Advanced topic: Zero-Knowledge Proofs

CSCI 3130 Formal Languages and Automata Theory

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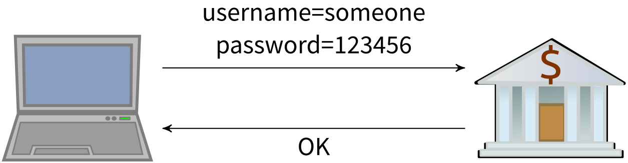
Authentication

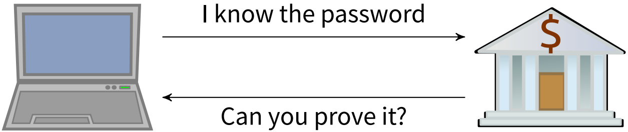


username=someone
password=123456

OK

- Server knows your password
- They may impersonate you at other websites where you use the same password

Zero-knowledge authentication



I know the password

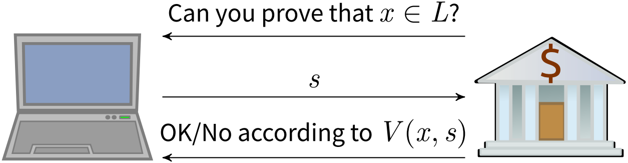
Can you prove it?

Can you convince the server that you know your password, without revealing it?

NP and proofs

Recall that a language L is in NP if

there is a polynomial-time verifier V such that
$x \in L$ if and only if V accepts (x, s) for some s



Can you prove that $x \in L$?

s

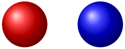
OK/No according to $V(x, s)$

s is a proof that $x \in L$

Verifier V is convinced that $x \in L$, but verifier also knows a lot more

A protocol for non-color-blindness

You want to convince me you are not color-blind



I pull at random either a red ball or a blue ball and show it to you

You say red or blue

We repeat this 10 times

If you got all the answers right
I am convinced you know red from blue

Interaction and knowledge

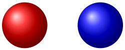
What knowledge did I gain from this interaction?

I learned that you can tell red from blue

But I also learned the colors of the balls

If I were color-blind
Then I used you to gain some knowledge

A different protocol



I pull at random either a red ball or a blue ball and show it to you

We repeat 10 times

Each time, you say “same color as previous” or “different color from previous”

If you got all the answers right
I am convinced you know red from blue

But I did not gain other knowledge!

Zero-knowledge

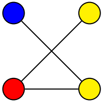
Suppose I am color-blind but you are not

In the first experiment, I cannot predict your answer ahead of time

In the second one, I know what you are supposed to say, so I do not gain knowledge when you say it

Graph Coloring

Task: Assign one of 3 colors to the nodes so that every edge has different colors at its endpoints

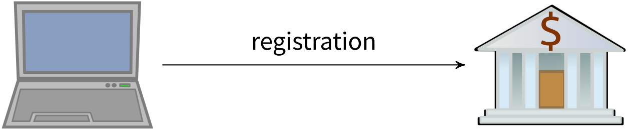


$$\text{3COL} = \{\langle G \rangle \mid \text{Graph } G \text{ has a valid } 3\text{-coloring}\}$$

3COL is NP-complete

Goldreich–Micali–Wigderson proposed a zero-knowledge procotol for 3COL

GMW protocol: Choosing a password



password is a random string of colors

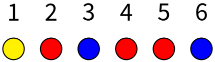
$\Sigma = \{$🔵, 🔴, 🟡$\}$

e.g. password $=$ 🟡🔴🔵🔴🔴🔵

GMW protocol: Commitment phase

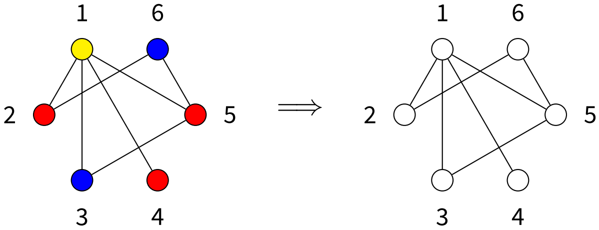
Instead of sending the password to the server

you construct a graph with vertices colored as in password



Put some (random) edges between vertices of different colors

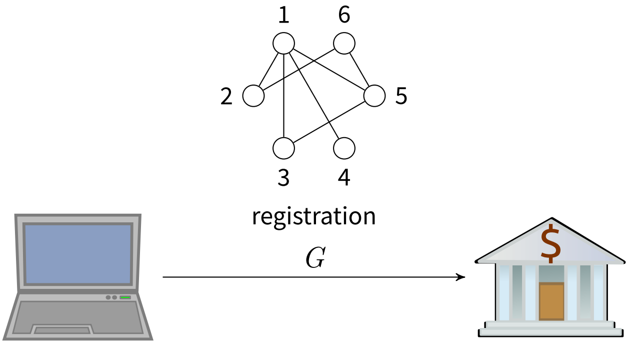
Delete the colors of the vertices



GMW protocol: Commitment phase

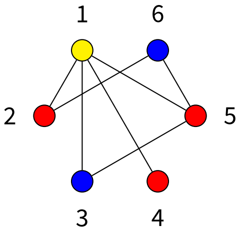
Your real password is the coloring, which you hide from the server

You give the server a graph G that you know how to color, but the server doesn't



Since 3COL is NP-hard, the server shouldn't be able to figure out your coloring (password) from G

GMW protocol: Login phase



You randomly permute the colors

You lock each of the colors in an imaginary box

Send the locked boxes to server

The server picks a random edge and asks for the keys to the related boxes

You send the two requested keys

The server unlocks two boxes and checks the colors are different

Repeat all of the above steps 1000 times

If colors are always different, login succeeds

GMW protocol: Security

Why can’t an impostor log in instead of you?
An impostor does not know how to color the graph

Some edge will be colored improperly
When the server asks to see this edge, impostor will be detected

GMW protocol: Zero-knowledge

Why doesn't the server learn your password?

When you send the password, the server can only see some locked boxes
The server then asks you to unlock some boxses

Colors in the password were shuffled, so server will only see two random colors

Hidden details

How do you send boxes and keys over the internet?

Commitment scheme!

Other proposed applications

1. Zero-knowledge voting
2. Zero-knowledge nuclear warhead verification