LR(0) Parsers

CSCI 3130 Formal Languages and Automata Theory

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Parsing computer programs

```
if (n == 0) { return x; }
```

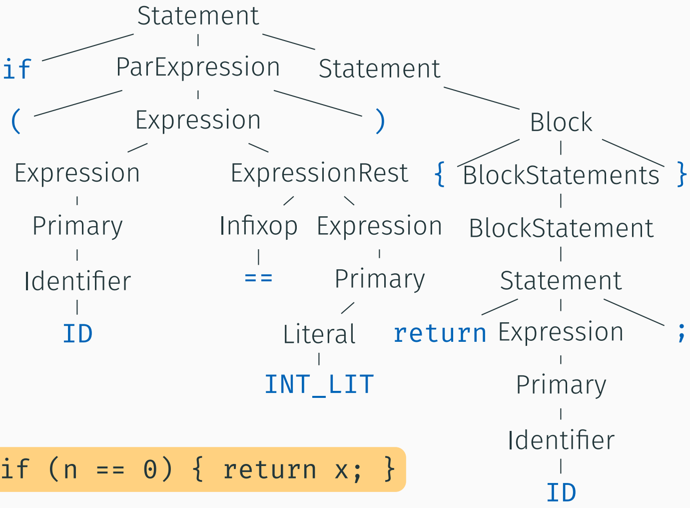
First phase of `javac` compiler: lexical analysis

`if` `(` `ID` `==` `INT_LIT` `)` `{` `return` `ID` `;` `}`

The alphabet of Java CFG consists of tokens like

$$\Sigma = \{\texttt{if}, \texttt{return}, \texttt{(}, \texttt{)}, \texttt{\{}, \texttt{\}}, \texttt{;}, \texttt{==}, \texttt{ID}, \texttt{INT\_LIT}, \ldots\}$$

Parse tree of a Java statement



```
if (n == 0) { return x; }
```

CFG of the java programming language

```
Identifier:
   IdentifierChars but not a Keyword or BooleanLiteral or
   NullLiteral
Literal:
   IntegerLiteral
   FloatingPointLiteral
   BooleanLiteral
   CharacterLiteral
   StringLiteral
   NullLiteral
Expression:
   LambdaExpression
   AssignmentExpression

AssignmentOperator:
   (one of) = *= /= %= += -= <<= >>= >>>= &= ^= |=
```

from `http://java.sun.com/docs/books/jls/second_edition/html/syntax.doc.html#52996`

Parsing Java programs

```
class Point2d {
    /* The X and Y coordinates of the point--instance variables */
    private double x;
    private double y;
    private boolean debug;    // A trick to help with debugging

    public Point2d (double px, double py) { // Constructor
        x = px;
        y = py;

        debug = false;     // turn off debugging
    }

    public Point2d () {   // Default constructor
        this (0.0, 0.0);                    // Invokes 2 parameter Point2D constructor
    }
    // Note that a this() invocation must be the BEGINNING of
    // statement body of constructor

    public Point2d (Point2d pt) {             // Another consructor
        x = pt.getX();
        y = pt.getY();
    }
  ...
}
```

Simple Java program: about 1000 tokens

Parsing algorithms

How long would it take to parse this program?

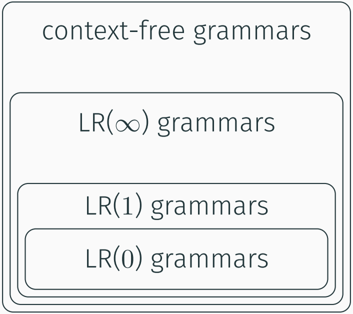
try all parse trees	$\geqslant 10^{80}$ years
CYK algorithm	hours

Can we parse faster?

CYK is the fastest known general-purpose parsing algorithm for CFGs

Luckily, some CFGs can be rewritten to allow for a faster parsing algorithm!

Hierarchy of context-free grammars



context-free grammars

LR(∞) grammars

LR(1) grammars

LR(0) grammars

Java, Python, etc have LR(1) grammars

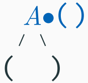
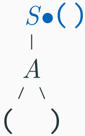
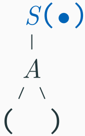
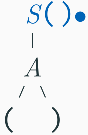
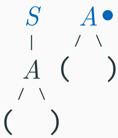
We will describe LR(0) parsing algorithm

A grammar is LR(0) if LR(0) parser works correctly for it

LR(0) parser: overview

$$S \to SA \mid A$$
$$A \to (\,S\,) \mid (\,)$$

input: ( )( )

1 •( )( )	2 (•)( )	3 ( )•( )
4 	5 	6 
7 	8 	

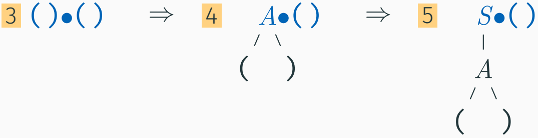
LR(0) parser: overview

$$\begin{aligned} S &\to SA \mid A \\ A &\to (\,S\,) \mid (\,) \end{aligned}$$

input: ( )( )

Features of LR(0) parser:

- Greedily reduce the recently completed rule into a variable
- Unique choice of reduction at any time



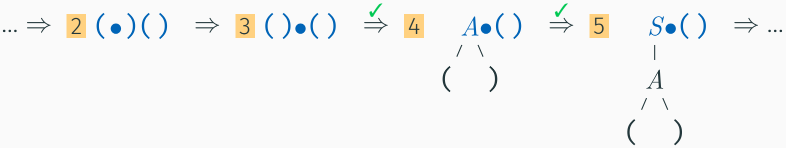
LR(0) parsing using a PDA

To speed up parsing, keep track of partially completed rules in a PDA P

In fact, the PDA will be a simple modification of an NFA N

The NFA accepts if a rule $B \to \beta$ has just been completed

and the PDA will reduce β to B



✓: NFA N accepts

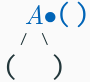
NFA acceptance condition

$$S \to SA \mid A$$
$$A \to (S) \mid ()$$

A rule $B \to \beta$ has just been completed if

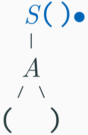
Case 1 input/buffer so far is exactly β

Examples: 3 ( )•( ) and 4



Case 2 Or buffer so far is $\alpha\beta$ and there is another rule $C \to \alpha B\gamma$

Example: 7



This case can be chained

Designing NFA for Case 1

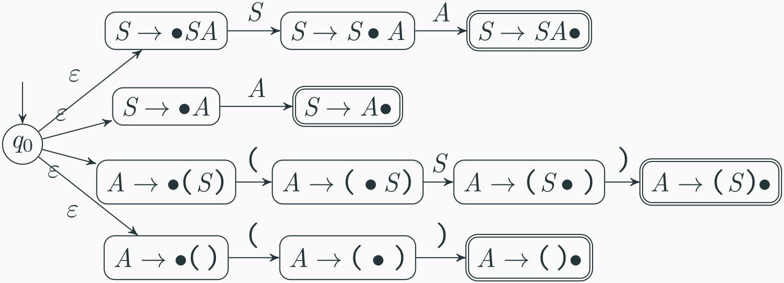
$$S \to SA \mid A$$
$$A \to (S) \mid ()$$

Design an NFA N' to accept the right hand side of some rule $B \to \beta$

Designing NFA for Case 1

$$S \to SA \mid A$$
$$A \to (\,S\,) \mid (\,)$$

Design an NFA N' to accept the right hand side of some rule $B \to \beta$



Designing NFA for Cases 1 & 2

$$S \to SA \mid A$$
$$A \to (\,S) \mid (\,)$$

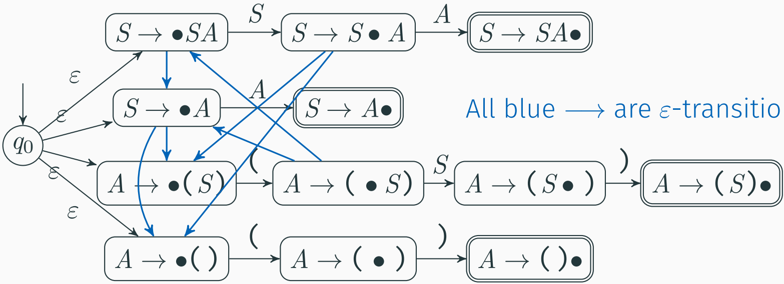
Design an NFA N to accept $\alpha\beta$ for some rules $C \to \alpha B \gamma, \quad B \to \beta$
and for longer chains

Designing NFA for Cases 1 & 2

$S \to SA \mid A$
$A \to (\,S\,) \mid (\,)$

Design an NFA N to accept $\alpha\beta$ for some rules $C \to \alpha B\gamma, \quad B \to \beta$
and for longer chains

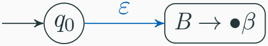
For every rule $C \to \alpha B\gamma$, $B \to \beta$, add $\boxed{C \to \alpha \bullet B\gamma} \xrightarrow{\varepsilon} \boxed{B \to \bullet\beta}$



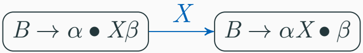
All blue ⟶ are ε-transitions

Summary of the NFA

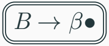
For every rule $B \to \beta$, add



For every rule $B \to \alpha X \beta$ (X may be terminal or variable), add



Every completed rule $B \to \beta$ is accepting

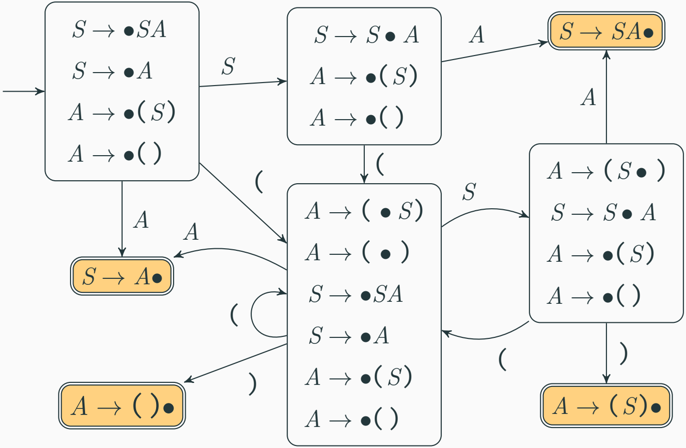


For every rule $C \to \alpha B \gamma$, $B \to \beta$, add

The NFA N will accept whenever a rule has just been completed

Equivalent DFA D for the NFA N

Dead state (empty set) not shown for clarity



Observation: every accepting state has only one rule:
a completed rule, and such rules appear only in accepting states

LR(0) grammars

A grammar G is LR(0) if its corresponding D_G satisfies:

Every accepting state has only one rule:
a completed rule of the form $B \rightarrow \beta\bullet$
and completed rules appear only in accepting states

Shift state:

no completed rule

$$\boxed{\begin{array}{l} S \rightarrow S \bullet A \\ A \rightarrow \bullet(\,S\,) \\ A \rightarrow \bullet(\,) \end{array}}$$

Reduce state:

has (unique) completed rule

$$\boxed{A \rightarrow (\,S\,)\bullet}$$

Simulating DFA D

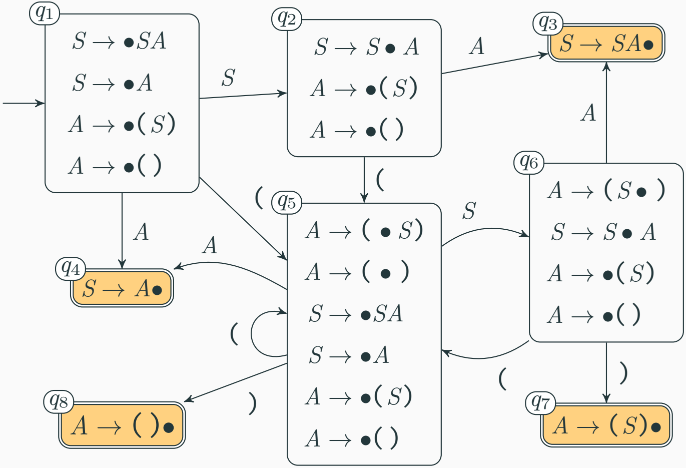
Our parser P simulates state transitions in DFA D

After reducing ( ) to A, what is the new state?

Solution: keep track of previous states in a stack

go back to the correct state by looking at the stack

Let’s label D’s states



LR(0) parser: a “PDA” P simulating DFA D

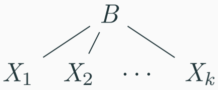
P’s stack contains labels of D’s states to remember progress of partially completed rules

At D’s non-accepting state q_i

1. P simulates D’s transition upon reading terminal or variable X
2. P pushes current state label q_i onto its stack

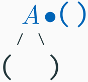
At D’s accepting state with completed rule $B \to X_1 \dots X_k$

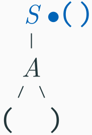
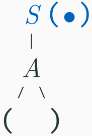
1. P pops k labels $q_k, \dots, q_1$ from its stack
2. constructs part of the parse tree



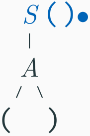
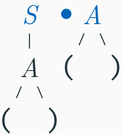
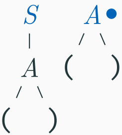
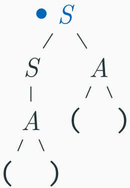
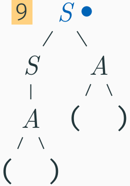
3. P goes to state q_1 (last label popped earlier), pretend next input symbol is B

Example

		state	stack
1	$\bullet(\,)(\,)$	q_1	$\$$
2	$(\bullet)(\,)$	q_5	$\$1$
3	$(\,)\bullet(\,)$	q_8	$\$15$
	 $\bullet A(\,)$	q_1	$\$$
4	 $A\bullet(\,)$	q_4	$\$1$
	 $\bullet\ S(\,)$	q_1	$\$$

		state	stack
5	 $S\bullet(\,)$	q_2	$\$1$
6	 $S(\bullet)$	q_5	$\$12$

Example

	state	stack
7 	q_8	$\$125$
	q_2	$\$1$
8 	q_3	$\$12$
	q_1	$\$$
9 	q_2	$\$1$

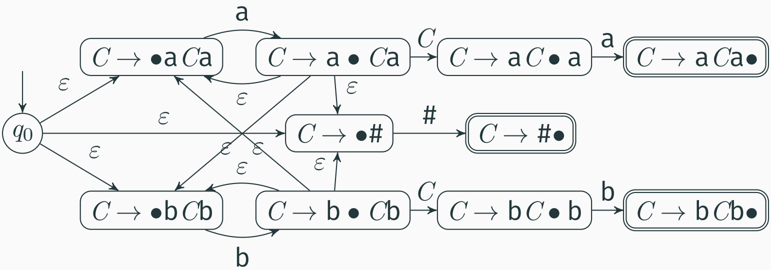
parser's output is the parse tree

Another LR(0) grammar

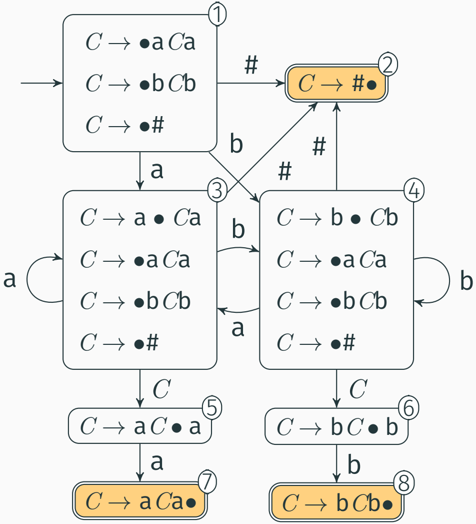
$L = \{w\#w^R \mid w \in \{\mathsf{a}, \mathsf{b}\}^*\}$

$C \to \mathsf{a}\,C\mathsf{a} \mid \mathsf{b}\,C\mathsf{b} \mid \#$

NFA N:



Another LR(0) grammar



$C \to \mathsf{a} C \mathsf{a} \mid \mathsf{b} C \mathsf{b} \mid \#$

input: ba#ab

stack	state	action
$	1	S
$1	4	S
$14	3	S
$\$14\underline{3}$	2	R
$143	5	S
$\$1\underline{435}$	7	R
$14	6	S
$\$1\underline{46}$	8	R

Deterministic PDAs

PDA for LR(0) parsing is deterministic

Some CFLs require non-deterministic PDAs, such as

$$L = \{ww^R \mid w \in \{\mathsf{a}, \mathsf{b}\}^*\}$$

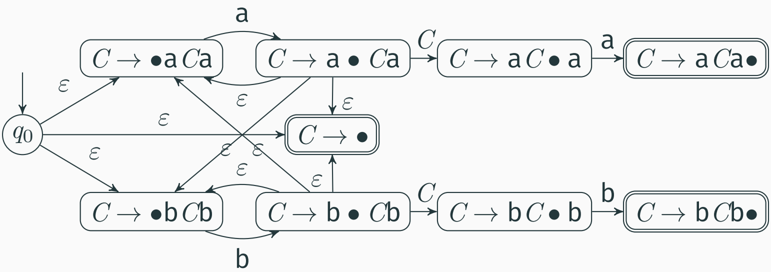
What goes wrong when we do LR(0) parsing on L?

Example 2

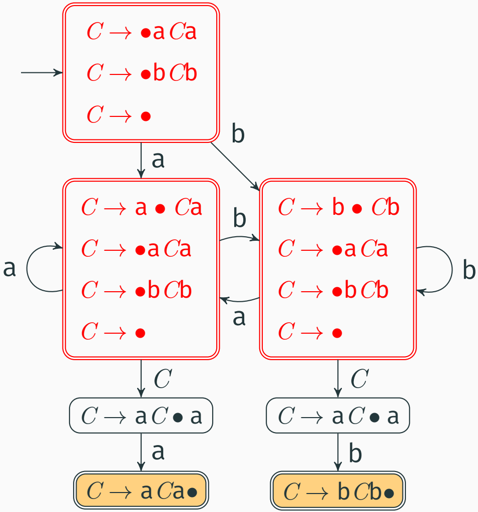
$L = \{ww^R \mid w \in \{\mathsf{a}, \mathsf{b}\}^*\}$

$C \to \mathsf{a}C\mathsf{a} \mid \mathsf{b}C\mathsf{b} \mid \varepsilon$

NFA N:



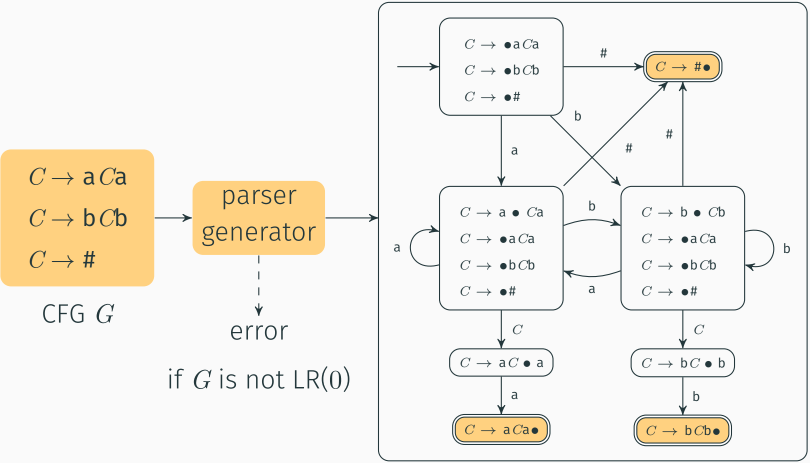
Example 2



$C \to \mathsf{a} C \mathsf{a} \mid \mathsf{b} C \mathsf{b} \mid \varepsilon$

shift-reduce conflicts

Parser generator



CFG G

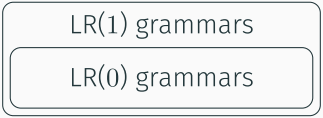
if G is not LR(0)

“PDA” for parsing G

Motivation: Fast parsing for programming languages

LR(1) Grammar: a few words

LR(0) grammar revisited



LR(0) parser: **L**eft-to-right read, **R**ightmost derivation, **0** lookahead symbol

$$S \to SA \mid A$$
$$A \to (\,S\,) \mid (\,)$$

Derivation

$S \Rightarrow SA \Rightarrow S(\,) \Rightarrow A(\,) \Rightarrow (\,)(\,)$

Reduction (derivation in reverse)

$(\,)(\,) \rightarrowtail A(\,) \rightarrowtail S(\,) \rightarrowtail SA \rightarrowtail S$

LR(0) parser looks for rightmost derivation

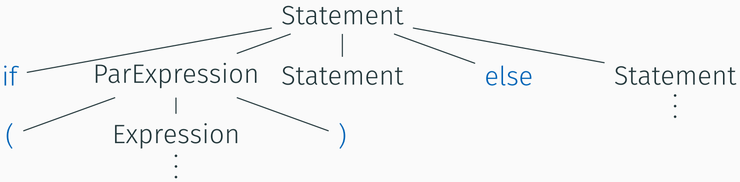
Rightmost derivation = Leftmost reduction

Parsing computer programs

```
if (n == 0) { return x; }
```

Parsing computer programs

```
if (n == 0) { return x; }
else { return x + 1; }
```



CFGs of most programming languages are not LR(0)

LR(0) parser cannot tell apart

`if` …`then` from `if` …`then` …`else`

LR(1) grammar

LR(1) grammars resolve such conflicts by one symbol lookahead

States in NFA N

LR(0):	LR(1):
$A \to \alpha \bullet \beta$	$[A \to \alpha \bullet \beta, a]$

States in DFA D

	LR(0):	LR(1):
shift-reduce conflicts	forbidden	some allowed
reduce-reduce conflicts	forbidden	some allowed if resolvable with lookahead symbol a

We won't cover LR(1) parser in this class; take CSCI 3180 for details