CMSC5724: Quiz 2

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Problem 1 (50%). Let P be a set of 4 points: $A = (0, 2)$, $B = (2, 0)$, $C = (1, 0)$ and $D = (-2, 0)$ where B and C have label 1, while A and D have label -1. Run Margin Perceptron on P with $\gamma_{guess} = \frac{6}{\sqrt{13}}$. Recall that the algorithm maintains a vector $\boldsymbol{w}$ that describes a linear classifier. Show the value of $\boldsymbol{w}$ after every adjustment and the violation point used to do the adjustment.

Solution: At the beginning of Margin Perceptron, $\boldsymbol{w} = (0, 0)$.

Iteration 1. As $\boldsymbol{w} \cdot \boldsymbol{A} > 0$, we update $\boldsymbol{w}$ to $\boldsymbol{w} - \boldsymbol{A} = (0, 0) - (0, 2) = (0, -2)$.

Iteration 2. As $\boldsymbol{w} \cdot \boldsymbol{B} < 0$, we update $\boldsymbol{w}$ to $\boldsymbol{w} + \boldsymbol{B} = (0, -2) + (2, 0) = (2, -2)$.

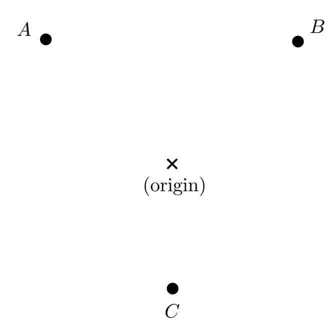
Iteration 3. As the distance between C and the line $\boldsymbol{w} \cdot \boldsymbol{x} = 0$ is $\frac{1}{\sqrt{2}} < \gamma_{guess}/2$, we update $\boldsymbol{w}$ to $\boldsymbol{w} + \boldsymbol{C} = (2, -2) + (1, 0) = (3, -2)$.

Iteration 4. No more violation. The final $\boldsymbol{w}$ is $(3, -2)$.

Problem 2 (50%). Define a *linear classifier* in 2D space as:

$$h(x, y) = \begin{cases} 1 & \text{if } ax + by \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where a and b are real-valued coefficients. Let $\mathcal{H}$ be the set of all linear classifiers. Prove: $\mathcal{H}$ cannot shatter the set of points A, B, and C shown in the figure below.



Solution. If $\mathcal{H}$ can shatter $\{A, B, C\}$, there is a classifier $h \in \mathcal{H}$ that assigns label 1 to all three points. Then, h must assign label 1 to every point inside the triangle ABC. Let Γ an infinitestimally small circle centered at the origin. All the points on Γ are assigned 1 by h. This is not possible because every linear classifier must assign -1 to half of Γ.