DEPARTMENT OF PHYSICS, CHINESE UNIVERSITY OF HONG KONG
PHYS3021 QUANTUM MECHANICS I

SAMPLE QUESTIONS FOR WEEK 8 EXERCISE CLASSES (23-27 October 2017)

Read me: TA will discuss the **SAMPLE QUESTIONS** (SQs) and answer your questions in exercise classes every week. The SQs are designed to review what you have learnt, tell a physics story, enrich our discussions, and help you work out the upcoming Problem Set. **Your time table should allow you to attend one exercise class session. The Exercise Classes are an integrated part of the course.** You are encouraged to work out (or think about) the SQs before attending exercise class and ask the TA questions. Over the semester, you are welcome to seek help from TAs/me.

SQ19 - Numerical solving TISE is practical, effective, and necessary
SQ20 - 1D bound state wavefunctions of symmetric $U(x)$
SQ21 - A special/useful way to crank out a quantity (current density) for a wavefunction

SQ19 *Numerical solutions to TISE is a good way to explore quantum physics*

Background: We saw that even the one-dimensional finite square well problem **cannot be solved analytically**. We can at best write down a set of equations, but at the end we need to solve them numerically. And more..., there are many situations (potential energy functions $U(x)$) that the only equation we have in hand is the TISE plus the requirements that a quantum mechanical wavefunction must satisfy (well-behaved). Therefore, numerical solution to TISE becomes a practical and effective method in handling quantum mechanical problems.

TA: Look up on the web and **find a demo tool** that can illustrate the bound state energies and the number of bound states in a 1D finite square well as the width and/or the depth vary. If you find a place with a good library of physics related demos/programs, introduce it (give the link(s)) to students.

Use the tool to **explore how the energies** (y-axis on a plot) **vary with the well width** (x-axis) as the well width increases, for fixed well depth U_0. **Point out** the key features.

Optional: If the tool can handle ugly shaped $U(x)$ (not symmetrical, not piecewise continuous, etc.), **demonstrate** such a case.

SQ20 *1D bound state wavefunctions of symmetric $U(x)$*

Background: We saw the bound state solutions to three different 1D problems so far. They are the 1D infinite well, 1D finite square well, and 1D harmonic oscillator. For each case, we see that (i) the eigenvalues of **different eigenfunctions** are **different**, i.e. E_n are all different; and (ii) the symmetry of the bound state wavefunctions alternates, i.e., the ground state is symmetric/even (about the center of the potential energy function), the 1st excited state is anti-symmetric/odd, the 2nd excited state is symmetric/even, and so on. Property (i) is called the **non-degenerate** case in which there is **only one eigenfunction associated with each eigenvalue**. More explicitly, it means that we will **NOT** see cases where $\psi_n(x)$ (corresponding formally to E_n) and $\psi_m(x)$ (corresponding formally to E_m) with $E_n = E_m$. This could happen, especially in 2D and 3D problems.

Inspecting the $U(x)$ for the three cases, a common property is $U(x) = U(-x)$. That is to say, $U(x)$ **is symmetric** about the center of the function (which is taken to be $x = 0$). Here, TA will show that the even and odd properties of the energy eigenfunctions $\psi_n(x)$ follow from $U(x) = U(-x)$ **and** only one energy eigenfunction is associated with each eigenvalue.

(a) **Show** that if $\psi_n(x)$ is an energy eigenfunction associated with the eigenvalue E_n, the fact that $U(x) = U(-x)$ implies that $\psi_n(-x)$ is also an eigenfunction associated with the same eigenvalue E_n.

(b) Imposing the non-degenerate property, **show** that the energy eigenfunctions are either even or odd about $x = 0$.

SQ21 **A special/useful way to crank out a quantity for a given wavefunction**

Consider the following rather strange-looking manipulation that takes in a $\Psi(x,t)$ and gives the following expression as the output:

$$J(x,t) = \frac{q}{2m}\frac{\hbar}{i}\left[\left(\frac{d\Psi(x,t)}{dx}\right)^{*}\Psi(x,t) - \Psi^{*}(x,t)\left(\frac{d\Psi(x,t)}{dx}\right)\right] \tag{1}$$

At the moment, **don't worry** what this operator represents. [TA: need not explain what it means.]

Here, we illustrate the output of Eq. (1) by examples.

(a) Take any 1D infinite well energy eigenstate (any $\psi_n(x)$, doesn't matter which one) and evaluate Eq. (1).

(b) Take the 1D harmonic oscillator ground state (see Problem 2.5) and evaluate Eq. (1).

(c) Take any eigenfunction of the momentum operator $\sim e^{ikx}$, which is also the energy eigenfunction of free (**travelling**) electron and evaluate Eq. (1).

[Remarks: In part (c), if q is the magnitude e of electron charge and m is the electron mass, then the resulting expression gives how fast a charge moves. For the states in part (a) and part (c), they ain't travelling.]