## DEPARTMENT OF PHYSICS, CHINESE UNIVERSITY OF HONG KONG
## PHYS3021 QUANTUM MECHANICS I

### SAMPLE QUESTIONS FOR WEEK 9 EXERCISE CLASSES (30 October - 3 November 2017)

**Read me:** TA will discuss the **SAMPLE QUESTIONS** (SQs) and answer your questions in exercise classes every week. The SQs are designed to review what you have learnt, tell a physics story, enrich our discussions, and help you work out the upcoming Problem Set. **Your time table should allow you to attend one exercise class session. The Exercise Classes are an integrated part of the course.** You are encouraged to work out (or think about) the SQs before attending exercise class and ask the TA questions. Over the semester, you are welcome to seek help from TAs/me.

**Mid-Term Examination Announcement**
**Date/Time/Venue: 4 November 2017 (Saturday), 10am - 12 noon, SC L1**
**Coverage:** *Everything from the beginning to the end of Chapter VII (inclusive of Ch. VII) on key features in 2D and 3D problems, including those discussed in class notes, lectures, sample questions, and problem sets.*

SQ22 - Harmonic Oscillator is important because it shows up near equilibrium separation
SQ23 - 3D Isotropic Harmonic Oscillator and Degeneracy

SQ22 *Harmonic Oscillator is important because it shows up near equilibrium separation*

We discussed in class notes that the parabolic potential energy function $U(x) \sim x^2$ characterizing a harmonic oscillator shows that in the vicinity of the minimum of any $U(x)$. Here is a famous example. Consider a diatomic molecule, i.e. a molecule formed by two atoms through a bond (spring). The **Morse potential** is known to be a good approximation to the intermolecular potential energy $U(l - l_0)$, where $l_0$ is the equilibrium separation, i.e. bond length, between the two atoms at which $U$ is a minimum. The Morse potential energy function has the form

$$U(l) = D_e \left(1 - e^{-\beta(l-l_0)}\right)^2 \tag{1}$$

where $D_e$ and $\beta$ are two parameters that fix the precise form of the $U$. They take up different values for different molecules.

(a) By considering the value of $U$ at $l = l_0$, at $l \gg l_0$ value, and at $l \to 0$, **sketch** $U(l)$. [TA: "Sketch" means a leisure figure (no need to be accurate in the numbers) that illustrates the key features]

(b) Physically, the parameter $D_e$ is the **dissociation energy** of the molecule measured from the minimum of $U(l)$. TA: **Explain** what this statement is about.

(c) In our discussion on harmonic oscillator, we set the minimum at $x = 0$. To do so for $U(l)$, we define $x = l - l_0$, i.e. look at the deviation from the equilibrium separation. The Morse potential becomes

$$U(x) = D_e \left(1 - e^{-\beta x}\right)^2 \tag{2}$$

TA: **Show that** $U(x)$ behaves like a harmonic oscillator near the equilibrium separation and **find an expression** for the spring constant $K$ of the bond. [Optional: Find the first anharmonic term in the potential energy function.]

SQ23 *3D Isotropic Harmonic Oscillator - Degeneracy*

Consider a 3D harmonic oscillator of the form

$$U(x, y, z) = \frac{1}{2} m\omega_0^2 \left(x^2 + y^2 + z^2\right). \tag{3}$$

Note that the same $\omega_0$ applies to the three directions. This is called an **isotropic** 3D oscillator. More generally, the angular frequencies in the three directions are different, thus **anisotropic** 3D oscillator.

Using the results $E_n = (n + \frac{1}{2})\hbar\omega_0$ and $\psi_n(x)$ for the energy eigenvalues and eigenfunctions of a 1D oscillator, **write down** the energy eigenfunctions and eigenvalues for a 3D isotropic harmonic oscillator.

**Order** the lowest eight allowed energies (different values) in increasing energy and **identify the degeneracy** for each value.

SQ00 *Answer students' questions, especially those arisen from revisions for the mid-term exam.*