PHYS4031 STATISTICAL MECHANICS

SAMPLE QUESTIONS FOR DISCUSSION IN WEEK 3 EXERCISE CLASSES (19, 21 September 2016)

You may want to think about (work out) them before attending exercise class.
Concepts and techniques covered:
SQ6 - How $W(E,N)$ depends on E for unbounded single-particle spectrum
SQ7 - Why a swinging pendulum slows down in air? **Key idea connecting Stat. Mech. to Second Law of thermodynamics (MUST READ/DO)**

SQ6 **Counting $W(E,V,N)$ is the first step of getting $S(E,V,N)$ and all thermodynamic quantities.**

Summary on first approach in Stat Mech: Chapter III discussed the fundamental principles of equilibrium stat mech. Practically, we take on the following strategy in handling *equilibrium isolated systems*: (i) Count the number of accessible microstates $W(E,V,N)$, (ii) $S(E,V,N) = k \ln W(E,V,N)$, (iii) take derivatives to find $1/T$, p/T and μ/T and thus T, p, and μ, (iv) hence see how the quantities are related (equation of state). Done!

In Ch.III class notes, we showed that for $N = 3$ distinguishable particles A,B,C (say) in an isolated system with $E = 3\epsilon$ total energy, there are $W = 10$ microstates. Later in the chapter, for $N = 6$ and $E = 6\epsilon$, there are $W = 462$ microstates. The point is that even E/N is the same for the two systems, W increases rapidly when we scale up the system. This is an important point that you should remember.

Here, the TA will show that for the spectrum of single-particle states under consideration, a system of higher energy carries many more microstates.

Pay attention to the set up in the question: There are 3 particles A,B,C. Each particle can take on $0, 1, 2, 3, 4, \ldots$ units of energy. There are 6 units of energy to be shared by the 3 particles. **Find** the number of microstates $W(N = 3, E = 6)$.

If there are 9 units of energy to be shared by the 3 particles, **what** is $W(N = 3, E = 9)$?

Remarks: TA should guide students' attention to the dependence of $W(E)$ on E **and** the special nature of the spectrum of the single-particle states, i.e., one particle can take on any units of energy (thus **unbounded**). In the three cases, secondary school physics tells you that the system with $E = 9$ units is the hottest and the one with $E = 3$ units is the coldest. In Stat Mech (microcanonical ensemble), this sense of hottest is **derived** by calculating the temperature from $S(E)$.

SQ7 **(All students must try!) Why does a swinging pendulum come to rest? Essential ideas of Stat. Mech. give sensible results.**

The second law is needed because the first law cannot cover all the physics. For example, we know that when a pendulum is lifted up to a certain height and released, it will swing, gradually slow down and eventually stop. You never see, however, a pendulum at rest suddenly starts swinging. **There is a one-way tendency for physical processes, from which we have a sense of an arrow of time!** This is a fact of life (and we keep on getting older, not younger)! But the first law does not say anything about it. We need the second law. This is why we see **inequalities** in the second law, i.e. $dS \geq 0$ in an isolated system. With inequalities, then we can give a direction to processes. The same situation of the pendulum, when dealt with in Mechanics, will be referred to as damping due to friction, in which the energy of the swinging pendulum is turned into heat (air molecules move slightly more rapidly).

A key idea in Stat Mech is that for an isolated system at equilibrium, all compatible microstates are equally probable. It implies that if a system is restricted to a situation corresponding to only a small number of microstates initially (lift up the pendulum and all air molecules at rest) and then let go, the system will evolve to explore all possible microstates (thus redistributing the energy). We will say how this idea can be applied to understanding the slowing down of a swinging pendulum.

Here is a **toy model** (but the physics is real). Let P be an object (the pendulum). It is placed in a box with a few gas molecules. For simplicity, let there be only 4 indistinguishable molecules. At the beginning, the pendulum is given a "big energy" of 12 units (e.g. lift it up to 30 degrees and release it). The 4 molecules are assumed to have NO energy initially. [They may be moving at the beginning, but we take this as the zero reference. It doesn't matter here. The physics is more important.] Altogether, we have (1+4) **five objects** in the box and **12 units of energy**.

In terms of a microstate description, the initial situation can be described as a particular microstate $(0, 0, 0, 0; P = 12)$ (meaning zero energy for the molecules and the pendulum P takes all the 12 units).

Now as the pendulum swings, the gas molecules will collide with the pendulum and then there will be exchange of energies.

TA: List ALL the possible ways that the 12 units of energy can be shared by the five objects. Note that (to prepare for $N = 10^{24}$ particles) the 4 molecules here are indistinguishable. Therefore, if there are 2 units of energy in the gas, then a situation of 2-unit in one molecule AND zero in the other three is counted as one microstate; and another situation of two molecules each having one unit of energy and two other having none is counted as another microstate. OK? Think about all the possible division of the energies and **give a table** showing the number of microstates for every division of energy.

As the pendulum swings, the system evolves to explore the microstates. Eventually, at equilibrium, the fundamental postulate says *all compatible (or accessible) microstates are equally probable.* **Give** the probability of finding each partition of energy between the pendulum and the gas molecules, i.e., for P $= 0, 1, 2, \ldots, 12$.

Finally, call the students' attention to the following points.

- The initial situation is actually a highly unlikely microstate. [So the moment the pendulum is released, the system (P + 4 molecules) is out of equilibrium.]
- Thus, as the system evolves (collisions (seen as damping)), the system explores ALL possible microstates. As these microstates are all equally probable, the division of energy with the highest number of microstates will appear most often. (Even) In our toy model, the share of the pendulum's energy drops. Point out what does the most probable situation refer to (which situation corresponds to the highest number of microstates and thus the highest probability to be seen).
- Probabilistically, for our tiny system (4 molecules and P), a pendulum at rest could suddenly start swinging. Sort out the probability of P having 6 or more units of energy.
- But for an ordinary system with $N \sim 10^{24}$ gas molecules, the fate of slowing down and coming to a stop becomes deterministic!
- Encourage students to think (and re-think) what is being used in this SQ to understand a daily-life phenomenon with a direction. This is nearly ALL of STAT MECH in understanding why there is an arrow of time.

One may think that the pendulum consists of a large number of atoms inside it and therefore we should count them in. Well, these atoms move together (swing together) inside the pendulum and their collective motions give the mechanical (kinetic) energy of the pendulum. The random motions of the gas molecules, however, are the heat energy.

Extension: In Mechanics, we would have introduced a damping term (something like $\sim \gamma m v$) in describing the motion of the pendulum. Similarly, an oil drop would attain its terminal velocity as it drops due to friction (damping) of the air molecules. Then comes the question on how one can connect what we did here (treating all the particles instead of focusing on the pendulum) with damping term in mechanics. This is beyond our scope of *Equilibrium Statistical Mechanics*. Instead, this is a topic in **Non-equilibrium statistical mechanics** - the next course on the subject. To explore the topic, google it or search the library catalog on non-equilibrium statistical mechanics and you may find many books written on it. Standard topics are Brownian motion, Langevin equation, Navier Stokes equation, etc.

[*Reference:* The idea in this SQ is taken from the book *Great Ideas in Physics* by Alan Lightman.]