CSCI3160: Regular Exercise Set 3

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Problem 1. Let S be a set of n intervals $\{[s_i, f_i] \mid 1 \leq i \leq n\}$, satisfying $f_1 \leq f_2 \leq ... \leq f_n$. Denote by S' the set of intervals in S that are disjoint with $[s_1, f_1]$. Prove: if $T' \subseteq S'$ is an optimal solution to the activity selection problem on S', then $T' \cup \{[s_1, f_1]\}$ is an optimal solution to the activity selection problem on S.
(Note: This completes the induction step of the correctness proof discussed in the class.)

Problem 2. Describe how to implement the activity selection algorithm discussed in the lecture in $O(n \log n)$ time, where n is the number of input intervals.

Problem 3. Prof. Goofy proposes the following greedy algorithm to "solve" the activity selection problem. Let S be the input set of intervals. Initialize an empty T, and then repeat the following steps until S is empty:

- (Step 1) Add to T the interval $I = [s, f]$ in S that has the smallest s-value.
- (Step 2) Remove from S (i) the interval I, and (ii) all the intervals that overlap with I.

Finally, return T as the answer.

Prove: the above algorithm does not guarantee an optimal solution.

Problem 4.** Prof. Goofy is giving another try! This time he proposes a more sophisticated greedy algorithm. Again, let S be the input set of intervals. Initialize an empty T, and then repeat the following steps until S is empty:

- (Step 1) Add to T the interval $I \in S$ that overlaps with the *fewest* other intervals in S.
- (Step 2) Remove from S the interval I as well as all the intervals that overlap with I.

Finally, return T as the answer.

Prove: the above algorithm does not guarantee an optimal solution.