CSCI3160: Regular Exercise Set 9

Prepared by Yufei Tao

Problem 1*. Prove the correctness of Dijkstra's algorithm (when the edges have non-negative weights).

Problem 2. Consider again your proof for Problem 1. Point out the place that requires edge weights to be non-negative.

Problem 3* (SSSP in a DAG). Consider a simple acyclic directed graph $G = (V, E)$ where each edge $e \in E$ has an arbitrary weight $w(e)$ (which can be negative). Solve the SSSP problem on G in $O(|V| + |E|)$ time.

Problem 4. Let $G = (V, E)$ be a simple directed graph where each edge $e \in E$ carries a weight $w(e)$, which can be negative. It is guaranteed that G has no negative cycles. Prove: given any vertices $s, t \in V$, at least one shortest path from s to t is a simple path (i.e., no vertex appears twice on the path).

Problem 5**. Let $G = (V, E)$ be a simple directed graph where the weight of an edge (u, v) is $w(u, v)$. Prove: the following algorithm correctly decides whether G has a negative cycle.

algorithm negative-cycle-detection
1. pick an arbitrary vertex $s \in V$
2. set λ to the sum of all the positive edge weights in G
3. initialize $dist(s) = 0$ and $dist(v) = \lambda$ for every other vertex $v \in V$
4. **for** $i = 1$ **to** $|V| - 1$
5.   relax all the edges in E
6. **for** each edge $(u, v) \in E$
7.   **if** $dist(v) > dist(u) + w(u, v)$ **then**
8.   **return** "there is a negative cycle"
9. **return** "no negative cycles"