

Solving Finite Domain Constraint Hierarchies by Local Consistency and Tree Search*

Stefano Bistarelli[†]

Dipartimento di Scienze
Università “D’Annunzio” di Pescara, Italy
bista@sci.unich.it

Istituto di Informatica e Telematica, C.N.R.
Pisa, Italy
stefano.bistarelli@iit.cnr.it

Philippe Codognet

Univ. of Paris 6, LIP6, Paris Cedex 05, France
Philippe.Codognet@lip6.fr

H.K.C. Hui and J.H.M. Lee

Dept of Comp. Sci. & Eng.
The Chinese Univ. of Hong Kong, Hong Kong
{kchui,jlee}@cse.cuhk.edu.hk

Abstract

We provide a reformulation of the constraint hierarchies (CHs) framework based on the notion of *error indicators*. Adapting the generalized view of local consistency in semiring-based constraint satisfaction problems (SCSPs), we define *constraint hierarchy k-consistency* (CH-*k*-C) and give a CH-2-C enforcement algorithm. We demonstrate how the CH-2-C algorithm can be seamlessly integrated into the ordinary branch-and-bound algorithm to make it a finite domain CH solver. Experimentation confirms the efficiency and robustness of our proposed solver prototype. Unlike other finite domain CH solvers, our proposed method works for both local and global comparators. In addition, our solver can support arbitrary error functions.

1 Introduction

The Constraint Hierarchy (CH) framework [Borning *et al.*, 1992] is a general framework for the specification and solutions of over-constrained problems. Originating from research in interactive user-interface applications, the CH framework attracts much effort in the design of efficient solvers in the real number domain [Badros *et al.*, 2001; Hosobe *et al.*, 1996]. To extend the benefit of the CH framework to also discrete domain applications, such as timetabling and resource allocation, the paper takes a step towards a general and efficient finite domain CH solver, based on consistency techniques and tree search.

*A more complete and detailed version of this paper can be obtained from the authors.

[†]Part of this research has been carried out while the author was visiting the Department of Computer Science and Engineering, The Chinese University of Hong Kong, China

2 A Reformulation of Constraint Hierarchies

We formulate the CH framework [Borning *et al.*, 1992] (in particular in the definition of comparators and solution set) using error indicators [Bistarelli *et al.*, 1999].

We denote an error value by ξ , possibly with subscripts. Let $I = \{\xi_1, \dots, \xi_N\}$ be a poset (partially ordered set), each element ξ_j of which is an *error indicator*. Given a constraint hierarchy $H = \{H_0, \dots, H_n\}$ where n is the number of non-required levels, and for all $i \in \{0, \dots, n\}$, $H_i = \{c_1^i, \dots, c_{k_i}^i\}$ with k_i being the number of constraints in level i . An error indicator ξ_θ of a valuation θ for a set of variables V is a tuple of error values such that $\xi_\theta = \langle \langle \xi_{\theta_1}^0, \dots, \xi_{\theta_{k_0}}^0 \rangle, \dots, \langle \xi_{\theta_1}^n, \dots, \xi_{\theta_{k_n}}^n \rangle \rangle$ and $\forall a \in \{0, \dots, n\}, \forall b \in \{1, \dots, k_a\}, \xi_{\theta_b}^a = e(c_b^a \theta)$ if $\text{vars}(c_b^a) \subset V$ and $\xi_{\theta_b}^a = 0$ if $\text{vars}(c_b^a) \not\subset V$. Error indicators provide a measure of the “badness” of valuations with respect to H .

The comparator predicate *better* in the original CH formulation is redefined using a *partial order*, denoted by \prec . We define \prec to be irreflexive and transitive over I . Hence, it preserves the meaning of *better*. Intuitively, $\xi^i \prec \xi^j$ means ξ^j is “better” than ξ^i in I . In general, \prec will not provide a total ordering. For convenience, we define \preceq such that $\forall \xi^i, \xi^j \in I, \xi^i \preceq \xi^j \rightarrow (\xi^i \prec \xi^j) \vee (\xi^i = \xi^j)$.

We can redefine *l-b* in the original formulation as a partial order \prec_{l-b} as follows. Given any two valuations θ and σ , and the corresponding error indicators ξ_θ and ξ_σ , \prec_{l-b} is defined as:

$$\begin{aligned} \xi_\theta \prec_{l-b} \xi_\sigma &\equiv \exists l > 0 \text{ such that } \forall i \in \{0, \dots, l-1\}, \\ &\forall j \in \{1, \dots, k_i\}, \xi_{\theta_j}^i = \xi_{\sigma_j}^i \\ &\wedge \exists a \in \{1, \dots, k_l\}, \xi_{\sigma_a}^l < \xi_{\theta_a}^l \\ &\wedge \forall b \in \{1, \dots, k_l\}, \xi_{\sigma_b}^l \leq \xi_{\theta_b}^l. \end{aligned}$$

The intuitive meaning of $\xi_\theta \prec_{l-b} \xi_\sigma$ is that valuation σ is *locally-better* than valuation θ .

Similarly, we can define g - b \prec_{g-b} , and its instances w - s - b \prec_{w-s-b} , w - c - b \prec_{w-c-b} , and l - s - b \prec_{l-s-b} respectively. Given any two valuations θ and σ , and the corresponding error indicators $\vec{\xi}_\theta$ and $\vec{\xi}_\sigma$:

$$\vec{\xi}_\theta \prec_{g-b} \vec{\xi}_\sigma \equiv \exists l > 0 \text{ such that } \forall i \in \{0, \dots, l-1\}, \\ g(\langle \xi_{\theta 1}^i, \dots, \xi_{\theta k_i}^i \rangle) = g(\langle \xi_{\sigma 1}^i, \dots, \xi_{\sigma k_i}^i \rangle) \\ \wedge g(\langle \xi_{\sigma 1}^l, \dots, \xi_{\sigma k_l}^l \rangle) < g(\langle \xi_{\theta 1}^l, \dots, \xi_{\theta k_l}^l \rangle),$$

where g is a suitable *combining function* for error values.

Notice that by definition, all local/global comparators ignore constraints in hierarchy levels greater than or equal to l .

We are now ready to define the solution set S_H of a CH H with variables V by:

$$S_0 = \{\theta \mid \text{vars}(\theta) = V, \xi_{\theta i}^0 = 0 \text{ for all } i \in \{1, \dots, k_0\}\} \\ \text{and } S_H = \{\theta \in S_0 \mid \forall \sigma \in S_0, \vec{\xi}_\theta \not\prec \vec{\xi}_\sigma\}.$$

The following lemma gives the monotonicity of the introduced comparators, which are collectively denoted by \prec_{better} and \preceq_{better} in the rest of the paper.

Lemma 1 *Given any two error indicators $\vec{\xi}^I$ and $\vec{\xi}^{II}$. If for all a, b we have $\xi_b^{IIa} \leq \xi_b^{Ia}$, then $\vec{\xi}^I \preceq_{better} \vec{\xi}^{II}$.*

Notice that the above lemma let us compare valuation for both local and global comparators (because the \preceq_{better} order implies all the orders induced from any specific comparator) and for arbitrary error functions.

We also introduce the notion of a *hierarchy problem* which is a CH augmented with an *Error Indicator Store*.

Definition 1 *A hierarchy problem $P = \langle H, I_H \rangle$ is a constraint problem, where H is a CH with variables V and I_H is a set containing error indicator stores $\vec{\xi}_{x=d}$ for all variables $x \in V$ and for all $d \in D(x)$. Each $\vec{\xi}_{x=d}^1$ is used for keeping an estimate of the errors of valuations involving $\{x \mapsto d\}$.*

Definition 2 *A valuation θ is a solution of $P = \langle H, I_H \rangle$ if (1) θ is a solution of H and (2) $\vec{\xi}_\theta \preceq_{better} \vec{\xi}_{x=d}$ for all $\vec{\xi}_{x=d} \in I_H$.*

In other words, solutions of $P = \langle H, I_H \rangle$ are solutions of H which have a “worse” error than the estimates provided in I_H . By the definition, the solutions of H always contain those of $\langle H, I_H \rangle$. Equality holds when the error estimates provided in I_H fails to “filter” out any solutions of H .

Theorem 1 *Consider a CH H and the associated hierarchy problem $P = \langle H, I_H \rangle$.*

- $S_P \subseteq S_H$, and
- $S_P = S_H$ if $\vec{\xi}_\theta \preceq_{better} \vec{\xi}_{x=d}$ for all $(x \mapsto d) \in \theta$ and $\theta \in S_H$.

In particular, a hierarchy problem $\langle H, I_H \rangle$ must share the same solution as H if all $\vec{\xi}_{x=d} \in I_H$ contain only the error value 0 (*i.e.* no error information). This fact is useful in ensuring the correctness of our local consistency algorithm and the completeness of our branch-and-bound solver later.

¹Note the similarity of $\vec{\xi}_{x=d}$ in notation to $\vec{\xi}_\theta$ which denotes the error indicator of a valuation θ . Here, $\vec{\xi}_{x=d}$ is a *store* containing an estimate of the error associated with valuations containing $x \mapsto d$.

3 Summary of the Results

The notion of *constraint hierarchy k -consistency* (CH- k -C), is defined using error indicators which are structures isomorphic to the structure of a given CH used for storing the error information of the CH problem (similar notion was defined by Bistarelli *et al.* [1999]). In particular, we give an algorithm for enforcing CH-2-C of a CH problem. While classical consistency algorithms [Mackworth, 1977] aim to reduce the size of constraint problems, our CH-2-C algorithm works by explicating error information that is originally implicit in CH problems. The space complexity of the CH-2-C algorithm is simply of $O(n_v n_d n_c)$ in the worst case where n_c is the number of labeled constraints, n_v is the number of variables, and n_d is the size of the largest variable domain.

Incorporating a CH-2-C enforcement algorithm in a branch-and-bound algorithm, we obtain a general finite domain CH solver, which works for arbitrary comparators. Search space is pruned by utilizing the error information generated by the CH-2-C algorithm. Unlike other finite domain CH solvers, our proposed solver is applicable to arbitrary comparators.

We compare the performance of our proposed solver with generate-and-test, basic branch-and-bound, and the reified constraint approach by Lua [2000]. Experiments confirm the efficiency and robustness of our research prototype, which brings us one step towards practical finite domain CH solving.

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