Exercises: Tangent and Gradient

Problem 1. Let $\boldsymbol{f}(t) = [3, 4] + t[1, 2]$. Give a tangent vector of the curve at the point corresponding to $\boldsymbol{f}(2)$.

Problem 2. Let $\boldsymbol{f}(t) = [\sin(t), \cos(t^3), 5t^2]$. Give a tangent vector of the curve at the point corresponding to $\boldsymbol{f}(2)$.

Problem 3. Give a tangent vector of point $(2, \sqrt{2})$ on the ellipse $x^2 + \frac{y^2}{2} = 5$.

Problem 4. Let $\boldsymbol{f}(t) = [t^2, -2t, -t^3]$. Give a tangent vector of the curve at point $(9, -6, -27)$.

Problem 5. Compute the following gradients:

- $\nabla f(3, 4)$ where $f(x, y) = (4x + 3)(2y - 1)$.
- $\nabla f(3, 4, 5)$ where $f(x, y, z) = 3x^2yz$.

Problem 6. Let $g(x, y) = (f(x, y))^c$. Prove that $\nabla g(x, y) = c(f(x, y))^{c-1} \nabla f(x, y)$.

Problem 7. Let $f(x, y, z) = 3x^2yz$. Let $\boldsymbol{u} = [1/3, 1/3, 1/3]$. Compute directional derivative of $f(x, y, z)$ in the direction of $\boldsymbol{u}$ at point $(5, 2, 3)$.

Problem 8. Let $f(x, y, z) = 3x^2yz$. Find the unit vector $\boldsymbol{u}$ that maximizes the directional derivative of $f(x, y, z)$ in the direction of $\boldsymbol{u}$ at point $(5, 2, 3)$.