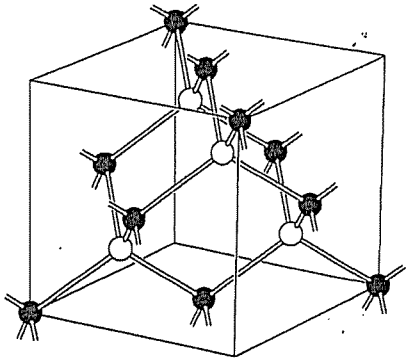
XIII. Topics in Semiconductor Physics

A. Materials

		E_g (room temp.)
Group IV	Si	1.1eV
	Ge	0.7eV
	α Sn	0.08eV
III-V	GaAs	1.4eV
	GaP	2.3eV
	GaSb	0.7eV
	InAs	0.4eV
	InP	1.3eV
II-VI	CdS	2.6eV
	CdTe	1.5eV
	ZnS	3.6eV
	ZnTe	2.3eV
IV-VI	PbS	0.4eV
	PbTe	0.3eV

B. Crystal Structure

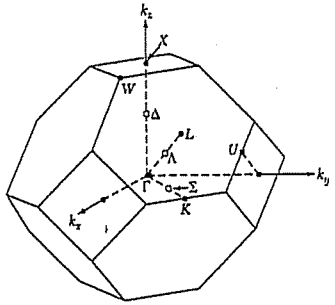


- fcc
- Each lattice point is decorated by two atoms
- Si, Ge: two identical atoms (diamond structure)
- GaAs, etc.: 1 Ga, 1As for each lattice point (zinc-blende structure)

Each atom has 4 nearest neighbors forming a tetrahedral (sp^3 hybridization), covalent bonds

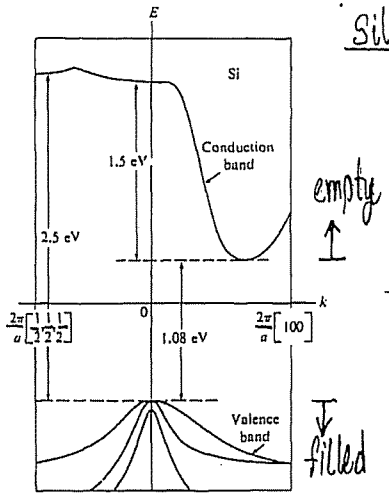
- reciprocal Lattice is a bcc

1st B.Z.



C. Some Band structure

Silicon

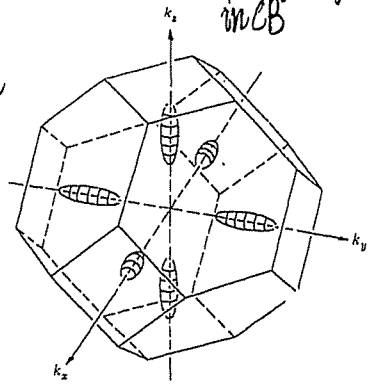


indirect gap

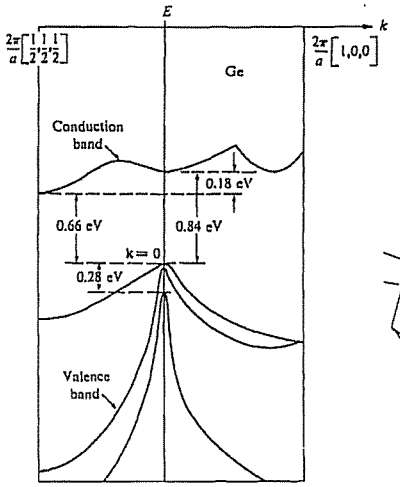
empty ↑

↓ filled

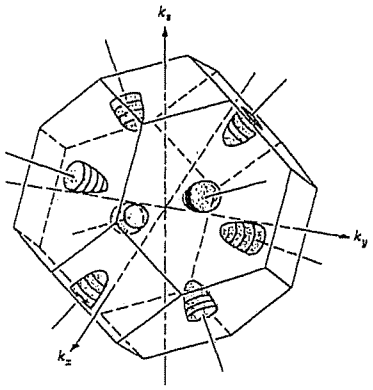
1st B.Z. showing constant energy surface in CB



Germanium

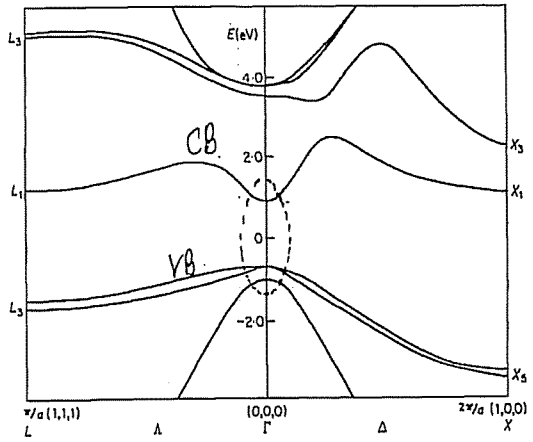


indirect ↑ gap ↓



GaAs

direct ↑ gap ↓

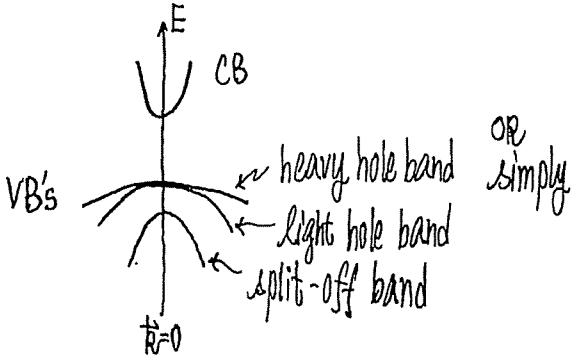


↑ empty

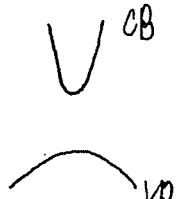
↓ filled

Physics is governed by the band structure around $\vec{k}=0$ (Γ).

In studying semiconductor physics, the <u>model band structure</u> is: (near $\vec{k}=0$)



OR simply



Key physics has to do with <u>some electrons near bottom of CB</u> and <u>some missing electrons near top of VB.</u>

Tight-binding viewpoint:

Each atom : one s-orbital + three p-orbitals

4 atomic orbitals/atom

AND 4 electrons (e.g. $3s^2 3p^2$ (Si))

Two atoms per primitive cell

8 atomic orbitals per primitive cell

tight-bind to form 8 bands

⇓

2N states/band × 8 bands

= 16 N states

Total # electrons = $4 \cdot 2 \cdot N = 8N$

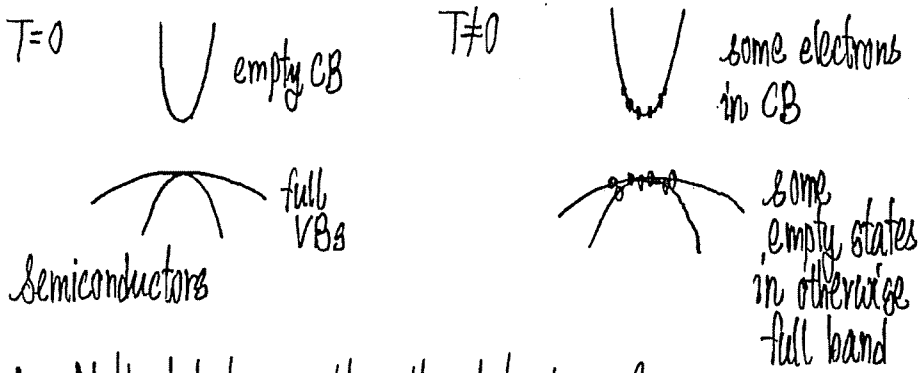
(4: per atom; 2: 2 atoms/cell; N: # cells)

8N electrons completely fill 4 bands

8 bands → 4 bands empty

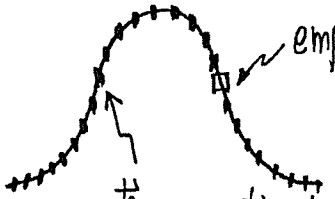
→ 4 bands filled ↕ Gap

D. Concept of Holes[+] (useful in semiconductors)



∴ Motivated to consider the behavior of a nearly filled band with a few empty states

We are interested in filled band with one empty state.



$\vec{k}_{empty}$ = $\vec{k}$-value of the empty state

$\vec{k}_{lonely}$ = $\vec{k}$-value of an occupied state without a cancelling partner

- System : Many electrons
- Behavior of "hole" : results of a large number of electrons!

"Hole" : { NOT simply the empty state
NOT simply the uncancelled state

[+] Kittel: Ch.8

(i) Consider the total crystal momentum of the electrons

$$\sum_{\text{occupied states } \vec{k}'} \hbar\vec{k}' = \sum_{\substack{\text{occupied states } \vec{k}' \\ \text{except } \vec{k}_{\text{lonely}}}} \hbar\vec{k}' + \hbar\vec{k}_{\text{lonely}} = \hbar\vec{k}_{\text{lonely}}$$

$$= \sum_{\text{occupied states } \vec{k}'} \hbar\vec{k}' + \hbar\vec{k}_{\text{empty}} - \hbar\vec{k}_{\text{empty}}$$

$$= \sum_{\text{all } \vec{k} \in BZ} \hbar\vec{k} - \hbar\vec{k}_{\text{empty}} = -\hbar\vec{k}_{\text{empty}}$$

($\sum_{\text{all } \vec{k} \in BZ} \hbar\vec{k} = 0$)

∴ Crystal momentum of a filled band with one empty state at $\vec{k}_{\text{empty}}$ = $\hbar\vec{k}_{\text{lonely}}$ (looking at the electrons)

= $-\hbar\vec{k}_{\text{empty}}$ (as if due to the empty state)

Total Crystal momentum in band = Due to many ($\sim 10^{22}-1$) electrons (electron point of view)

But the result looks simple!

It looks as if the contribution comes from one new particle, that we call a hole, with crystal momentum $\hbar\vec{k}_h$.

$$\boxed{\hbar\vec{k}_h = -\hbar\vec{k}_{\text{empty}}} \quad (1)$$

($\hbar\vec{k}_h$: crystal momentum of hole; $-\hbar\vec{k}_{\text{empty}}$: negative of crystal momentum of empty state)

(ii) What is the semiclassical equation of motion $\frac{d}{dt}(\hbar\vec{k}_h) = ?$ under the influence of a uniform electric field $\vec{\mathcal{E}}$?

Physically, the force $-e\vec{\mathcal{E}}$ acts on the electrons (occupied state)

⇒ shift in $\vec{k}$-values of the occupied states in $-\hat{x}$-direction (left)

⇒ $\vec{k}_{\text{empty}}$ also shifts in $-\hat{x}$ direction

⇒ $\vec{k}_{\text{lonely}} = \vec{k}_h$ shifts in $+\hat{x}$ direction (see figure on next page)

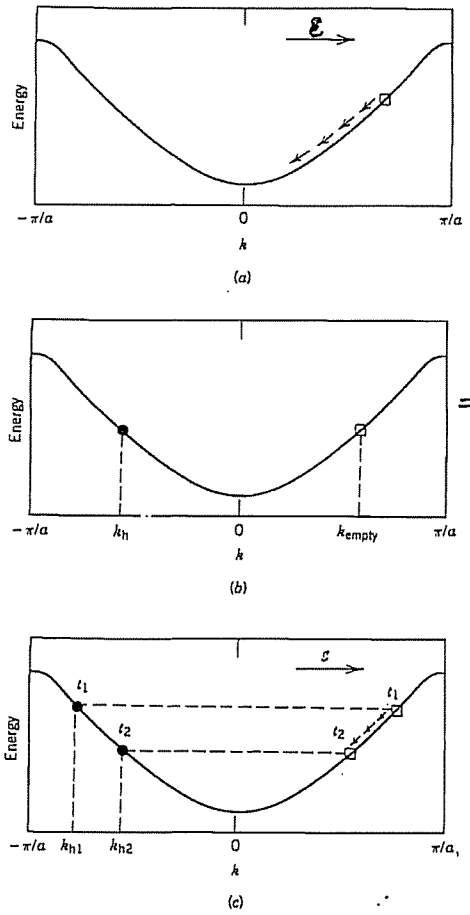
∴ the new particle (the hole) satisfies

$$\boxed{\frac{d}{dt}(\hbar\vec{k}_h) = +e\vec{\mathcal{E}}} \quad (2)$$

Regarding $e\vec{\mathcal{E}}$ as the electric force on a hole, the hole behaves as if it is a particle with charge $+e$

∴ $\boxed{e_h = +e}$ (3)

Nearly Full band with one empty state



An electron energy band for a one-dimensional crystal. The square marks a state that is initially empty in an otherwise filled band. (a) When an electric field is turned on in the direction shown, the states indicated by arrows successively become empty as electrons make transitions to the left. (b) The band is completely filled except for the state marked by the square. Except for the electron in the state marked by a circle, each electron can be paired with another so the sum of their crystal momenta vanishes. The total crystal momentum for the band and the crystal momentum of the hole are both $\hbar k_h$. (c) The empty state and unpaired electron for two times $t_2 > t_1$ when an electric field is in the direction shown. The change in k_h is in the direction of the field.

- Electron dynamics: it is the electrons that shift,
- effectively, the empty state moves in the direction of the arrows

Total crystal momentum $= \hbar \vec{k}_h = -\hbar \vec{k}_{empty}$

Two different time $t_2 > t_1$

$\frac{d}{dt}(\hbar \vec{k}_h) = +e\vec{\mathcal{E}}$

(iii) Consider total electric current $\vec{J}_e$ for a band with one empty state

$$\vec{J}_e = \sum_{\text{occupied states } \vec{k}'} (-e)\,\vec{v}(\vec{k}')$$

contributions due to occupied states (electrons)

$\vec{v}(\vec{k}')$: velocity of an electron in Bloch state $\vec{k}'$

general expression of $\vec{J}_e$ due to electrons

$$\vec{J}_e = -e\vec{v}(\vec{k}_{lonely})$$ electron viewpoint

$$= -e\vec{v}(-\vec{k}_{empty})$$ using $\vec{k}_{lonely} = -\vec{k}_{empty}$

$$= (-e)(-\vec{v}(\vec{k}_{empty}))$$ using property of band structure $\mathcal{E}(\vec{k})$

$$= +e\vec{v}(\vec{k}_{empty})$$ ← still electron viewpoint

$\vec{v}(\vec{k}_{empty})$ is velocity of Bloch state with $\vec{k} = \vec{k}_{empty}$

OR

$$\vec{J}_e = \sum_{\text{full band } (\text{all } \vec{k} \in BZ)} (-e)\vec{v}(\vec{k}) - (-e)\vec{v}(\vec{k}_{empty})$$

$= 0$

$$= +e\vec{v}(\vec{k}_{empty})$$

Switch to hole picture: Write $\vec{J}_h = \vec{J}_e = +e\vec{v}(\vec{k}_{empty})$ due to hole

$$\equiv e_h \vec{v}_h(\vec{k}_h)$$

$$\therefore \boxed{\vec{v}_h(\vec{k}_h) = \vec{v}(\vec{k}_{empty}) = \frac{1}{\hbar}\vec{\nabla}_{\vec{k}}\mathcal{E}(\vec{k})} \quad (4) \quad (\vec{k} = \vec{k}_{empty})$$

$$\vec{v}_h(\vec{k}_h) = \vec{v}(\vec{k}_{empty}) = \frac{1}{\hbar}\vec{\nabla}_{\vec{k}}\mathcal{E}(\vec{k})$$

(band of electronic states)

hole picture ↑ ; ↑ electron picture

One can also write $\vec{v}_h(\vec{k}_h)$ entirely in the hole picture

Recall: $\vec{k}_h = -\vec{k}_{empty}$

$$\vec{v}_h(\vec{k}_h) = \frac{1}{\hbar}\vec{\nabla}_{-\vec{k}_h}\mathcal{E}(-\vec{k}_h)$$

$$= \frac{1}{\hbar}\vec{\nabla}_{-\vec{k}_h}\mathcal{E}(\vec{k}_h)$$ ↓ symmetry of energy bands (usually OK)

$$= -\frac{1}{\hbar}\vec{\nabla}_{\vec{k}_h}\mathcal{E}(\vec{k}_h)$$ ↓ Calculus

$$= \frac{1}{\hbar}\vec{\nabla}_{\vec{k}_h}\left(-\mathcal{E}(\vec{k}_h)\right)$$ ↓ rewrite

← RHS still involves electron picture since $\mathcal{E}(\vec{k}_h)$ is the electronic eigenvalue of a Bloch state with $\vec{k} = \vec{k}_h$

Switch to hole picture

Write as:

$$\vec{v}_h(\vec{k}_h) = \frac{1}{\hbar}\vec{\nabla}_{\vec{k}_h}\mathcal{E}_h(\vec{k}_h)$$

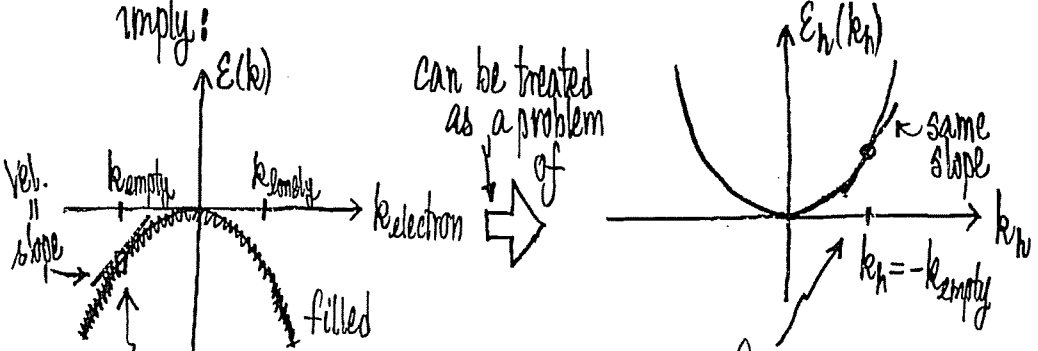
($\mathcal{E}_h(\vec{k}_h)$: hole dispersion relation)

in analogy to $\vec{v}(\vec{k}) = \frac{1}{\hbar}\vec{\nabla}_{\vec{k}}\mathcal{E}(\vec{k})$ for electrons

∴ The hole behaves as if it has an energy dispersion relation (or hole energy band)

$$\boxed{\mathcal{E}_h(\vec{k}_h) = -\mathcal{E}(\vec{k}_h) = -\mathcal{E}(\vec{k}_{empty})} \quad (5)$$

Sounds complicated! In pictorial form, properties (1), (4), (5) imply:



One empty state in otherwise full band

treat many electrons → give same physical results! ← treat one hole

(-e charge) (+e charge)

(iv) It follows that (from figures above) the reciprocal effective mass tensor of a hole is the negative of the element in the reciprocal effective mass tensor of an electron corresponding to the $\vec{k}_{empty}$ state.

The reciprocal effective mass tensor of a hole is:

$$\left(\frac{1}{m_h^*}\right)_{ij} = \frac{-1}{\hbar^2}\frac{\partial^2 \mathcal{E}(\vec{k})}{\partial k_i \partial k_j} \qquad i,j = x,y,z \qquad (6)$$

i.e., negative of the element of the reciprocal effective mass tensor for the empty state.

Summary

A missing electron in a state $\vec{k}$ behaves as:

- charge $+e$, crystal momentum $\hbar\vec{k}_h = -\hbar\vec{k}$
- $\vec{v}_h(\vec{k}_h) = \frac{1}{\hbar}\vec{\nabla}_{\vec{k}}\mathcal{E}(\vec{k})$, $\left(\frac{1}{m_h^*}\right)_{ij} = -\frac{1}{\hbar^2}\frac{\partial^2 \mathcal{E}(\vec{k})}{\partial k_i \partial k_j}$

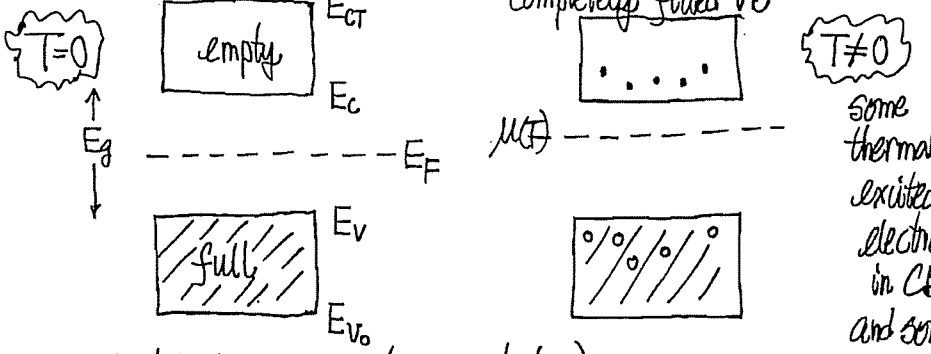
Notes[†] Refs: Kittel Ch.8, Christman Ch.9

- charge enters when one needs to write down $\vec{F}_{\text{ext on hole}}$
- $\hbar\vec{k}_h$ enters into the semi-classical equation of motion
- Eq. of motion and $\vec{v}_h(\vec{k}_h)$ give: how $\vec{k}_h$ changes due to $\vec{F}_{\text{ext on hole}}$ and then how $\vec{v}_h$ changes (i.e. acceleration)
- Alternatively, one can invoke the effective mass $\frac{1}{m_h^*}$ to relate acceleration and $\vec{F}_{\text{ext on hole}}$.

[†] These points are also valid in considering electron dynamics.

E. The Chemical Potential μ of a Semiconductor

Semiconductor: $T=0$ ↗ empty CB
↳ completely filled VB



(Intrinsic semiconductor, no dopings)

Result:

$$\mu(T) = \frac{E_c + E_v}{2} + \frac{3}{4}kT\ln\left(\frac{m_h^*}{m_e^*}\right) \qquad \text{(key result)}$$

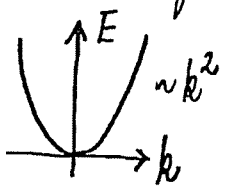
- At $T=0$,

$$\mu(0) = E_F = \frac{E_c+E_v}{2} = \frac{E_g+E_v+E_v}{2} = E_v + \frac{1}{2}E_g$$

$\Rightarrow$ E_F is in middle of the gap.

E_{CT} = energy of top of CB
E_c = bottom of CB
E_g = gap (see figure)
E_v = top of VB
E_{v_0} = bottom of VB

• Recall: Background knowledge



3D parabolic band

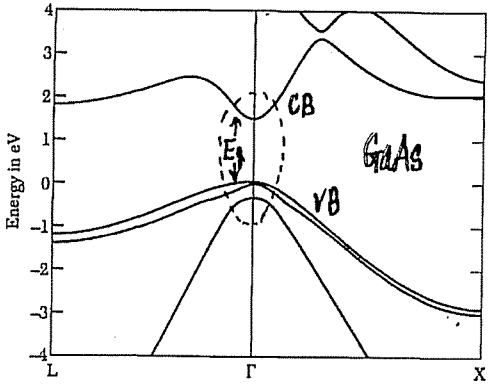
DOS is

$$g(E) = \frac{V}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2}\sqrt{E}$$

$$= \frac{4\pi V(2m^*)^{3/2}}{h}\sqrt{E}$$

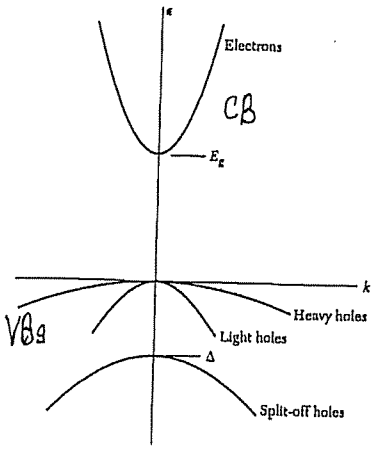
with m^* describing the curvature.

Band Structure of semiconductors could be complicated!



Direct Gap, E_g = Band Gap

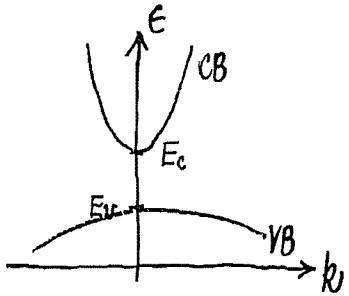
Model semiconductor bands



Near band gap

For our purpose:

2-band model (parabolic)



CB: $\epsilon_{CB}(k) = E_c + \frac{\hbar^2k^2}{2m_e^*}$ ← bottom of CB

m_e^*: electron effective mass near bottom of CB ($m_e^* > 0$)

VB: $\epsilon_{VB}(k) = E_v - \frac{\hbar^2k^2}{2m_h^*}$ ← top of VB

m_h^*: hole effective mass near top of VB ($m_h^* = -m^* > 0$)

$E_g = E_c - E_v$

↑ energy gap or band gap

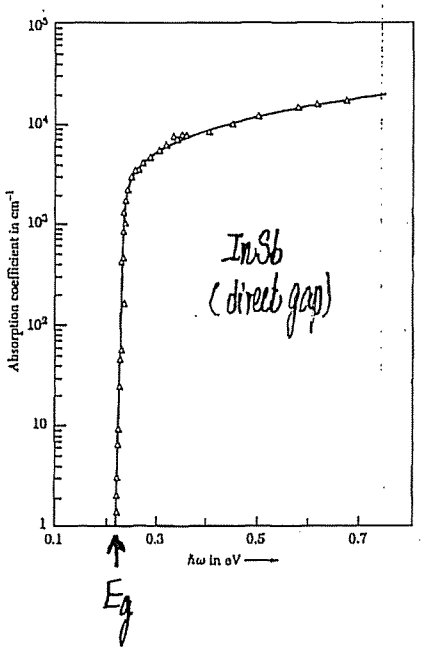
Crystal	Gap	E_g, eV 0 K	E_g, eV 300 K	Crystal	Gap	E_g, eV 0 K	E_g, eV 300 K
Diamond	*i*	5.4		SiC(hex)	*i*	3.0	—
Si	*i*	1.17	1.11	Te	*d*	0.33	—
Ge	*i*	0.744	0.66	HgTe[a]	*d*	−0.30	
αSn	*d*	0.00	0.00	PbS	*d*	0.286	0.34–0.37
InSb	*d*	0.23	0.17	PbSe	*i*	0.165	0.27
InAs	*d*	0.43	0.36	PbTe	*i*	0.190	0.29
InP	*d*	1.42	1.27	CdS	*d*	2.582	2.42
GaP	*i*	2.32	2.25	CdSe	*d*	1.840	1.74
GaAs	*d*	1.52	1.43	CdTe	*d*	1.607	1.44
GaSb	*d*	0.81	0.68	SnTe	*d*	0.3	0.18
AlSb	*i*	1.65	1.6	Cu_2O	*d*	2.172	—

i = indirect gap

d = direct gap

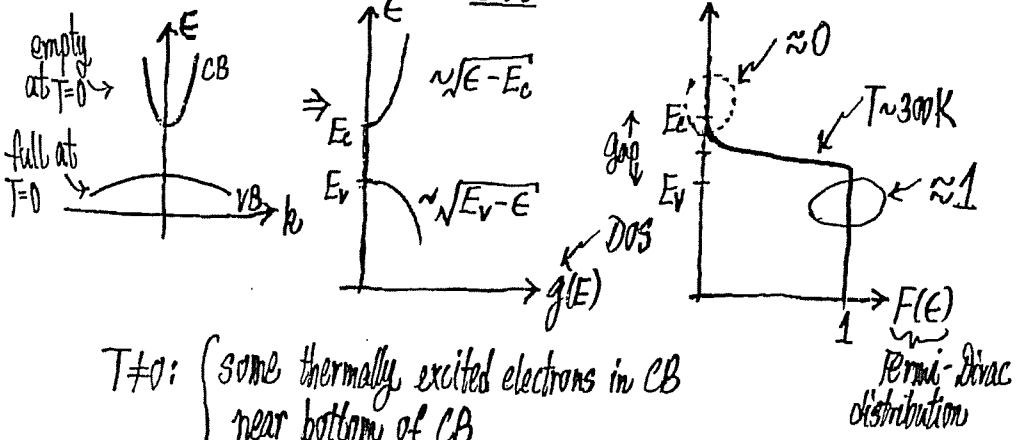
Crystal	Electron m_e^*/m	Heavy hole m_{hh}^*/m	Light hole m_{lh}^*/m	Split-off hole m_{so}^*/m	Spin-orbit Δ, eV
InSb	0.015	0.39	0.021	(0.11)	0.82
InAs	0.026	0.41	0.025	0.08	0.43
InP	0.073	0.4	(0.078)	(0.15)	0.11
GaSb	0.047	0.3	0.06	(0.14)	0.80
GaAs	0.066	0.5	0.082	0.17	0.34
Cu_2O	0.99	—	0.58	0.69	0.13

m_e^* and m_h^* can be measured by cyclotron resonance measurements



E_g can be measured by absorption experiments (room temperature)

Thermal excitation:



DOS

$T \neq 0$: { some thermally excited electrons in CB near bottom of CB
{ some missing electrons (holes) in VB near top of VB

Q: Given temperature T, can we calculate the number of electrons in CB (n_e) and the number of holes in VB (p_v) per unit volume?

Key points to note:

Typically, $E_g \sim 1eV$, $kT \sim \frac{1}{40} eV$ (room temperature)

- Anticipating that μ is somewhere in the middle of the gap, we have $E_c - \mu \sim eV$; $\mu - E_v \sim eV$

$$\Rightarrow \frac{\epsilon - \mu}{kT} \gg 1 \quad ; \quad \frac{\mu - \epsilon}{kT} \gg 1$$

for ϵ in CB — for ϵ in VB

$$F(\epsilon) \approx e^{-(\epsilon-\mu)/kT} \ll 1 \quad ; \quad F(\epsilon) \approx 1 - e^{-(\mu-\epsilon)/kT} \approx 1$$

states in CB have a tiny chance of being occupied

states in VB have a tiny chance of being unoccupied

Q: What is $\mu(T)$ and $\lim_{T\to 0} \mu(T) \equiv E_F$?

At $T=0$, all electrons are in VB and CB is empty.

N_e = total number of electrons in the full valence band

$$= \int_{E_{V_0}}^{E_V} \underbrace{g_{VB}(\mathcal{E})}_{\text{DOS of VB}} d\mathcal{E}$$

<u>At $T\neq 0$, some electrons are thermally excited to CB</u>

N_c = number of electrons in CB

$$= \int_{E_c}^{E_{CT}} g_{CB}(\mathcal{E}) \frac{1}{e^{\beta(\mathcal{E}-\mu)}+1} d\mathcal{E}$$

$g_{CB}(\mathcal{E})$ = DOS of CB

$$\approx \int_{E_c}^{E_{CT}} g_{CB}(\mathcal{E})\, e^{-\beta(\mathcal{E}-\mu)} d\mathcal{E}$$

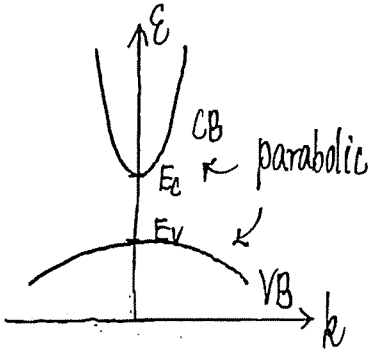
$$\approx \int_{E_c}^{\infty} g_{CB}(\mathcal{E})\, e^{-\beta(\mathcal{E}-\mu)} d\mathcal{E}$$

- extend upper limit to $+\infty$
- exponential tail $\Rightarrow$ negligible error

Q: What is $\mu(T)$?

Yet to be determined!

2-band model



$$g_{CB}(\mathcal{E}) = \frac{V}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2}\sqrt{\mathcal{E}-E_c}$$

$$g_{VB}(\mathcal{E}) = \frac{V}{2\pi^2}\left(\frac{2m_h^*}{\hbar^2}\right)^{3/2}\sqrt{E_V-\mathcal{E}}$$

$\therefore \frac{N_c}{V} \equiv n_c$ = electron number density (concentration) in CB

$$= \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2} \int_{E_c}^{\infty} \underbrace{\sqrt{\mathcal{E}-E_c}}_{\text{from DOS}}\, \underbrace{e^{-\beta(\mathcal{E}-\mu)}}_{\text{from } F_{FD}} d\mathcal{E}$$

Put $x=\beta(\mathcal{E}-E_c)$

$$= \frac{1}{2\pi^2}\left(\frac{2m_e^* kT}{\hbar^2}\right)^{3/2} e^{-\beta(E_c-\mu)} \underbrace{\int_0^{\infty} \sqrt{x}\, e^{-x} dx}_{\frac{\sqrt{\pi}}{2}\ \text{(just a number)}}$$

$$\Rightarrow \boxed{n_c(T) = 2\left(\frac{m_e^* kT}{2\pi\hbar^2}\right)^{3/2} \underbrace{e^{-(E_c-\mu)/kT}}_{\ll 1} \equiv n_c^* \cdot e^{-(E_c-\mu)/kT}}$$

only a small electron concentration in CB at $T\neq 0$

This is why doping with impurities, instead of thermal excitations, is used to introduce more electrons in CB.

Similarly, n_V = Electron concentration in VB (it is nearly full)

$$= \frac{1}{V}\int_{E_{V_0}}^{E_V} g_{VB}(\varepsilon)\,\frac{1}{e^{\beta(\varepsilon-\mu)}+1}\,d\varepsilon$$

$$= \underbrace{\frac{1}{V}\int_{E_{V_0}}^{E_V} g_{VB}(\varepsilon)\,d\varepsilon}_{\text{all states in VB (full at } T=0)} - \frac{1}{V}\int_{E_{V_0}}^{E_V}\frac{g_{VB}(\varepsilon)}{e^{\beta(\mu-\varepsilon)}+1}\,d\varepsilon$$

$$= N_e$$

$$\Rightarrow n_V = \frac{N_e}{V} - \underbrace{\frac{1}{V}\int_{E_{V_0}}^{E_V}\frac{g_{VB}(\varepsilon)}{e^{\beta(\mu-\varepsilon)}+1}\,d\varepsilon}_{\text{number density of Missing electrons in VB}}$$

(n_V: electron number density in VB; $\frac{N_e}{V}$: electron number density)

↳ call it p_V = hole concentration in VB

$$\therefore p_V = \frac{1}{V}\int_{E_{V_0}}^{E_V}\frac{g_{VB}(\varepsilon)}{e^{\beta(\mu-\varepsilon)}+1}\,d\varepsilon$$

$$\approx \frac{1}{V}\int_{-\infty}^{E_V} e^{-\beta(\mu-\varepsilon)}\,g_{VB}(\varepsilon)\,d\varepsilon = 2\left(\frac{m_h^* kT}{2\pi\hbar^2}\right)^{3/2} e^{\beta(E_V-\mu)}$$

$$\Rightarrow \boxed{p_V(T) = n_V^* \cdot e^{\beta(E_V-\mu)} = n_V^* \cdot e^{-\beta(\mu-E_V)}}$$

$$\boxed{n_V^* \equiv 2\left(\frac{m_h^* kT}{2\pi\hbar^2}\right)^{3/2}}$$

Key results:

$$n_c(T) = 2\left(\frac{m_e^* kT}{2\pi\hbar^2}\right)^{3/2} e^{-(E_c-\mu)/kT}$$

$$p_V(T) = 2\left(\frac{m_h^* kT}{2\pi\hbar^2}\right)^{3/2} e^{(E_V-\mu)/kT}$$

- These equations are valid for intrinsic (pure) and extrinsic (doped) semiconductors.
- $\mu(T)$ is yet-to-be-determined.

$$n_c \cdot p_V = \underbrace{4\left(\frac{kT}{2\pi\hbar^2}\right)^3 (m_e^* m_h^*)^{3/2} e^{-E_g/kT}}_{\text{depends on } E_g \text{ only, independent of } \mu} \equiv n_{i,\text{intrinsic}}^2 = n_i^2$$

valid for pure and doped semiconductors.

E_F and $\mu(T)$ for intrinsic (pure) semiconductors

(i) Intrinsic (pure) semiconductors†

$n_c = p_v$ ⇕ this equality fixes $\mu(T)$ — electrons in CB must come from missing ones in VB

$$2\left(\frac{m_e^* kT}{2\pi\hbar^2}\right)^{3/2} e^{-(E_c-\mu)/kT} = 2\left(\frac{m_h^* kT}{2\pi\hbar^2}\right)^{3/2} e^{(E_v-\mu)/kT}$$

Solve for $\mu(T)$:

$$\boxed{\mu(T) = \underbrace{\frac{E_c+E_v}{2}}_{\text{mid-gap}} + \frac{3}{4}kT\ln\left(\frac{m_h^*}{m_e^*}\right)}$$

(ii) At $T=0$,

$$E_F = \lim_{T\to 0}\mu(T) = \frac{E_c+E_v}{2} = \frac{E_c-E_v+2E_v}{2}$$

$$= E_v + \frac{E_g}{2}$$

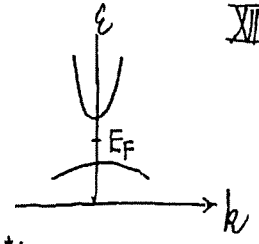
E_F is at mid-gap

† The intrinsic regime is where thermal excitations from VB to CB dominate. Thus, doped semiconductors at sufficiently high temperatures also satisfy the intrinsic regime.

(iii) $T \neq 0$, since $m_e^* < m_h^*$

$$\ln\frac{m_h^*}{m_e^*} > 0$$

$$\Rightarrow \mu(T) = E_v + \frac{E_g}{2} + \underbrace{\frac{3}{4}kT\ln\left(\frac{m_h^*}{m_e^*}\right)}_{>0}$$



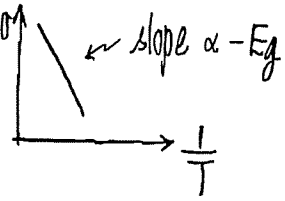
In intrinsic semiconductors, $\mu(T)$ shifts towards bottom of CB (i.e., up from mid-gap). (Why?)

(iv) Given E_g, m_e^*, m_h^*, T:

$$n_e = p_v = 2\left(\frac{kT}{2\pi\hbar^2}\right)^{3/2}(m_e^* m_h^*)^{3/4}\underbrace{e^{-E_g/2kT}}_{\text{dominating T-dependence}}$$

Recall: $\sigma = \frac{ne^2\tau}{m}$ (actually both electrons and holes contribute)

$$\log\sigma = \log f(T) - \frac{E_g}{2k_B}\frac{1}{T}$$



(v) Estimate:

$m_e^* = m_h^* = m_e$ (not true)

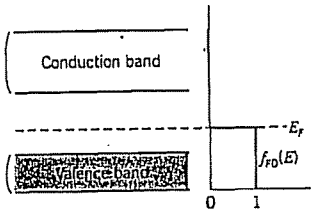
$E_g \sim 1\,eV$, $T = 300\,K$

$n_e = p_v \sim 10^{15}\,cm^{-3} \sim 10^{21}\,m^{-3}$ (metals: $10^{28}\,m^{-3}$)

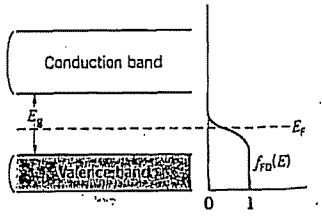
(vi) Why does a difference of 2-3 eV in E_g change a material from semiconductor to insulator?

Ans: Since E_F is near mid-gap, what matters is the tail of the Fermi-Dirac distribution

$T=0$



Band structure in which E_F lies in the gap between bands.



When $E_g \gg kT$, the conduction band is still unpopulated. This situation is characteristic of an insulator.

$T \neq 0$ Insulator

E_g is so large that basically, there is no electron in CB at room temperature

e.g. $E_g \sim 5$eV (insulator)

$T = 300K$

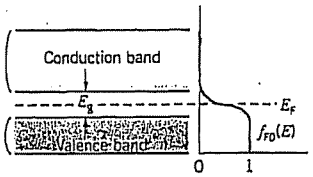
$e^{-E_g/kT} \approx 10^{-44}$

⇓

No electrons in CB

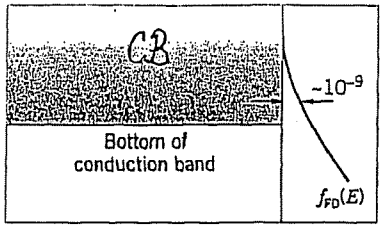
e.g. $E_g \sim 1$eV

$T \neq 0$ semiconductor



Band structure of a semiconductor. The gap is much smaller than in an insulator, so there is now a small population of the conduction band.

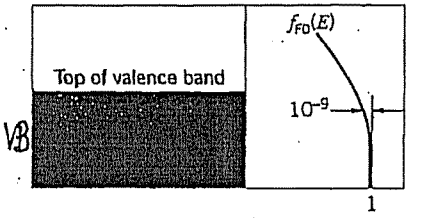
Zoom in



The tail of the Fermi-Dirac distribution function near the bottom of the conduction band. On the scale of this diagram, the "1" of $f_{FD}(E)$ is 1000 km off to the right and E_F is about 1 m below the bottom of the page.

(Still has about 10^{11} electrons in CB in cm^3)

Zoom in



The Fermi-Dirac distribution function near the top of the valence band, showing the small fraction of empty states.

$(1-10^{-9})$ prob. of being unoccupied

$E_g \sim 1$eV, $T = 300K$ $\Big\}$ $e^{-E_g/kT} \approx 10^{-9}$

• Only 1 electron out of 10^9 contributes to conduction

[Silicon at room temperature, about 10^{11} electrons/cm^3, i.e. only about 1 electron out of 10^{10} contributes!]

[But, if $E_g \sim 5$eV, practically NO electrons in CB.]