Variants of Turing Machines

CSCI 3130 Formal Languages and Automata Theory

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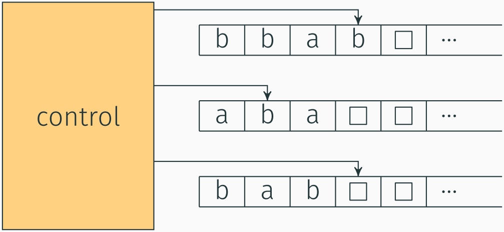
Fall 2021

Chinese University of Hong Kong

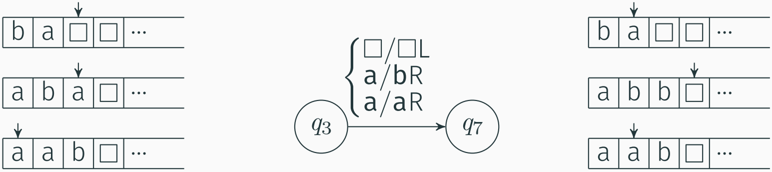
Multitape Turing machine



Transitions may depend on the contents of all cells under the heads

Different tape heads can move independently

Multitape Turing machine

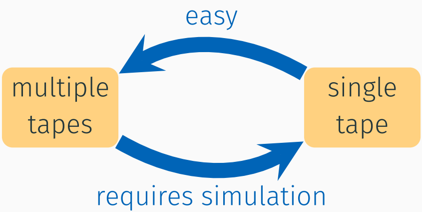


Multiple tapes are convenient

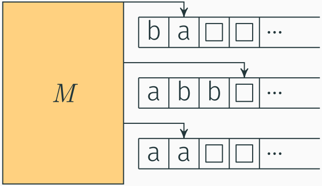
Some tapes can serve as temporary storage

How to argue equivalence

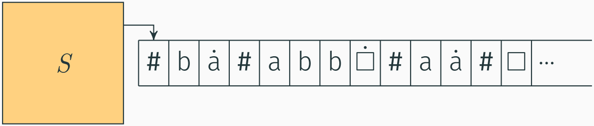
Multitape Turing machines are equivalent to single-tape Turing machines



Simulating multitape Turing machine



$\Gamma = \{a, b, \square\}$



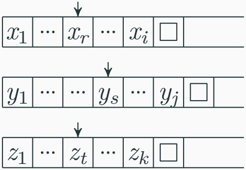
$\Gamma = \{a, b, \square, \dot{a}, \dot{b}, \dot{\square}, \#\}$

Simulating multitape Turing machine

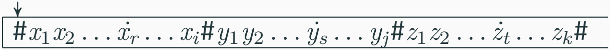
We show how to simulate a multitape Turing machine on a single tape Turing machine

To be specific, let's simulate a 3-tape TM

Multitape TM M

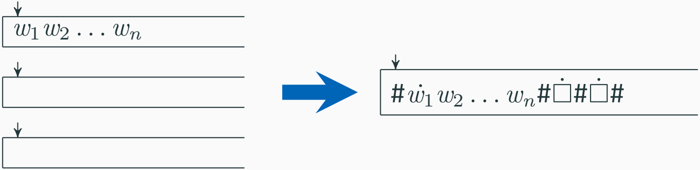


Single tape TM S



Simulating multitape Turing machine

Single-tape TM: Initialization



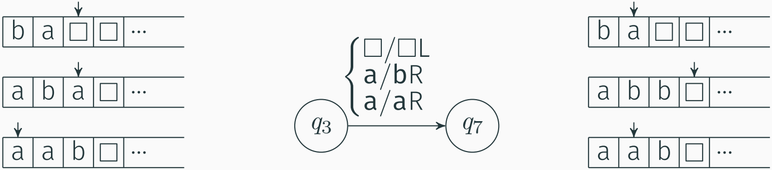
S: On input $w_1 \dots w_n$:

Replace tape contents by $\# \dot{w}_1 w_2 \dots w_n \# \dot{\square} \# \dot{\square} \#$

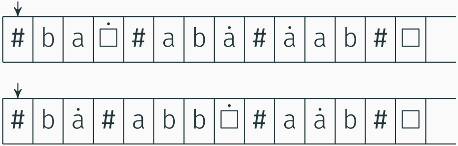
Remember that M is in state q_0

Single-tape TM: Simulating multitape TM moves

Suppose Multitape TM M moves like this:



We simulate the move on single-tape TM S like this



S given input $w_1 \dots w_n$:

Replace tape contents by $\#\dot{w}_1 w_2 \dots w_n\#\dot{\square}\#\dot{\square}$

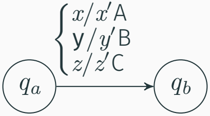
Remember (in state) that M is in state q_0

S simulates a step of M:

Make a pass over tape to find $\dot{x}$, $\dot{y}$, $\dot{z}$

$$\#x_1 x_2 \dots \dot{x} \dots x_i \# y_1 y_2 \dots \dot{y} \dots y_j \# z_1 z_2 \dots \dot{z} \dots z_k \#$$

If M at state q_a has transition



update state/tape accordingly

If M reaches accept (reject) state, S accepts (rejects)

Simulation

To simulate a model M by another model N:

Say how the state and storage of N is used to represent the state and storage of M

Say what should be initially done to convert the input of N

Say how each transition of M can be implemented by a sequence of transitions of N