Collaborating on homework is encouraged, but you must write your own solutions in your own words and list your collaborators. Copying someone else's solution will be considered plagiarism and may result in failing the whole course.

Please answer clearly and concisely. Explain your answers. Unexplained answers will get lower scores or even no credits.

(1) (36 points) For each of these languages, give both a context-free grammar *and* a pushdown automaton. Briefly explain how your CFG and PDA work. Your CFG should be relatively simple and contain at most 6 variables. For this question, your PDA cannot be directly converted from your CFG (as in Lecture 11).

(a) $L_1 = \{x\#y \mid x \text{ is a suffix of } y^R, \text{ and } y \in \{\texttt{a}, \texttt{b}\}^*\},\ \Sigma = \{\texttt{a}, \texttt{b}, \#\}$
Recall that w^R is w written backwards, and u is a suffix of v if $v = tu$ for some string t.
For example bb#bba is in L_1, but a#ba is not.

(b) $L_2 = \{\texttt{0}^i\texttt{1}^{2i} \mid i \geqslant 0\}, \Sigma = \{\texttt{0}, \texttt{1}\}$

(c) $L_3 = \{\texttt{a}^i\texttt{b}^j\texttt{c}^k \mid i < j \text{ or } j < k\}, \Sigma = \{\texttt{a}, \texttt{b}, \texttt{c}\}$

(2) (24 points) Consider the following context-free grammar G that describes simple boolean expressions involving the operators $\star$ (multiplication), - (unary minus), and variables x and y:

$$E \to E\star E \mid \texttt{(}-E\texttt{)} \mid \texttt{x} \mid \texttt{y}$$

The alphabet of G consists of $\star$, -, (,), x, y.

(a) Convert G to Chomsky Normal Form.

(b) Apply the Cocke–Younger–Kasami algorithm to obtain a parse tree for the following string: (-x)$\star$y. Show the table of variables that generate every substring. Also draw the parse tree you get.

(c) Show that G is ambiguous. Also give a CFG G' that describes the same language as G but is not ambiguous. (Your G' needs not be in Chomsky Normal Form.)

(3) (40 points) Consider the following languages. For each of the languages, say whether the language is (1) regular, (2) context-free but not regular, or (3) not context-free. Explain your answer (give a DFA or argue why one exists, give a CFG or PDA, apply the appropriate pumping lemma or give pairwise distinguishable strings).

(a) $L_1 = \{\texttt{a}^n\#\texttt{a}^n\#\texttt{a}^n \mid n \geqslant 0\}, \Sigma = \{\texttt{a}, \#\}$

(b) $L_2 = \{\texttt{a}^i\texttt{b}^{2i}\texttt{a}^{2i}\texttt{b}^i \mid i \geqslant 0\}, \Sigma = \{\texttt{a}, \texttt{b}\}$

(c) $L_3 = \{u\#v \mid v \in \{\texttt{c}, \texttt{d}\}^* \text{ and } u \text{ is a suffix of } v\}, \Sigma = \{\texttt{c}, \texttt{d}, \#\}$
Recall that u is a suffix of v if $v = tu$ for some string t.

(d) $L_4 = \{w \in \{\texttt{a}, \texttt{b}\}^* \mid w \text{ has exactly three times as many b's as a's}\}$
For example, babb $\in L_4$ but aba $\notin L_4$