CSCI 3130: Formal Languages and Automata Theory — Assignment 4

The Chinese University of Hong Kong, Fall 2022 — due 11:59 p.m. Sunday November 13

Collaborating on homework is encouraged, but you must write your own solutions in your own words and list your collaborators. Copying someone else's solution will be considered plagiarism and may result in failing the whole course.

Please answer clearly and concisely. Explain your answers. Unexplained answers will get lower scores or even no credits.

(1) (35 points) Consider the following context-free grammar G:

$$S \to \texttt{(}S\texttt{)} \mid S\,\texttt{()} \mid \texttt{()}$$

It generates expressions like `()`, `(())()`, and so on.

(a) Every partially completed rule of the form $A \to \alpha \bullet \beta$ is known as an *item*. Write all items in the grammar G and construct an NFA for the valid item updates.

(b) Convert the NFA to a DFA. Which of the states are shift states and which are reduce states? Are there any conflicts?

(c) Using the DFA, show an execution of the LR(0) parsing algorithm on the input

`(()())`

Show the stack of states, stack of processed input, and remaining input throughout the execution.

(d) Now consider the following extended context-free grammar G':

$$S \to \texttt{()}\,S \mid \texttt{()}$$

Show that G' is not an LR(0) grammar by giving the DFA of valid item updates.

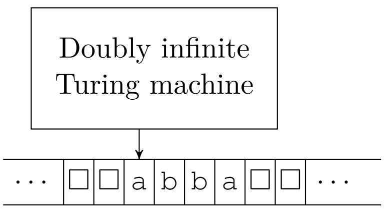
(2) (25 points) This is the only question in this course concerning the state diagram of a Turing machine.

In this problem, you will design a Turing machine for the following language. Briefly explain how your Turing machine works (insufficient explanation may get zero points).

$$L = \{\texttt{a}^i\texttt{b}^j\texttt{c}^k \mid i \geqslant j \geqslant k \geqslant 1\}.$$

Give both a high-level description and a state diagram of your Turing machine.

(3) (30 points) A **Turing machine with doubly infinite tape** is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right. The tape is initially filled with blanks except for the portion that contains the input. Computation is defined as usual except that the head never encounters an end to the tape as it moves leftward.



You will argue that this type of Turing machine is equivalent to the usual one-side-unbounded Turing machine.

(a) Write a formal definition of a doubly infinite tape Turing machine. A formal definition of an automaton will look like page 17/slide 16 of Lecture 14 or page 9/slide 7 of Lecture 10.

(b) Show how to simulate a usual Turing machine on a doubly infinite tape Turing machine. You need to specify

- how the tape of the doubly infinite Turing machine will be used to represent the usual Turing machine.
- how the doubly infinite Turing machine tape should be set up initially;
- what the doubly infinite Turing machine should do when the usual Turing machine performs a transition (you may specify in 1-2 sentences the general idea, omitting the tedious details);
- what the doubly infinite Turing machine should do when the usual Turing machine accepts/rejects.

(c) Show how to simulate a doubly infinite Turing machine on a usual Turing machine. Again you should specify simulation details similar to those in part (b). *Hint:* You may want to simulate the doubly infinite Turing machine with some machine M that is not the usual Turing machine, but can itself be simulated by a usual Turing machine.

(4) (10 points) Argue that the collection of decidable languages is closed under the star operation.

In other words, given a Turing machine/algorithm M that decides a language L, how do you construct a Turing machine/algorithm M' that decides L^*?