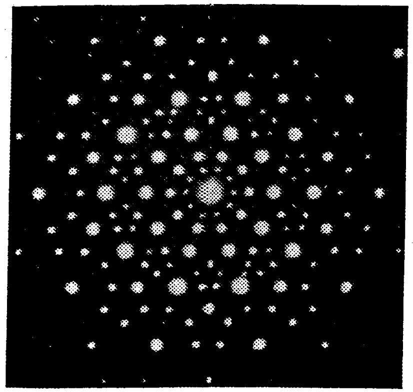
Appendix B: Quasi-Crystals

- As discussed in Appendix A, we don't expect to see signals corresponding to 5-fold symmetry in crystals.

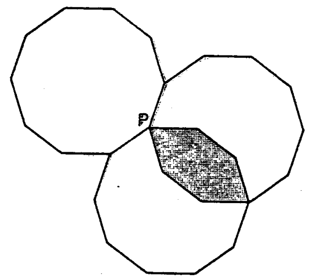
  [Recall: This stems from the restricted types of lattices, based on the requirement of discrete translational symmetry.]

- But solid state physics is full of surprise!
- In 1984, X-ray diffraction pattern with 10-fold symmetry was observed in a rapidly cooled Al (86%)-Mn(14%) alloy

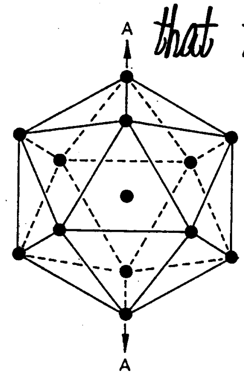


D. Schechtman et. al., Phys. Rev. Lett. 53, 1951 (1984)

Just as pentagons cannot fill a 2D space without leaving holes or having overlapping pentagons, 2D units of 10-fold symmetry (decagons) cannot tile a 2D area.

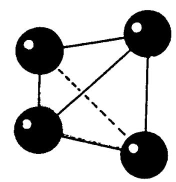


- Futher investigations indicated that atoms tend to form (slightly distorted) tetrahedra and these tetrahedra form the buiding blocks of the solid that take on the form of an icosahedron.



The icosahedron possesses 5-fold axes.

(*a*) The icosahedron: the line AA is one of six five fold symmetry axes that pass through the 12 vertices

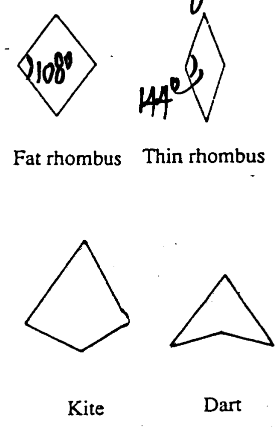


(*b*) Tetrahedral arrangement of atoms; if 20 slightly distorted tetrahedra share a common vertex the icosahedral arrangement of atoms of (*a*) is obtained

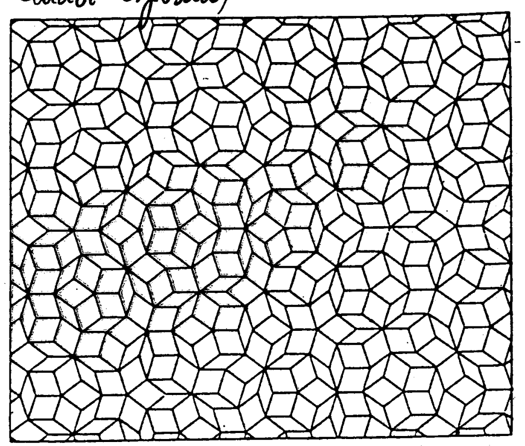
- The question of how to describe the structure of quasi-crystals has led to many fascinating ideas.

  It is believed that they are generalizations of "Penrose tiling" in 2D.

Penrose Tiling (2D Quasi-crystals)

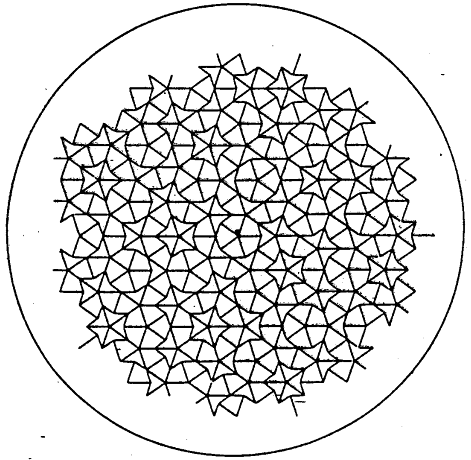


(a)



(b)

Two examples of Penrose tiling. The patterns are quasi-periodic, and built up using two tile types as in (a). Pattern (b) is made up of fat and thin rhombuses, while pattern (c) is made using the kite and the dart.



(c)

- They have 5-fold rotational symmetry.
- But they are NOT periodic.

- These systems do NOT have discrete translational symmetry, but the systems contain regular decagons all with the same orientation – quasi-crystals.

Quasi-periodic 1D lattice:

- Constructed with definite rules
- but need more than one basic unit

e.g. Fibinnaci lattice

L L L L ...

S L S L S L S L ...

L S L L S L L S L L S L ...

S L L S L S L S S L S L L S L S L L S L ...

⋮

∞ times

- There are many other fascinating ideas.

  e.g. Is it possible to generate the Fibinnaci quasi-periodic lattice by projections from a periodic 2D square lattice? How?

  Is it possible to generate the 2D Penrose tilings by projections from a periodic lattice in high dimensions?

  How electronic states are modified in QCs.

  ⋮

Refs.

M. Senechal, "Quasicrystals and Geometry" (1995)

D. R. Nelson, Science 229, 233 (1985) [an early review article]

D. R. Nelson, "Defects and Geometry in Condensed Matter Physics" (2002)

刘有延、傅秀军：「准晶体」(非线性科学丛书 1999)

References:

- Kittel: Chapter 1
- Christman: Chapters 1, 2
- Hook and Hall: Sec. 1.1 - 1.3
- 黄昆: Sec. 1.1-1.3, 1.5, 1.7