so popular.____________
10. Light-to-voltage optical sensors contains **photodiode/amplifier** to sense light intensity change.

A1

1. amplifier
2. negative, positive
3. 70.7
4. cosine
5. greater, two
6. gyroscopes
7. sample-and-hold circuit
8. two-pole low-pass filter
9. open-source hardware; cheap; modular...
10. photodiode

Q2 **(15%)** **Determine** the output voltage (i.e. the mathematical expression of $V_{out}(t)$) for the integrator circuit of Figure 1a if the input is a square wave of amplitude $\pm A$ and period T shown in Figure 1b. Try to **sketch** the waveform of output. Assume $T = 100ms$, $C_F = 0.1\mu F$, $R_s = 100k\Omega$ and ideal op-amp. The square wave starts at $t = 0$ and therefore $V_{out}(0) = 0$.



(a) (b)

Figure 1: (a) Op-amp integrator; (b) Input of a square wave.

A2 We write the expression for the output of the integrator:

$$\begin{aligned} v_{out}(t) &= -\frac{1}{R_s C_F}\int_{-\infty}^{t} v_s(t')dt' \\ &= -\frac{1}{R_s C_F}\left(\int_{-\infty}^{0} v_s(t')dt' + \int_{0}^{t} v_s(t')dt'\right) \\ &= -\frac{1}{R_s C_F}\int_{0}^{t} v_s(t')dt'. \end{aligned} \tag{1}$$

Next, we note that we can integrate the square wave in a piecewise fashion by observing that $v_s(t) = A$ for $0 \le t < T/2$ and $v_s(t) = -A$ for $T/2 \le t < T$. We consider the first half of the waveform:

$$\begin{aligned} v_{out}(t) &= -\frac{1}{R_s C_F}\int_{0}^{t} v_s(t')dt' \\ &= -100At \qquad 0 \le t < T/2, \end{aligned} \tag{2}$$

and

$$\begin{aligned} v_{out}(t) &= v_{out}(\frac{T}{2}) - \frac{1}{R_s C_F}\int_{T/2}^{t} v_s(t')dt' \\ &= -100A\frac{T}{2} + 100A(t - \frac{T}{2}) \\ &= -100A(T - t) \qquad T/2 \le t < T. \end{aligned} \tag{3}$$

Since the waveform is periodic, the above result will repeat with period T , as shown in Figure 2.

Q3 **(15%)** Assume op-amps are ideal. Given $R_1 = 0.2M\Omega$, $R_2 = 0.5M\Omega$, $R_3 = 2M\Omega$, $R_4 = 5k\Omega$, $R_5 = 2M\Omega$, and $C_1 = 2\mu F$, $C_2 = 0.5\mu F$, derive the differential equation

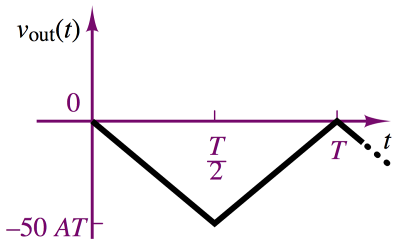


Figure 2: The sketch of output of integrator.

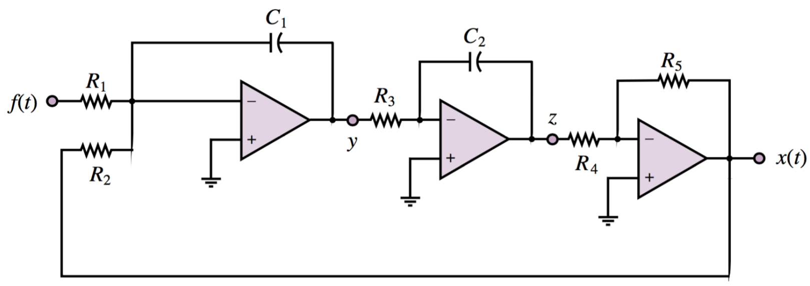


Figure 3: Analog computer simulation of unknown system.

corresponding to the analog computer simulator of Figure 3, i.e. the mathematical relationship between f and x. Note that $f(t)$ is input signal, y and z are outputs of corresponding op-amps.

A3 We start the analysis from the right-hand side of the circuit, to determine the intermediate variable z as a function of $x(t)$:

$$x = -\frac{R_5}{R_4}z = -400z. \tag{4}$$

Next, we move to the left to determine the relationship between y and z:

$$z = -\frac{1}{R_3C_2}\int y(t')dt', \tag{5}$$

or

$$y = -\frac{dz}{dt}. \tag{6}$$

Finally, we determine y as a function of x and f:

$$\begin{aligned} y &= -\frac{1}{R_2C_1}\int x(t')dt' - \frac{1}{R_1C_1}\int f(t')dt' \\ &= -\int [x(t') + 2.5f(t')]dt', \end{aligned} \tag{7}$$

or

$$\frac{dy}{dt} = -x - 2.5f. \tag{8}$$

Substituting the expressions into one another and eliminating the variables y and z, we obtain the differential equation in x:

$$\begin{aligned} x &= -400z \\ \frac{dx}{dt} &= -400\frac{dz}{dt} = 400y \\ \frac{d^2x}{dt^2} &= 400\frac{dy}{dt} = -400(x + 2.5f). \end{aligned} \tag{9}$$

Q4 (15%)

For the DAC circuit shown in Figure 4 (using an ideal op-amp), what value of R_F will givean output range of $-15 \leq V_0 \leq 0V$? Assume that logic $0 = 0V$ and logic $1 = 5V$.

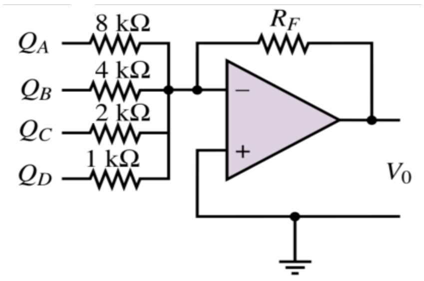


Figure 4: RF-DAC.

A4 We have the above equation:

$$\frac{-V_0}{R_F} = (\frac{Q_A}{8k\Omega} + \frac{Q_B}{4k\Omega} + \frac{Q_C}{2k\Omega} + \frac{Q_D}{1k\Omega}) \times 5V \tag{10}$$

when input equals 0000, $V_0 = 0V$, when input equals 1111, $V_0 = -15V$. So we get:

$$R_F = \frac{15 \times 8}{15 \times 5} = \frac{8}{5} K\Omega \tag{11}$$

Q5 (15%)

Assume the linear estimate system equation is $\boldsymbol{x}_{t+1} = \boldsymbol{A}\boldsymbol{x}_t + \boldsymbol{B}\boldsymbol{u}_{t+1} + \boldsymbol{w}_t$. Given a second-autoregression random series:

$$x(t) = 2.14x(t-1) - 0.50x(t-2) + u(t) + \omega_t \tag{12}$$

Kalman Filter is used to estimate $x(t)$ (Here $x(t)$ is a scalar). Try to give the formulations of state transition matrices $\boldsymbol{A}$, $\boldsymbol{B}$, and noise vector $\boldsymbol{w}_t$.

A9

$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 \\ -0.50 & 2.14 \end{pmatrix} \tag{13}$$

$$\boldsymbol{B} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{14}$$

or

$$\boldsymbol{B} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{15}$$

$$\boldsymbol{w}_t = \begin{pmatrix} 0 \\ \omega_t \end{pmatrix} \tag{16}$$