PHYS4031 STATISTICAL MECHANICS

SAMPLE QUESTIONS FOR DISCUSSION IN WEEK 1 EXERCISE CLASSES (5, 7 September 2016)

What are exercise classes? Exercise Classes are an *integrated part* of PHYS4031. **You should arrange your time table to attend one exercise class per week.** Sample Questions (SQs) will be given to students prior to exercise classes during lectures. Students should read (or try to solve) the questions. TAs will discuss SQs in the exercise classes in a week (repeated in two sessions per week). These SQ's are designed to resemble some Problem Set Questions (thus help you do your homework), to illustrate the principles discussed in lectures with examples, to fill in some background (e.g. those covered in previous course such as PHYS3031), and to enhance your understanding of the subject.

Below are the SQ's for Week 1 Exercise Classes. You may want to think about the SQ's before attending exercise class.

SQ1 **Math Skill 1: Let's count - an essential skill in Stat. Mech.**

Statistical Mechanics is the microscopic theory of thermodynamics. Very soon (or already seen in PHYS3031), you will see that a key idea in Stat. Mech. is about the **number of microstates** corresponding to a given **macrostate**. Simply put, macrostates are the thermodynamic states specified by only a few variables. For a macrostate, e.g. given (E, V, N) for a gas, where E is the energy (think about it as the U in thermodynamics), V is the volume, and N is the number of particles (atoms/molecules), there can be many ways for the N particles to share the energy E. What many are there? It follows that **counting** is an important skill to do stat. mech. calculations. This SQ reminds you of some ways in counting.

(a) There is something called $N!$ (N factorial). Give the connection between $N!$ and the ways to arrange N **distinguishable objects** in order.

(b) In the most recent 2016 Olympics volley ball woman's tournament, there were 6 teams (Teams A-F for examples) in a pool in the preliminary round. The teams played against each other once. List all the games and find the total number of games needed to be arranged in the preliminary round. Relate the result to ${}_6C_2 = C(6,2) = {}^6C_2 = \binom{6}{2}$, where all the notations carry the same mathematical meaning.

(c) What is ${}_NC_n$? How is it related to (i) the number of ways that n objects can be selected from a set of N different objects (order of selection is not important) as well as (ii) the number of ways for the partition of N distinguishable objects into two groups, one of size n and another of size $(N-n)$? [Remark: We need this in later discussions.]

(d) Generalizing part (c), the N objects are partitioned into r groups (instead of just 2 groups), with n_1 in group 1, n_2 in group 2, $\ldots$, n_r in group r. How many ways can such a partition be done?

(e) Getting closer to thermodynamics, we want to divide 6 units of energy among 3 distinguishable particles, with no restriction on the units of energy per particle. How many ways can this be done? How is this related to ${}_NC_n$?

(f) Repeat part (e), only that there are now 9 units of energy in total. Note that everything being equal (3 distinguishable particles, etc.), a higher total energy implies an increase in the number of ways of distributing the energy among the 3 particles. This has a profound implication – if the entropy S is an increasing function of the number of ways (labelled by the symbol W), then we see that S increases with the total energy.

(g) Same problem dressed in physics costume. There are N one-dimensional quantum oscillators. Every oscillator has the same angular frequency ω. They can be identified, hence distinguished, by their locations. A marcostate is specified by a total energy. Let's say the total energy is given by

$$\begin{aligned} E &= M\hbar\omega + N\frac{\hbar\omega}{2} \\ &= M\hbar\omega + \text{G.S. energy} \end{aligned} \tag{1}$$

From Quantum Mechanics (PHYS3021 & PHYS3022 in Year 3), the last term is the ground state energy of the N oscillators. The first term is more interesting. Recall how the allowed energies are quantized in a harmonic oscillator. Find the number of ways $W(E, N)$ that the energy E can be distributed among the N oscillators? These $W(E, N)$ ways correspond to different microstates (different wavefunctions) for the macrostate specified by E.

[*Important remarks:* In Stat. Mech., once $W(E, N)$ is known, (Boltzmann said) the entropy is given by $S = k \ln W$ and then all the thermodynamic quantities can be calculated.

This problem is related to

(i) the Einstein model of solids and their heat capacity (solid state physics),

(ii) the shape of vibrational-rotational spectrum in a gas of molecules (discussed in molecular physics), and

(iii) Planck's thermal (black-body) radiation formula.

Using Quantum Mechanics, you also see that $W(E, N)$ is the **degeneracy** (how many different N-oscillator states) of the energy E for a quantum system consisting of N independent and distinguishable oscillators.]

SQ2 **Math Skill 1: Counting continued**

(a) The new horse racing season has just started (3 Sept 2016). With 14 horses in a race and assuming that the horses are equally capable (this is not true in reality), how many possible outcomes are there for the first three horses (called Trio) in a race where the order is NOT important?

(b) *Triple Trio* (or "3T") is the hottest bet in racing in Hong Kong. One needs to bet on the Trio in three different races. How many possible outcomes are there, assuming the 14 horses in each race are equally capable? What is the mathematical principle used here?

[*Important Remarks*: (i) For daily life, you can judge whether a HK$30,000,000 return for a HK$10 "3T" bet is reasonable. (ii) For stat. mech. purposes, we need the same mathematical principle in deriving results in **ensemble theories**!]

Useful Hints/Remarks: We need some counting skills in stat mech. But what we actually need is very basic. If you want to read more on (or review) how to count, see the relevant sections in

- Chapter 26 of *Mathematical Methods for Physics and Engineering* by Riley *et al.*
- Chapter 21 of *The Chemistry Maths Book* by Steiner

These books are listed under "Mathematical Methods" on the book list. They are put on reserve in the University Library (ground floor in reserved book section).