Sensor Drift Calibration via Spatial Correlation Model in Smart Building

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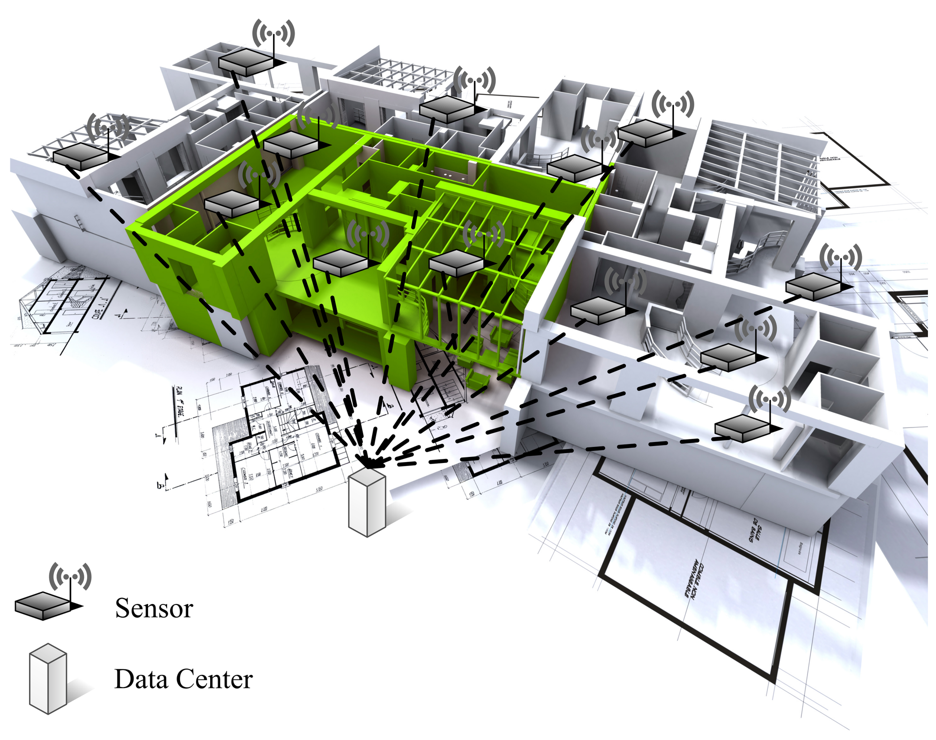
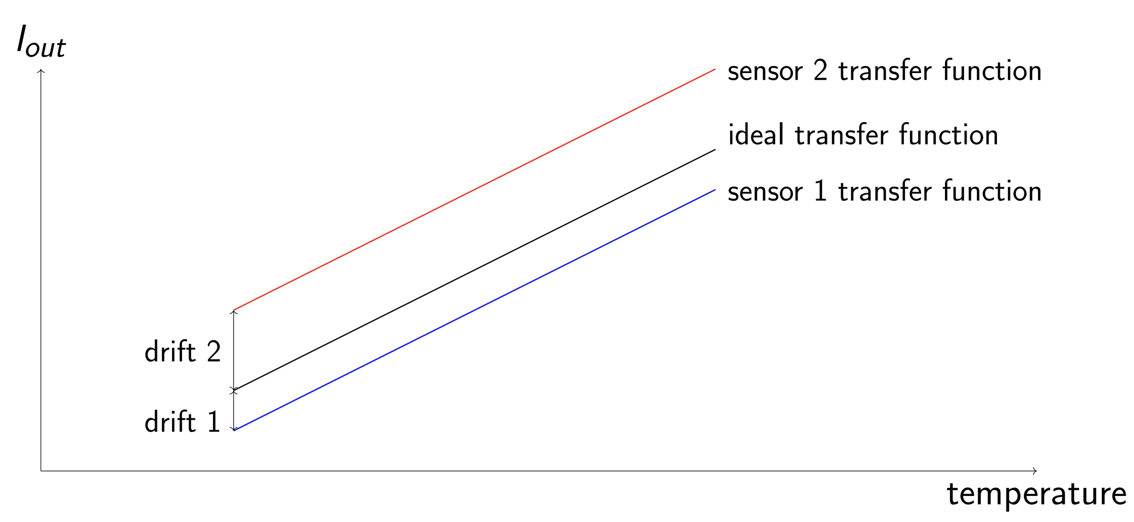
Introduction



Part Number	Temp. Range	Accuracy	Price
SMT172	−45 ∼ 130 °C	±0.25 °C	$ 35.13
AD590JH	−50 ∼ 150 °C	±0.5 °C	$ 17.91
TMP100	−55 ∼ 125 °C	±2.0 °C	$ 1.79
MCP9509	−40 ∼ 125 °C	±4.5 °C	$ 0.88
LM335A	−40 ∼ 100 °C	±5.0 °C	$ 0.75

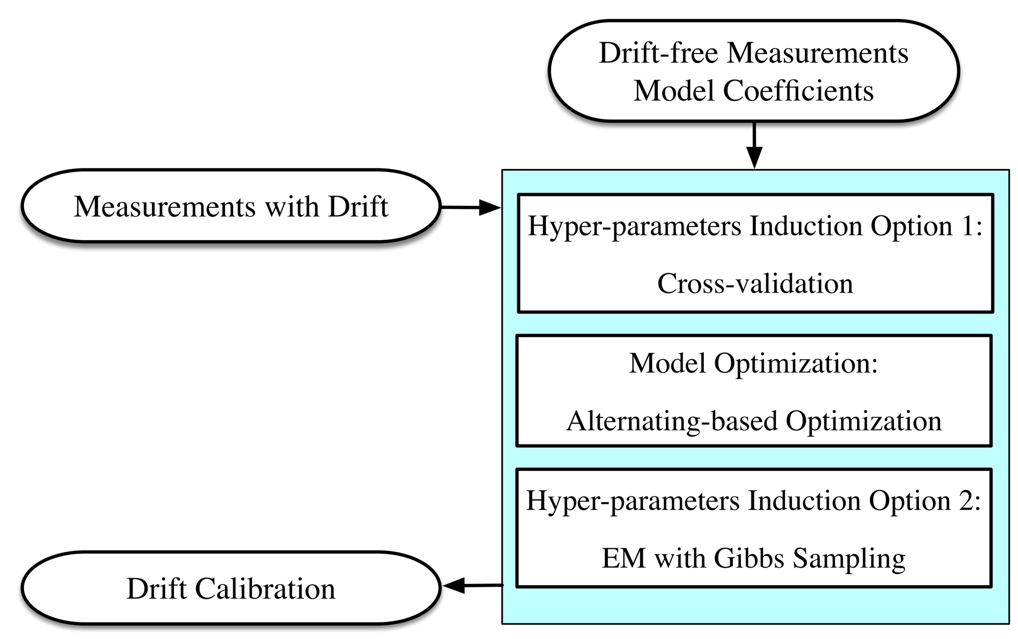
- Measurement from sensor without calibration is not accurate enough
- Looking for ways to do calibration

Spatial Correlation Model and Formulation



Sensor Drift Calibration

Given the measurement values sensed by all sensors during several time-instants, drifts will be accurately estimated and calibrated.



Input:

- $\hat{x}_i^{(k)}$: the measurement value sensed by ith sensor at kth time-instant.
- $a_{i,j}$: the drift-free model coefficient.

Output:

- ϵ_i: a time-invariant drift calibration.

Spatial Correlation Model:

- drift-free model: $x_i^{(k)} \approx \sum_{j=1, j\neq i}^{n} a_{i,j}x_j^{(k)} + a_{i,0}, k = 1, 2, \cdots, m_0$.
- drift-with model: $\hat{x}_i^{(k)} + \epsilon_i \approx \sum_{j=1, j\neq i}^{n} \hat{a}_{i,j}(\hat{x}_j^{(k)} + \epsilon_j) + \hat{a}_{i,0}, k = 1, 2, \cdots, m$.

Further Assumption:

- Likelihood: $\mathcal{P}(\hat{\mathbf{x}}|\hat{\mathbf{a}}, \boldsymbol{\epsilon}) \propto \exp(-\frac{\delta_0}{2}\sum_{i=1}^{n}\sum_{k=1}^{m}[\hat{x}_i^{(k)} + \epsilon_i - \sum_{j=1, j\neq i}^{n}\hat{a}_{i,j}(\hat{x}_j^{(k)} + \epsilon_j) - \hat{a}_{i,0}]^2)$.
- Prior distribution of model coefficients (Bayesian Model Fusion): $\mathcal{P}(\hat{\mathbf{a}}) \propto \exp\left(-\sum_{i=1}^{n}\sum_{j=0, j\neq i}^{n}\frac{\lambda}{2a_{i,j}^2}(\hat{a}_{i,j} - a_{i,j})^2\right)$.
- Prior distribution of calibration: $\mathcal{P}(\boldsymbol{\epsilon}) \propto \exp(-\frac{\delta_\epsilon}{2}\sum_{i=1}^{n}\epsilon_i^2)$.

Maximum-a-posteriori:

$$\min_{\hat{\mathbf{a}}, \boldsymbol{\epsilon}} \quad \delta_0\sum_{i=1}^{n}\sum_{k=1}^{m}[\hat{x}_i^{(k)} + \epsilon_i - \sum_{j=1, j\neq i}^{n}\hat{a}_{i,j}(\hat{x}_j^{(k)} + \epsilon_j) - \hat{a}_{i,0}]^2 + \lambda\sum_{i=1}^{n}\sum_{j=0, j\neq i}^{n}\frac{1}{a_{i,j}^2}(\hat{a}_{i,j} - a_{i,j})^2 + \delta_\epsilon\sum_{i=1}^{n}\epsilon_i^2.$$

Challenges:

- How to handle this Formulation
- How to determine hyper-parameters

Model Optimization

- With fixed one variable, Formulation is a convex unconstrained QP problem w.r.t. another variable;
- With the first-order optimality condition, sub-Formulation can be transferred as linear equations.

Alternating-based Optimization

Require: Sensor measurements $\hat{\mathbf{x}}$, prior $\mathbf{a}$ and hyper-parameters λ, δ_0, δ_ϵ.

```
Initialize â ← a and ϵ ← 0;
repeat
    for i ← 1 to n do
        Fix ϵ, solve linear equations (1) using Gaussian elimination to update â_i;
    end for
    Fix â, solve linear equations (2) using Gaussian elimination to update ϵ;
until Convergence
return â and ϵ.
```

$$\delta_0\sum_{k=1}^{m}(\hat{x}_t^{(k)} + \epsilon_t)\left[\sum_{j=1}^{n}\hat{a}_{i,j}(\hat{x}_j^{(k)} + \epsilon_j) + \hat{a}_{i,0}\right] + \lambda\frac{(\hat{a}_{i,t} - a_{i,t})}{a_{i,t}^2} = 0, \quad (1)$$

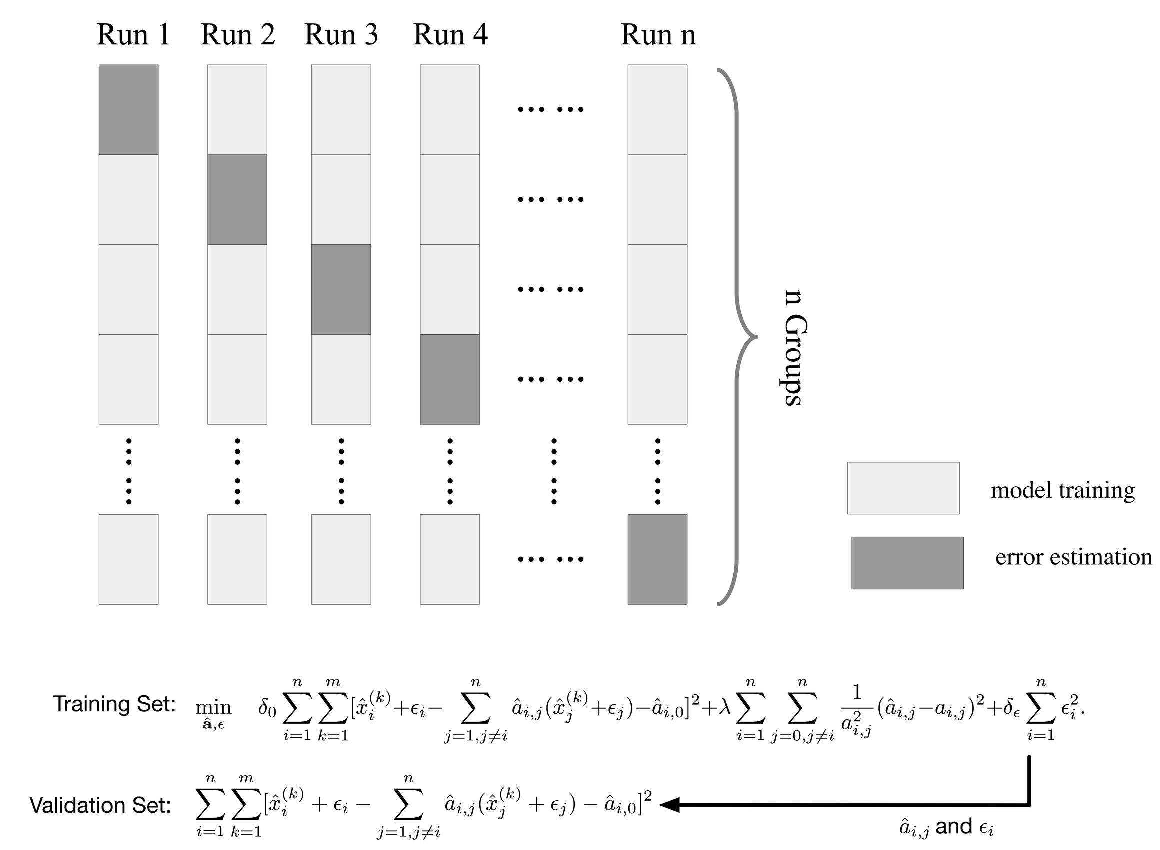
$$\delta_0\sum_{i=1}^{n}\sum_{k=1}^{m}\left[\hat{a}_{i,t}(\sum_{j=1}^{n}\hat{a}_{i,j}(\hat{x}_j^{(k)} + \epsilon_j) + \hat{a}_{i,0})\right] + \delta_\epsilon\epsilon_t = 0, \quad (2)$$

Estimation of Hyper-parameters

Comparison of Estimation for Hyper-parameters

- **Unsupervised Cross-validation:** simple, accurate but time-consuming.
- **Monte Carlo Expectation Maximization:** fast, flexible but no-accurate.

Unsupervised Cross-validation



Maximum Likelihood Estimation:

$$\max_{\delta_\epsilon, \delta_0, \lambda} \quad \mathcal{P}(\hat{\mathbf{x}}; \delta_0, \lambda, \delta_\epsilon).$$

E-Step

$$Q(\Omega|\Omega^{\text{old}}) = \int\int \mathcal{P}(\hat{\mathbf{a}}, \boldsymbol{\epsilon}|\hat{\mathbf{x}}; \Omega^{\text{old}}) \ln \mathcal{P}(\hat{\mathbf{x}}, \hat{\mathbf{a}}, \boldsymbol{\epsilon}; \Omega) d\hat{\mathbf{a}} d\boldsymbol{\epsilon} \approx \frac{1}{L}\sum_{l=1}^{L}\ln \mathcal{P}(\hat{\mathbf{x}}, \hat{\mathbf{a}}^{(l)}, \boldsymbol{\epsilon}^{(l)}; \Omega)$$

M-Step

$$\max_{\Omega} \quad \frac{1}{L}\sum_{l=1}^{L}\ln \mathcal{P}(\hat{\mathbf{x}}, \hat{\mathbf{a}}^{(l)}, \boldsymbol{\epsilon}^{(l)}; \Omega).$$

Conditional distribution of each variable

$$\hat{a}_{p,q} \sim \mathcal{P}(\hat{a}_{p,q}|\boldsymbol{\epsilon}, \hat{\mathbf{a}}_{/\hat{a}_{p,q}}, \hat{\mathbf{x}}; \delta_\epsilon, \delta, \lambda) = \mathcal{N}(\mu_{\hat{a}_{p,q}}, \sigma_{\hat{a}_{p,q}}^{-1}),$$
$$\epsilon_t \sim \mathcal{P}(\epsilon_t|\boldsymbol{\epsilon}_{/\epsilon_t}, \hat{\mathbf{a}}, \hat{\mathbf{x}}; \delta_\epsilon, \delta, \lambda) = \mathcal{N}(\mu_{\epsilon_t}, \sigma_{\epsilon_t}^{-1}),$$

Monte Carlo Expectation Maximization

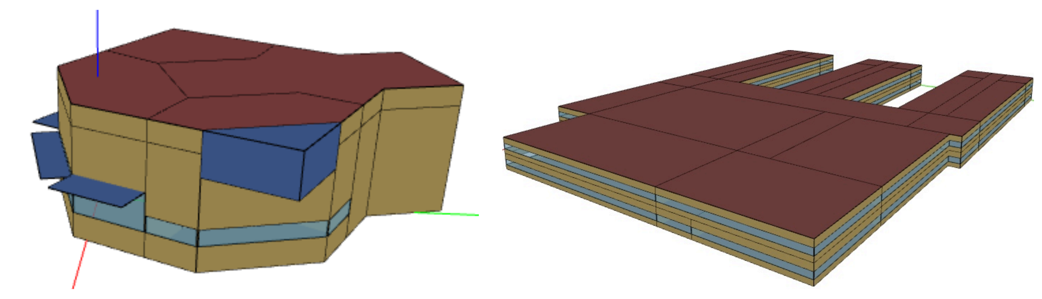
Require: Sensor measurements $\hat{\mathbf{x}}$, prior $\mathbf{a}$;

```
Initialize hyper-parameters Ω;
repeat
    Initialize samples Ψ^(0) by Alternating-based Optimization;
    for l ← 1 to L do
        for i ← 1 to n^2 + n do
            Sample ψ_i^(l) from the desired conditional distribution
            N(μ_ψi, σ_ψi) with ψ_1^(l), ..., ψ_(i-1)^(l), ψ_(i+1)^(l-1), ..., ψ_(n^2+n)^(l-1);
        end for
    end for
    Update hyper-parameters Ω;
until Convergence
return hyper-parameters Ω.
```

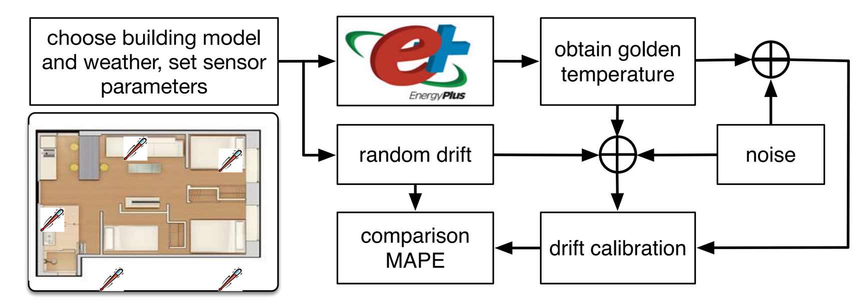
Experimental Results

Benchmark

- Hall
- Secondary School

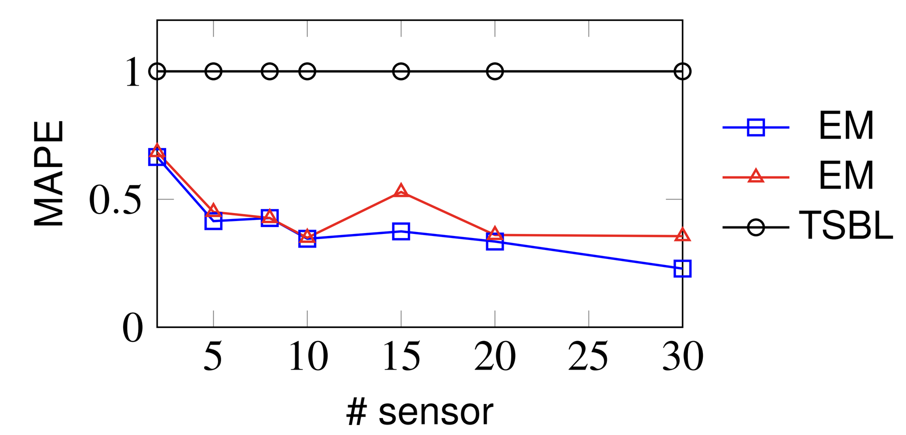


The generated simulation data

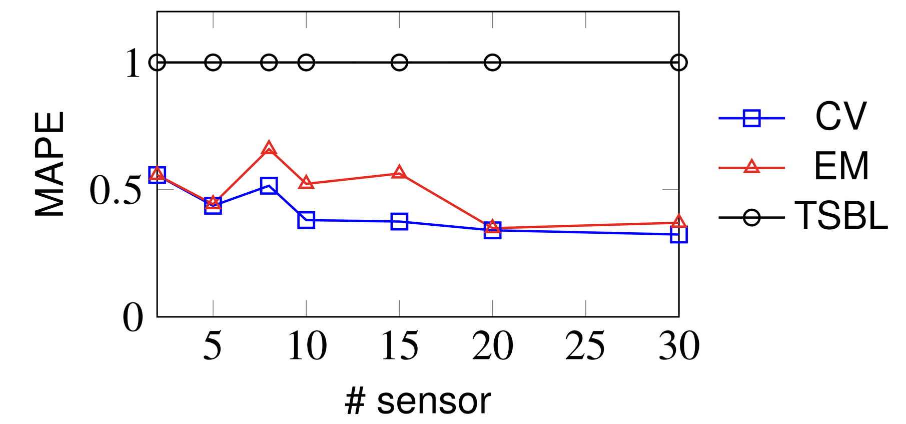


Accuracy

- Drift variance is set to 2.25; Benchmark: Hall.



- Drift variance is set to 2.78; Benchmark: Hall.



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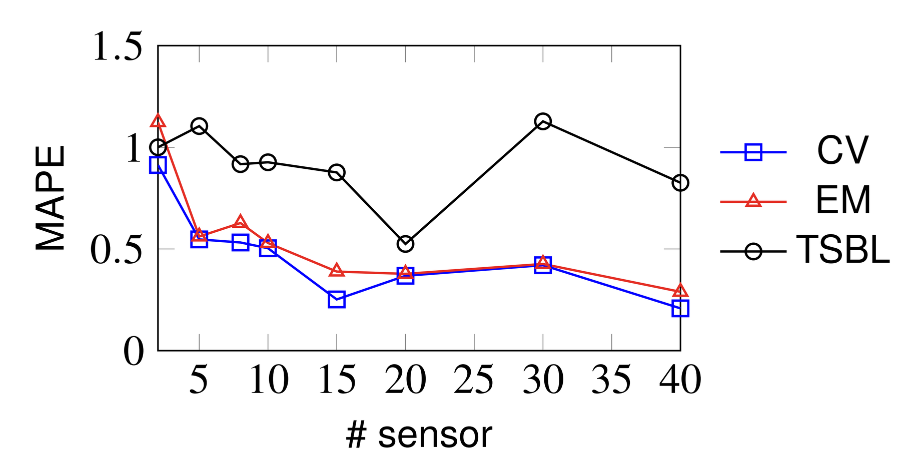

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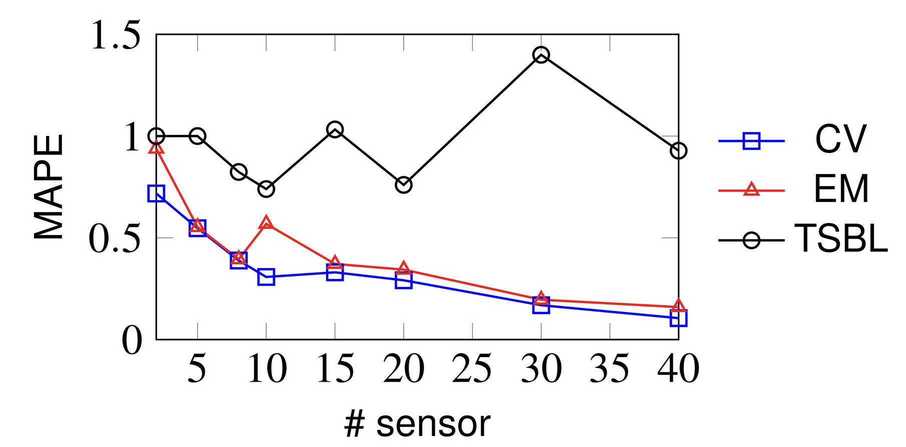
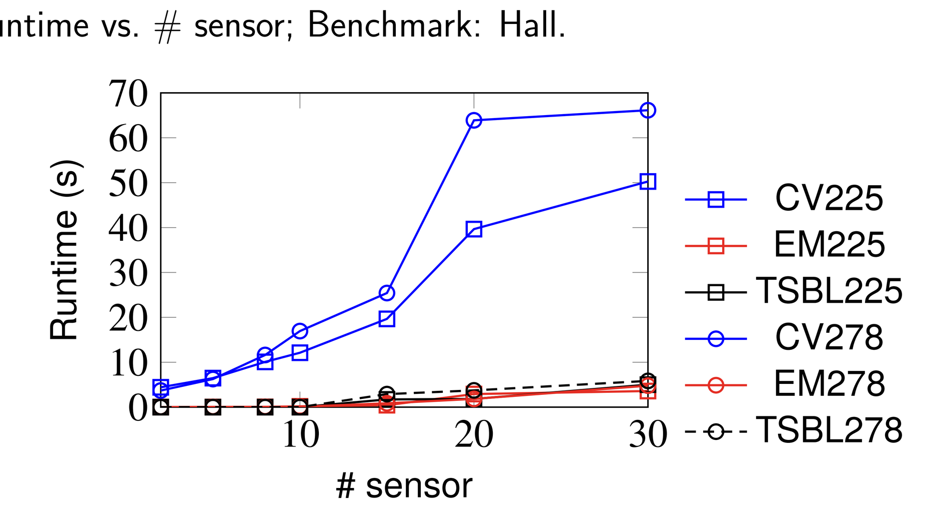
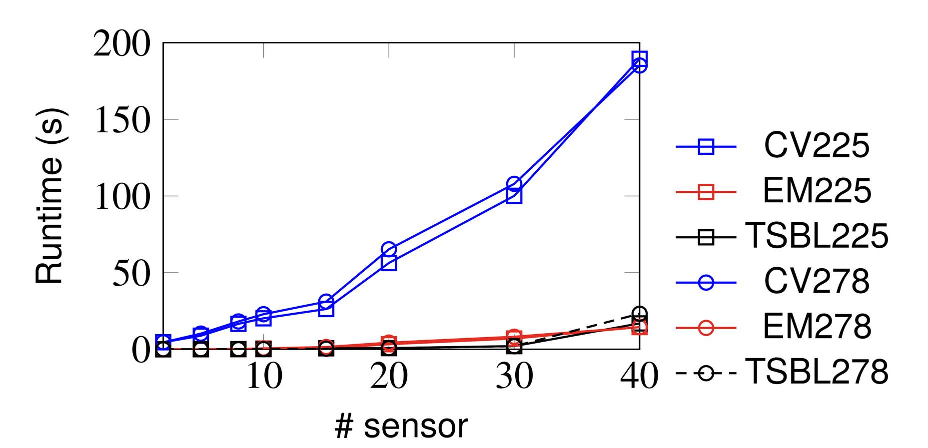


Runtime

- Runtime vs. # sensor; Benchmark: Hall.



- Runtime vs. # sensor; Benchmark: Secondary School.



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