

CSCI3160: Regular Exercise Set 12

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Problem 1. Consider the set cover algorithm discussed in the lecture. Prove: it achieves an approximation ratio of $h = \max_{S \in \mathcal{S}} |S|$, where \mathcal{S} is the input family of sets.

Remark: This means if all the sets in \mathcal{S} have constant sizes, then the approximation ratio is $O(1)$.

Problem 2. Let \mathcal{C}^* be an optimal universe cover for the set cover problem. Consider running the set cover algorithm discussed in the lecture. In particular, consider the moment right before the algorithm is to choose the i -th set S_i , having chosen already S_1, S_2, \dots, S_{i-1} . Let z_{i-1} be the number of elements in the universe that have not been covered by $S_1 \cup S_2 \cup \dots \cup S_{i-1}$. Let $s = |\{S_1, S_2, \dots, S_{i-1}\} \cap \mathcal{C}^*|$, i.e., s of the $i-1$ sets chosen by the algorithm are from \mathcal{C}^* . Prove: S_i has benefit at least $z_{i-1}/(|\mathcal{C}^*| - s)$ (namely, the i -th set picked by the algorithm covers at least $z_{i-1}/(|\mathcal{C}^*| - s)$ new elements).

Problem 3. Let R be a set of n red points in 2D space, and B be a set of n black points in 2D space. Fix an integer $\epsilon > 0$. A subset $S \subseteq R$ is a B -guarding set if, for every black point $b \in B$, there is at least one point $r \in S$ with $\text{dist}(r, b) \leq \epsilon$, where $\text{dist}(r, b)$ is the Euclidean distance between r and b . Let OPT be the smallest size of all B -guarding sets. Design a $\text{poly}(n)$ -time (i.e., polynomial in n) algorithm that returns a B -guarding set with size $\text{OPT} \cdot O(\log n)$; if no B -guarding sets exist, your algorithm must correctly declare so.

Problem 4. Let S be a set of n axis-parallel rectangles in 2D space (i.e., each rectangle has the form $[x_1, x_2] \times [y_1, y_2]$; you can assume that the x_1, x_2, y_1, y_2 values of the n rectangles are all distinct). A set P of points is an S -pinning set if every rectangle of S covers at least one point in P . Let OPT be the smallest size of all S -pinning sets. Design a $\text{poly}(n)$ -time algorithm that returns an S -pinning set with size $\text{OPT} \cdot O(\log n)$.

Problem 5 (Conversion from Set Cover to Hitting Set). Suppose that we have an algorithm \mathcal{A} for the hitting set problem that achieves an approximation ratio ρ . Use \mathcal{A} to design a ρ -approximate algorithm for the set cover problem.