PHYS3022 APPLIED QUANTUM MECHANICS

SAMPLE QUESTIONS FOR DISCUSSION IN WEEK 9 EXERCISE CLASSES (11 - 15 March 2019)

The Sample Questions are designed to serve several purposes. They either review what you have learnt in previous courses, supplement our discussions in lectures, or closed related to the questions in an upcoming Problem Set. Students should be able to do the homework problems independently after attending the exercise class. **You should attend one exercise class session.** You are encouraged to think about (or work out) the sample questions before attending exercise class and ask the TA questions.
Progress: In Week 8, we discussed the QM of stimulated absorption and emission, Einstein's 1917 A and B coefficients. Einstein's A-coefficient is related to the life time τ of an excited state. The line shape function due to natural broadening is a Lorentzian. The line width gives $1/\tau$ and thus the A coefficient from a spectral line. These provide the background of stimulated emission and life time, which are essential for understanding the principles of laser.

Mid-Term Examination – Please be reminded that the Mid-term Examination will be held on **16 March 2019 (Saturday) morning at 10am - 12noon in SC L1**. The coverage is everything from the beginning to the end of LMI-I Part 3 (inclusive) [covered in first 7 weeks], including what have been discussed in lectures, class notes, sample questions, and problem sets.

SQ20 - Quantum mechanics gives quantitatively the life time of hydrogen 2p state (see also SQ19)
SQ21 - Population inversion is an out of equilibrium situation

SQ20 *Life time of hydrogen 2p state - Quantum Mechanics is quantitative!*

In SQ19 (last week), TA calculated the vector $\overline{\mathbf{r}_{2p,1s}}$ for a hydrogen atom. This quantity gets into the transition rates and the A and B coefficient. The key point of this SQ is to illustrate that Quantum Mechanics allows us to **calculate** measurable quantities **quantitatively**. The example here is the life time of a hydrogen 2p state using the result in SQ19.

The flow of idea is as follows: QM gives the stimulated emission transition rate $\lambda_{2\to1}$. In 1917, Einstein introduced his B and A coefficients. The A-coefficient is related to spontaneous emission (difficult to handle within Schrödinger's QM) and the B-coefficient is related to stimulated processes. In QM, the formula of $\lambda_{2\to1}$ for stimulated processes gives a formula of the B-coefficient. Einstein gave a relation between the A-coefficient and the B-coefficient. Therefore, we can obtain a QM formula for the A-coefficient too. The life time of an excited state is how long on average it will last if it is "undisturbed". The life time τ of an excited state is related to spontaneous emission and the A-coefficient through $1/A$. In summary, $\lambda_{2\to1}$ (QM) $\to$ formula of B-coefficient $\to$ formula of A-coefficient via relation between A-coefficient and B-coefficient (Einstein) $\to$ lifetime $\tau = 1/A$.

Experimentally, the spectral line shape due to the finite life time of an excited time has a line width related to $1/\tau$ and thus τ can be obtained by spectroscopy.

Here, we consider the life time of a 2p state of hydrogen atom. We will make use of the result in SQ19 to get at the life time of the 2p state of $(2, 1, +1)$. For spontaneous emission, it can make a transition to the ground state (final state) of $(1, 0, 0)$. In this case, the matrix element involved is $\overline{\mathbf{r}_{1s,2p}}$ (from 2p to 1s) rather than $\overline{\mathbf{r}_{2p,1s}}$ evaluated in SQ19.

(a) Actually we did the matrix element in SQ19. By referring to SQ19, **give $\overline{\mathbf{r}_{1s,2p}}$** without doing any calculation. Note that it is a vector.

(b) What we need is $|\overline{\mathbf{r}_{1s,2p}}|^2$, which is a scalar. **Evaluate it** and give the answer in Bohr radius squared.

(c) To get at the Planck's formula, Einstein found a formula that relates the A and B coefficients (see class note). Using QM for the B-coefficient, a formula for A-coefficient is obtained. It consists of three factors: a bunch of constants, ω^3 dependence, and $|\overline{\mathbf{r}_{1s,2p}}|^2$ dependence. So, (i) **evaluate** the quantity $|\overline{\mu_{1s,2p}}|^2 = e^2|\overline{\mathbf{r}_{1s,2p}}|^2$ and give the result **in SI units**, i.e., in $\mathrm{C}^2\mathrm{m}^2$ where C is Coulomb and m is meter. This is related to the electric dipole moment squared. (ii) **Evaluate** ω_{21} (or simply call it ω) from the energy differences of the 2p and 1s states.

(d) For the 2p $(2, 1, +1)$ state, $(1, 0, 0)$ is the *only* allowed transition. This makes the calculation easier, as we don't need to consider several possible final state. The life time is given by $\tau = 1/A$. Thus we need to calculate A. The formula was given in class notes. There are some constants involving $\hbar$, c, and ϵ_0 in A. Plug in all the numbers to **find** A (in SI units) and the **lifetime** due to the electric dipole mechanism. The answer is a number in **seconds**. The result is worthy of remembering as it is typical of a state that can make a transition downward via electric dipole radiation. **You should appreciate that quantum mechanics works to give a precise number for a property of a quantum state.**

[Remarks: You just saw that typical life time is $\sim 10^{-9}$ s for states with allowed electric dipole transitions downward. If such a transition is forbidden (meaning "electric-dipole forbidden"), the life time becomes much longer. Physicists have manipulated atoms and measured some exceptionally long life time. For example, a metastable state in Mg was found to have a life time of 2050 seconds (see Jensen *et al.* Phys. Rev. Lett. **107**, 1130 (2011))! There are more extreme cases. When electric dipole transition is forbidden, then comes magnetic dipole, electric quadrupole, magnetic quadrupole, electric octupole processes, etc. An excited state in $^{172}\mathrm{Yb}^+$ ion was found to have a life time of 10 years via the electric octupole transition. See Roberts *et al.* in Phys. Rev. Lett. **78**, 1876 (1997). Of course, one needs to find a way to excite the atom to such a state before one can study it.]

SQ21 *Population inversion is an out of equilibrium situation*

We emphasized that one cannot populate a higher energy state 2 with more atoms than a lower state 1 by controlling the temperature with the system at thermal equilibrium (e.g. with a heat bath). It is always $N_2 > N_1$ for equilibrium cases. In fact, the ratio is given by

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/kT} = e^{-\hbar\omega/kT} \tag{1}$$

Let's consider two states that emits a line of wavelength of 488.0 nm (nanometer) and that a population inversion such that $N_2 = 2N_1$ is achieved. Somehow, some people still insist on using Eq. (1), which is equilibrium physics, for this out of equilibrium situation. TA: **Evaluate** the corresponding "temperature". The answer comes out to be something very strange.