CSCI 3130: Formal Languages and Automata Theory — Assignment 5

The Chinese University of Hong Kong, Fall 2021 — due 11:59pm Thursday November 25

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Collaborating on homework is encouraged, but you must write your own solutions in your own words and list your collaborators. Copying someone else's solution will be considered plagiarism and may result in failing the whole course.

Please answer clearly and concisely. Explain your answers. Unexplained answers will get lower scores or even no credits.

(1) (30 points) Prove that the following languages are decidable, by giving algorithms to decide them.

   (a) $L_1 = \{\langle R \rangle \mid \text{Every string of odd length can be generated by } R\}$
   Here $R$ is a regular expression over the alphabet $\{\mathtt{a}, \mathtt{b}\}$.

   (b) $L_2 = \{\langle D \rangle \mid D \text{ is a DFA that accepts an infinite number of strings}\}$

   (c) $L_3 = \{\langle G \rangle \mid G \text{ is a CFG that generates at least one string of odd length}\}$

(2) (40 points) For each of the following languages, say whether it is decidable. Justify your answer in about 5–10 sentences.

   (a) $L_1 = \{\langle M, t \rangle \mid \text{Turing machine } M \text{ accepts some input in at least } t \text{ steps}\}$

   (b) $L_2 = \{\langle M \rangle \mid \text{Turing machine } M \text{ infinite loops on some input}\}$

   (c) $L_3 = \{\langle M_1, M_2 \rangle \mid M_1, M_2 \text{ are Turing machines such that } L(M_1) \neq L(M_2)\}$

   (d) $L_4 = \{\langle M, t \rangle \mid \text{Turing machine } M \text{ accepts every input in at most } t \text{ steps}\}$

(3) (30 points) For each of the following variants of the Post Correspondence Problem (PCP), say if it is decidable or not. Justify your answer either by describing an algorithm to decide the language, or by reducing from PCP (over arbitrary alphabet).

   (a) $\text{PCP}_1$: PCP over the alphabet $\Sigma = \{\mathtt{1}\}$.

   (b) $\text{PCP}_1$: PCP over the alphabet $\Sigma = \{\mathtt{0}, \mathtt{1}\}$.

The *alphabet* of PCP is the set of symbols that can appear on the tiles. For example, here is an instance of PCP over the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$:

| | | | |
|---|---|---|---|
| ab | ab | d | b |
| bc | d | cd | d |