

# CSCI3160 Longest Common Subsequence (LCS)

Dynamic Programming:  
Accelerate the evaluation of recursive func

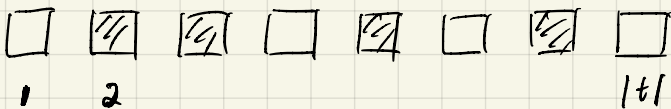
LCS  
Input:  $x$ : a string of length  $n$   
 $y$ : -----  $m$

Subsequence  
 $s$  is a subsequence of  $t$  if at least one of the following holds

- $s = t$  or
- we can convert  $t$  to  $s$  by deleting chars

$t = ABCDEF$

Subseqs of $t$	Nonexamples
$s = BDE$	CB
$s = B$	BDEG
$s = \emptyset$	



$opt(i, j) =$  LCS length of  $x[1:i]$  and  $y[1:j]$

$$opt(i, j) = \begin{cases} 0 \\ 1 + opt(i-1, j-1) \\ \max \{ \\ opt(i-1, j), \\ opt(i, j-1) \\ \} \end{cases}$$

if  $i=0$  or  $j=0$   
if  $i>0, j>0, x[i]=y[j]$   
if  $i>0, j>0, x[i] \neq y[j]$   
 $O(n^2)$

Thm: LCS thm

$z$  = an arbitrary LCS of  $x$  and  $y$   
 $k$  = the length of  $z$

① If  $x[n] = y[m] \Rightarrow$

- $z[k] = x[n] = y[m]$
- $z[1:k-1] =$  an LCS of  $x[1:n-1]$  and  $y[1:m-1]$

$x = BCBD \cancel{A} B$   
 $y = BD \cancel{C} A \cancel{B} A$

LCS of the trimmed  $x$  and  $y$  is  $BCAB$

② If  $x[n] \neq y[m] \Rightarrow$  at least one of the following correct:

- $z$  = an LCS of  $x[1:n-1]$  and  $y$
- $z$  = an LCS of  $x$  and  $y[1:m-1]$

$x = ABCBDAB$   
 $y = BDCAB \cancel{A}$   
 $z = BCAB$