

## CSCI3160: Special Exercise Set 6

Prepared by Yufei Tao

**Problem 1.** Define function  $f(x)$  — where  $x \geq 0$  is an integer — as follows:

- $f(0) = 0$
- $f(1) = 1$
- $f(x) = f(x - 1) + f(x - 2)$ .

Give an algorithm to calculate  $f(x)$  in  $O(n)$  time (you can assume that  $f(x)$  always fits in a word).

**Problem 2.** Let  $A$  be an array of  $n$  integers. Consider the following recursive function which is defined for any  $i, j$  satisfying  $1 \leq i \leq j \leq n$ :

$$f(i, j) = \begin{cases} 0 & \text{if } i = j \\ A[i] \cdot A[j] + \min_{k=i+1}^{j-1} f(i, k) & \text{if } i \neq j \end{cases}$$

Design an algorithm to calculate  $f(1, n)$  in  $O(n^3)$  time.

**Problem 3.** In Lecture Notes 7, we defined function  $f(i, j)$  based on strings  $x = \text{ABC}$  and  $y = \text{BDCA}$ . Calculate  $f(i, j)$  for all possible  $i$  and  $j$ .

**Problem 4.** In the rod-cutting problem, suppose that  $n = 5$  and the price array  $P$  is  $(2, 6, 7, 9, 10)$ . What is the maximum revenue achievable?

**Problem 5 (Textbook Problem 15.1-3).** Consider a modification of the rod-cutting problem in which, in addition to a price  $P[i]$  for each length  $i \in [1, n]$ , each cut incurs a fixed cost of  $c$ . The revenue associated with a solution is now the sum of the prices of the segments minus the total cost of making the cuts. Give a dynamic-programming algorithm to solve this modified problem in  $O(n^2)$  time.