Week 4 Tutorial Session

1. For any integer $k \geq 0$, define $L_k = \{ww \mid w \in \{0,1\}^k\}$.
   (a) Write down all strings in L_3.
   (b) Prove that any DFA for L_k has at least 2^k states.
   Hint: After reading the first half of the input, what should the DFA remember? Can you come up with a set of 2^k strings that are pairwise distinguishable by L_k?

2. For an integer $k \geq 1$, define L_k to be the set of strings (over $\Sigma = \{0,1\}$) that have a 1 at the kth-to-last position. For example, **1**00 and 0**1**01 are in L_3, but 0 and 011 are not.
   (a) Prove that every DFA for L_k has at least 2^k states.
   (b) Describe (e.g. with a diagram) an NFA for L_k that has at most $k+1$ states.

3. Let L be the set of strings over $\{0,1\}$ whose number of ones is a perfect square (e.g. $0, 1, 4, 9, 16, \ldots$). Prove that L is irregular.