CSCI3160: Quiz 2

Name: Student ID

Problem 1 (50%). Consider two strings $x = $ TCGTACAG and $y = $ CGCATGTA. For each $i \in [1, 8]$, define $x[1 : i]$ as the prefix of x with length i; similarly, for each $j \in [1, 8]$, define $y[1 : j]$ similarly with respect to y. For any $0 \leq i, j \leq 8$, define:

$$opt(i, j) = \begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0 \\ \text{the LCS length of } x[1 : i] \text{ and } y[1 : j] & \text{otherwise} \end{cases}$$

Fill in the table below the value of $opt(i, j)$ for all $0 \leq i, j \leq 8$.

Solution.

j

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	1	1	1	1
2	0	1	1	1	1	1	1	1	1
3	0	1	2	2	2	2	2	2	2
4	0	1	2	2	2	3	3	3	3
5	0	1	2	2	3	3	3	3	4
6	0	1	2	3	3	3	3	3	4
7	0	1	2	3	4	4	4	4	4
8	0	1	2	3	4	4	5	5	5

i (row label, left of table)

Problem 2 (50%). Let A be an array of n integers. Define:

$$cost(i, j) = \begin{cases} 1 & \text{if } i = j \\ \min_{k=i}^{j-1}(cost(i, k) + cost(k + 1, j) + A[i]A[k]A[j]) & \text{if } i < j \end{cases}$$

Describe an algorithm to compute $cost(1, n)$ in $O(n^3)$ time.

Solution. Compute $cost(i, j)$ — for all $1 \leq i \leq j \leq n$ — in $n - 1$ rounds as follows:

- Round 0: compute $cost(i, j)$ for all $1 \leq i \leq j \leq n$ satisfying $i = j$.

- Round 1: compute $cost(i, j)$ for all $1 \leq i \leq j \leq n$ satisfying $j - i = 1$.

- Round 2: compute $cost(i, j)$ for all $1 \leq i \leq j \leq n$ satisfying $j - i = 2$.

- ...

- Round $n - 1$: compute $cost(i, j)$ for all $1 \leq i \leq j \leq n$ satisfying $j - i = n - 1$.

This way, when we compute $cost(i, j)$, the values of $cost(i, k)$ and $cost(k + 1, j)$ are available for all $k \in [1, j - 1]$. We can therefore obtain $cost(i, j)$ in $O(n)$ time. The overall time complexity is $O(n^3)$ because there are $O(n^2)$ values to compute.