CSCI 3130: Formal Languages and Automata Theory | Homework 5
The Chinese University of Hong Kong, Fall 2019 | due 11:59pm Monday December 9

Collaborating on homework is encouraged, but you must write your own solutions in your own words and list your collaborators. Copying someone else's solution will be considered plagiarism and may result in failing the whole course.

Please answer clearly and concisely. Explain your answers. Unexplained answers will get lower scores or even no credits.

(1) (30 points) Prove that the following languages are decidable, by giving algorithms to decide them.

(a) $L_1 = \{\langle R \rangle \mid R$ does not generate any string containing `bbb` as a substring$\}$
Here R is a regular expression over alphabet $\{\texttt{a}, \texttt{b}\}$

(b) $L_2 = \{\langle D \rangle \mid D$ is a DFA that accepts an infinite number of strings$\}$

(c) $L_3 = \{\langle G \rangle \mid G$ generates at least one string containing `b`$\}$
Here G is a context-free grammar over alphabet $\{\texttt{a}, \texttt{b}\}$

(2) (40 points) For each of the following languages, say whether it is decidable. Justify your answer in about 5–10 sentences.

(a) $L_1 = \{\langle M, k \rangle \mid$ Turing machine M accepts all strings of length k and nothing else$\}$

(b) $L_2 = \{\langle M \rangle \mid$ Turing machine M is a decider (i.e. halts on every input)$\}$

(c) $L_3 = \{\langle G \rangle \mid$ Every string beginning with `bbb` can be generated by $G\}$
Here G is a context-free grammar over alphabet $\{\texttt{a}, \texttt{b}\}$

(d) $L_4 = \{\langle M, t \rangle \mid$ Turing machine M accepts some input in at most t steps$\}$

(3) (30 points) For each of the following variants of the Post Correspondence Problem (PCP), say if it is decidable or not. Justify your answer either by describing an algorithm to decide the language, or by reducing from PCP (over arbitrary alphabet).

(a) PCP_1: PCP over the alphabet $\Sigma = \{\texttt{1}\}$.

(b) PCP_2: PCP over the alphabet $\Sigma = \{\texttt{0}, \texttt{1}\}$.

The *alphabet* of PCP is the set of symbols that are allowed to appear on the tiles. Here is an instance of PCP over alphabet $\Sigma = \{\texttt{a}, \texttt{b}, \texttt{c}, \texttt{d}\}$:

ab	ab	d	b
bc	d	cd	d