DEPARTMENT OF PHYSICS, CHINESE UNIVERSITY OF HONG KONG
PHYS3021 QUANTUM MECHANICS I

SAMPLE QUESTIONS FOR WEEK 13 EXERCISE CLASSES (27 Nov - 1 Dec 2017)

Read me: TA will discuss the **SAMPLE QUESTIONS** (SQs) and answer your questions in exercise classes every week. The SQs are designed to review what you have learnt, tell a physics story, enrich our discussions, and help you work out the upcoming Problem Set. **Your time table should allow you to attend one exercise class session. The Exercise Classes are an integrated part of the course.** You are encouraged to work out (or think about) the SQs before attending exercise class and ask the TA questions. Over the semester, you are welcome to seek help from TAs/me.

SQ33 - Spin magnetic dipole moment and Stern-Gerlach experiment
SQ34 - More on spin angular momentum and measurement theory

SQ33 *The original Stern-Gerlach experiment*

We know that the spin angular momentum of an electron is characterized by the quantum number $s = 1/2$. The magnitude of the spin AM is always $\sqrt{3/4}\,\hbar$ and the z-component takes on either $+\hbar/2$ or $-\hbar/2$.

Just as a magnetic dipole moment $\vec{\mu}_L$ comes with the orbital AM $\vec{L}$, there is a **spin magnetic dipole moment** $\vec{\mu}_S$ associated with the **spin angular momentum** $\vec{S}$. The relation is

$$\vec{\mu}_S = -\frac{e}{m}\vec{S} \tag{1}$$

This is similar but NOT identical to the relation

$$\vec{\mu}_L = -\frac{e}{2m}\vec{L} \tag{2}$$

between $\vec{\mu}_L$ and $\vec{L}$. They are off by a factor of 2, the famous g-factor of 2 for spin that can be calculated to high accuracy by QED. It is, therefore, easier to take Eq. (1) as an experimental fact.

(a) With Eq. (1), **show that** the z-component of $\vec{\mu}_S$ can take on $\pm\mu_B$, where μ_B is the Bohr magneton. [Remark: Thus μ_B is a good number to remember that quantifies the typical strength of spin magnetic dipole moment.]

(b) The original Stern-Gerlach experiment used silver atoms. Although silver is more complicated that hydrogen, the key point is that the net angular momentum is a spin angular momentum with $s = 1/2$. In their experiment, each atom with a kinetic energy of about 3×10^{-20} J travelled a distance of 0.03 m through a non-uniform magnetic field of field gradient 2.3×10^3 T m^{-1}. **Estimate** the separation of the two beams 0.25 m beyond the magnet.

SQ34 *More on Spin angular momentum*

In class notes, we worked out the eigenvalues and eigenvectors of $\hat{S}_x$ and $\hat{S}_z$.

(a) **Find** the eigenvalues and normalized eigenvectors of $\hat{S}_y$.

(b) **Show** that the eigenvectors are orthogonal to each other.

(c) For a spin-half particle known to be in an eigenstate of $\hat{S}_z$, **find** $\langle \hat{S}_y \rangle$, $\langle \hat{S}_y^2 \rangle$, and hence (ΔS_y).

(d) For a beam of spins with the general state $(c, d)^T$, where T is the transpose so that it becomes a column vector (I don't know how to type the column) sending into Stern-Gerlach set up measuring S_y, i.e., SGY, **what can be said** about the measurement outcomes.

(e) If you take the exit beam corresponding to $+\hbar/2$ in the measurement in part (c) and send it into an apparatus SGZ, **what can be said** about the measurement outcomes.