CENG4480 Homework 2

Due: Nov. 13, 2018

Solutions

Q1 The circuit shown in Figure 1 represents a simple 4-bit digital-to-analog converter. Each switch is controlled by the corresponding bit of the digital number if the bit is 1 the switch is up; if the bit is 0 the switch is down. Let the digital number be represented by $b_3b_2b_1b_0$. Please answer the following two questions:

(1) Determine an expression relating v_o to the binary input bits.

(2) Use this converter, design another 4-bit digital-to-analog converter whose output is given by

$$v_o = -\frac{1}{10}(8b_3 + 4b_2 + 2b_1 + b_0)V. \tag{1}$$

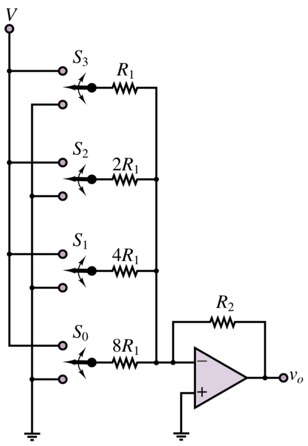

Figure 1: 4-bit DAC.

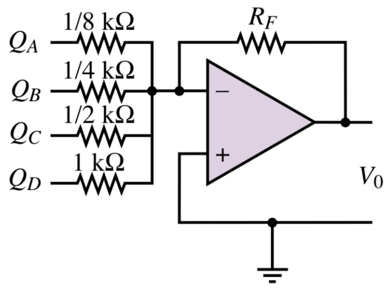

Figure 2: RF DAC.

A1 (1) Assuming the binary input bits are A_3, A_2, A_1, A_0. then we have:

$$-\frac{V_0}{R_2} = \frac{A_3V}{R_1} + \frac{A_2V}{2R_2} + \frac{A_1V}{8R_1} \tag{2}$$

$$V_0 = -\frac{(8A_3 + 4A_2 + 2A_1 + A_0)VR_2}{8R_1} \tag{3}$$

(2)

$$\frac{R_2}{8R_1} = \frac{1}{10} \tag{4}$$

then

$$\frac{R_2}{R_1} = \frac{4}{5} \tag{5}$$

Q2 For the DAC circuit shown in Figure 2 (using an ideal op-amp), what value of R_F will givean output range of $-10 \leq V_0 \leq 0V$? Assume that logic $0 = 0V$ and logic $1 = 5V$.

A2 We have the above equation:

$$\frac{-V_0}{R_F} = \frac{(8Q_A + 4Q_B + 2Q_C + Q_D) \times 5V}{1k\Omega} \tag{6}$$

when input equals 0000, $V_0 = 0V$, when input equals 1111, $V_0 = -10V$. So we get:

$$R_F = \frac{10}{15 \times 5} = \frac{2}{15} K\Omega \tag{7}$$

Q3 A simple Infra-Red Sensor system to detect passing human is presented as in Figure 3. A and B are IR Sensors which will generate different output voltages for different infra-red intensity, and higher voltage level corresponds to high light intensity.

(1) Explain how this system works for counting passing pedestrians.

(2) To increase counting accuracy, usually B is covered with materials that can reflect infra-red light. Explain why.

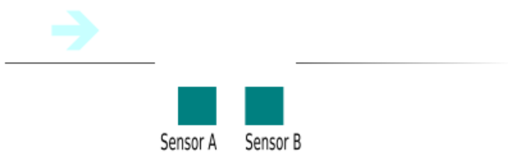

Figure 3: IR-System.

A3 (1) When pedestrians pass over IR Sensor, they will approach and deviate the sensor, which corresponds to voltage pulses V_A at the output of it. We can simply count pulse number for passing pedestrian.

(2) When Sensor B is covered with infra-red reflection materials, it can generate pulses V_B caused by non-infra-red wave. We can reduce wrongly counted number by subtract V_B from V_A to avoid counting noise signal.

Q4 Exemplify the working principles of sensors that measure: (1) Flow; (2) Temperature; (3) Pressure; (4) Motion; (5) Liquid Level.

A4 Refer to textbook "Principles and Applications of Electrical Engineering" Table 15.1

Q5 Briefly describe how PID affects motor control.

A5 Refer to lecture 07 slides, page 22-24.

1. **Proportional Gain** K_p: Larger K_p, faster response, but higher instability.
2. **Integral Gain** K_i: Larger K_i, eliminate steady state error, but larger overshoot.

3. **Derivative Gain** K_d: Larger K_d, reduce overshoot, but slower response.

Q6 Given a linear system

$$\begin{cases} \boldsymbol{x}_t = \boldsymbol{A}_{t-1}\boldsymbol{x}_{t-1} + \boldsymbol{\omega}_{t-1}, \\ \boldsymbol{z}_t = \boldsymbol{B}_t\boldsymbol{x}_t + \boldsymbol{v}_t, \\ \boldsymbol{v}_t = \boldsymbol{C}_{t-1}\boldsymbol{v}_{t-1} + \boldsymbol{n}_{t-1}, \end{cases} \tag{8}$$

where $\boldsymbol{\omega}_t$ and $\boldsymbol{n}_t$ are independent and obey Gaussian distribution zero-mean and covariance $\boldsymbol{Q}_t$ and $\boldsymbol{R}_t$, respectively. Please give the estimate equation and measurement equation of the system.

A6

$$\begin{pmatrix} \boldsymbol{x}_t \\ \boldsymbol{v}_t \end{pmatrix} = \begin{pmatrix} \boldsymbol{A}_{t-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{C}_{t-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_{t-1} \\ \boldsymbol{v}_{t-1} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\omega}_{t-1} \\ \boldsymbol{n}_{t-1} \end{pmatrix} \tag{9}$$

$$\boldsymbol{z}_t = \begin{pmatrix} \boldsymbol{B}_t & \boldsymbol{I} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_t \\ \boldsymbol{v}_t \end{pmatrix} \tag{10}$$

Q7 Given two Gaussian distributions $N(x_0; \mu_0, \sigma_0)$ and $N(x_1; \mu_1, \sigma_1)$, try to give the expectation and variance of a new distribution which is the product of these two Gaussian distributions.

A7 For detailed proof, refer to the first part of "Products and Convolutions of Gaussian Probability Density Functions"[1].

$$\mu_2 = \mu_0 + \frac{\sigma_0^2(\mu_1 - \mu_0)}{\sigma_0^2 + \sigma_1^2} \tag{11}$$

$$\sigma_2^2 = \sigma_0^2 - \frac{\sigma_0^4}{\sigma_0^2 + \sigma_1^2} \tag{12}$$

Q8 For the 4-bit R-2R DAC, calculate V_0 in terms of $V_{b,0} - V_{b,4}$ if V_{ref} is grounded (Figure 4).

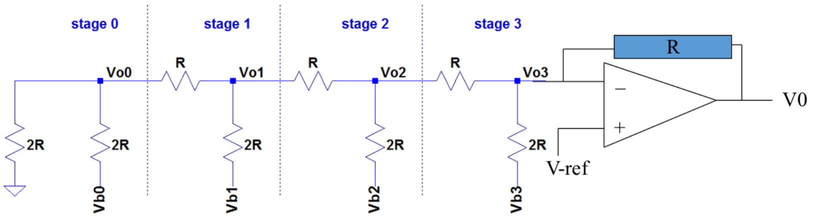

Figure 4: R-2R DAC.

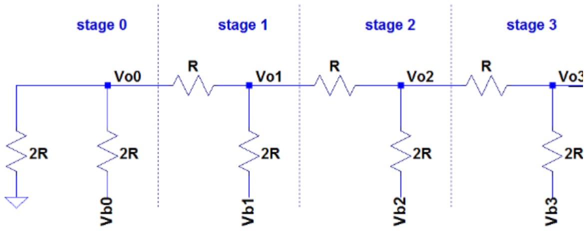

Figure 5: Load of R-2R ADC

A8 As shown in Figure 5, first we calculate the equivalence seen from V_{o3},

$$R_{eq} = R \tag{13}$$

Get contribution at V_{o3i} of each digital input V_{bi}, i=0,1,2,3 separately, it's easy to derive from Thevenin equivalent analysis,

$$V_{o30} = \frac{V_{b0}}{16} \tag{14}$$
$$V_{o31} = \frac{V_{b1}}{8} \tag{15}$$
$$V_{o32} = \frac{V_{b2}}{4} \tag{16}$$
$$V_{o33} = \frac{V_{b3}}{2} \tag{17}$$
$$\tag{18}$$

then, we have,

$$V_{o3} = \frac{V_{b0}}{16} + \frac{V_{b1}}{8} + \frac{V_{b2}}{4} + \frac{V_{b3}}{2} \tag{19}$$

Using the quality of op amp,

$$V_o = \frac{V_{b0}}{16} + \frac{V_{b1}}{8} + \frac{V_{b2}}{4} + \frac{V_{b3}}{2} \tag{20}$$

Q9 [UPDATED] Assume the liner estimate system equation is $\mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t + \mathbf{w}_t$. Given a second-autoregression random series:

$$x(t) = 2.32x(t-1) - 0.76x(t-2) + \omega_t \tag{21}$$

Kalman Filter is used to estimate $x(t)$ (Here $x(t)$ is a scalar). Try to give the formulations of state transition matrix $\mathbf{A}$ and noise vector $\mathbf{w}_t$.

A9

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -0.76 & 2.32 \end{pmatrix} \tag{22}$$

$$\mathbf{w}_t = \begin{pmatrix} 0 \\ \omega_t \end{pmatrix} \tag{23}$$

[1]The document can be accessed through: `http://www.tina-vision.net/docs/memos/2003-003.pdf`