

Exercises: Vector Derivative

Problem 1. Solve the following limits:

1. $\lim_{t \rightarrow 3} \mathbf{f}(t)$, where $\mathbf{f}(t) = [5t + 3, \frac{\sin(t-3)}{t-3}]$.
2. $\lim_{t \rightarrow 0} \mathbf{f}(t)$, where $\mathbf{f}(t) = [5t^2 + 3t, t^2, \frac{e^t - 1}{t}]$.
3. $\lim_{t \rightarrow 0} \mathbf{f}(t)$, where

$$\mathbf{f}(t) = \begin{cases} [5t^2 + 3t, t^2, \frac{e^t - 1}{t}] & \text{if } t \neq 0 \\ [10, 10, 10] & \text{otherwise} \end{cases}$$

Problem 2. Discuss the continuity of $\mathbf{f}(t)$ at $t = 0$.

1. $\mathbf{f}(t) = [5t^2 + 3t, t^2, \frac{e^t - 1}{t}]$.
2. $\mathbf{f}(t) = [5t^2 + 3t, t^2, \frac{e^t - 1}{t}]$ if $t \neq 0$; otherwise, $\mathbf{f}(t) = [10, 10, 10]$.
3. $\mathbf{f}(t) = [5t^2 + 3t, t^2, \frac{e^t - 1}{t}]$ if $t \neq 0$; otherwise, $\mathbf{f}(t) = [0, 0, 1]$.

Problem 4. Suppose that $\mathbf{f}(t) = [\sin(t), \cos(t^3), 5t^2]$. Answer the following questions:

1. Give the function $\mathbf{f}'(t)$.
2. Give the function $\mathbf{f}''(t)$ (which is the derivative of $\mathbf{f}'(t)$).
3. Give the function $\mathbf{f}'''(1)$ (where $\mathbf{f}'''(t)$ is the derivative of $\mathbf{f}''(t)$).

Problem 5. Suppose that $\mathbf{f}(t) = [t^2, \sin(t), 2t]$ and $\mathbf{g}(t) = 2t\mathbf{i} + \frac{1}{\sin(t)}\mathbf{j} + 3t^2\mathbf{k}$.

1. Give the function $h(t) = \mathbf{f}(t) \cdot \mathbf{g}(t)$.
2. Give the function $h'(t)$.
3. Give the function $\mathbf{f}'(t)$ and $\mathbf{g}'(t)$.
4. Verify that $h'(t) = \mathbf{f}'(t) \cdot \mathbf{g}(t) + \mathbf{g}'(t) \cdot \mathbf{f}(t)$.

Problem 6. Suppose that $\mathbf{f}(t) = [t, t^2, 1]$ and $\mathbf{g}(t) = [1, t, t^2]$.

1. Give the function $\mathbf{h}(t) = \mathbf{f}(t) \times \mathbf{g}(t)$.
2. Give the function $\mathbf{h}'(t)$.
3. Verify that $\mathbf{h}'(t) = \mathbf{f}'(t) \times \mathbf{g}(t) + \mathbf{f}(t) \times \mathbf{g}'(t)$.