Exercises

Problem 1. Consider the Voronoi diagram of a set P of points in $\mathbb{R}^2$. Prove: if a Voronoi vertex is incident to 4 Voronoi edges, then P has 4 points lying on the same circle.

Problem 2. P is a set of points in $\mathbb{R}^2$. Prove: if we take a point p from P uniformly at random, the number of Voronoi neighbors of p is $O(1)$ in expectation.

Problem 3. Let P be a set of points in $\mathbb{R}^2$. Consider any point q in $\mathbb{R}^2$ (which may not be in P). Let p_1 be the nearest neighbor of q and p_2 be the second nearest neighbor (i.e., p_2 has the second smallest distance to q among all the points in P). Prove: p_2 must be a Voronoi neighbor of p_1.

(Hint: Argue there is a circle passing p_1, p_2 and containing no points of P in the interior.)

Problem 4. Prove the following for the triangulation of a point set P in $\mathbb{R}^2$:

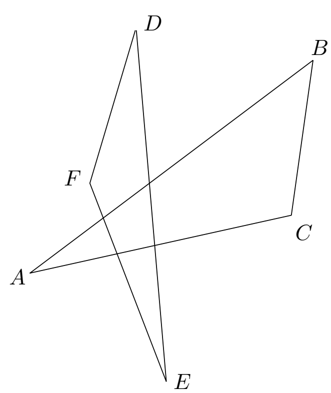
- Every bounded face of the triangulation is a triangle.

  (Hint: If not, you can always add a diagonal. Recall our argument on polygon triangulation.)

- Every triangulation of P contains $2n - 2 - k$ triangles where $n = |P|$ and k is the number of points on the convex hull boundary of P.

  (Hint: Induction.)

Problem 5. Let ABC and DEF be two triangles. No triangle contains any vertex of the other. We know that segment AB intersects with segment DE in the interior. Prove: a segment in $\{AC, BC\}$ must intersect a segment in $\{DF, EF\}$.



Remark: This completes our proof of the non-crossing lemma.