Intensive Course in Physics
Gravitational Waves

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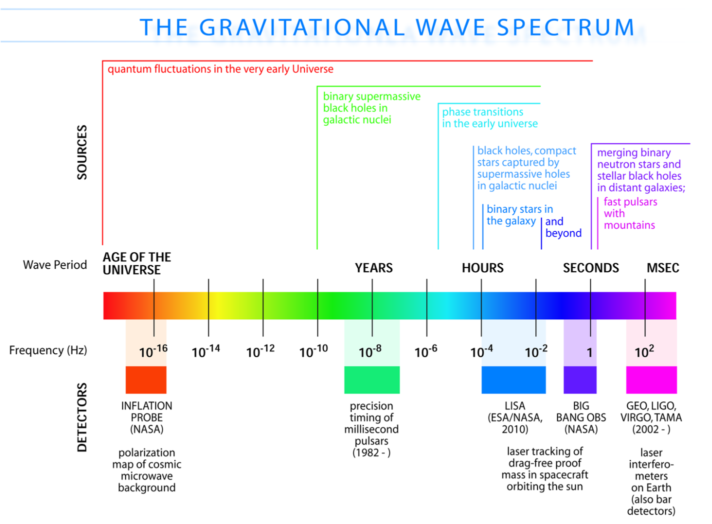
Chapter 3: Sources of Gravitational Waves

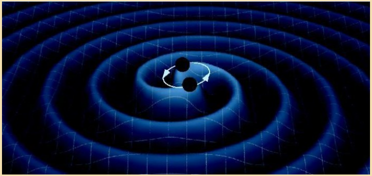
November 9, 2016

GRAVITATIONAL-WAVE SPECTRUM

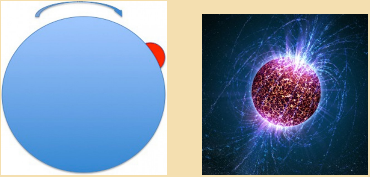


Astrophysical sources of gravitational waves

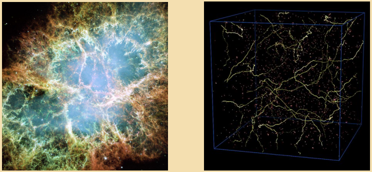
Binary mergers



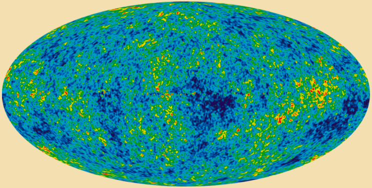
Continuous waves



Burst



Stochastic background



GWs from Ellipsoids

- Consider triaxial ellipsoid with

$$I(t=0) = \mathrm{diag}(I_1, I_2, I_3) \tag{1}$$

- Orbitting at frequency ω
- Gravitational waves emission

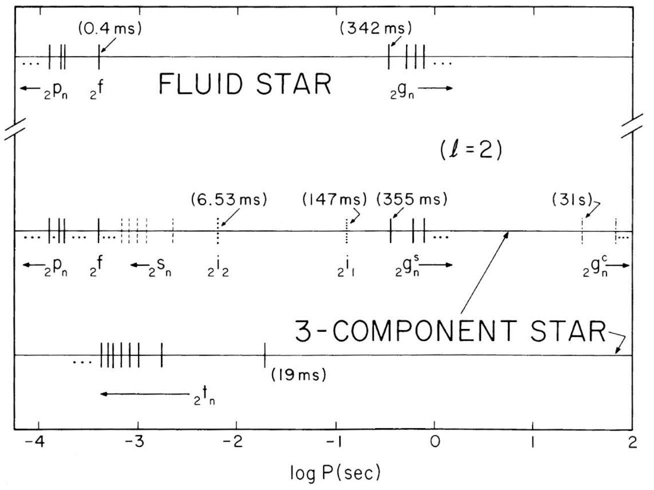
$$h_+ = \frac{1}{r}\frac{4G}{c^4}\,\omega^2\,(I_1 - I_2)\;\cos(2\omega t), \tag{2}$$

$$h_\times = \frac{1}{r}\frac{4G}{c^4}\,\omega^2\,(I_1 - I_2)\,\sin(2\omega t). \tag{3}$$

- Decrease of rotation frequency due to GW emission

$$\dot{\omega} = -\frac{32G}{5c^5} I_3^2 \epsilon^2 \omega^5. \tag{4}$$

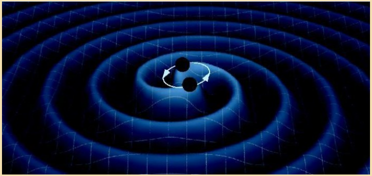
NEUTRON STAR OSCILLATIONS



McDermott et al. 1988

ASTROPHYSICAL SOURCES OF GRAVITATIONAL WAVES

Binary mergers



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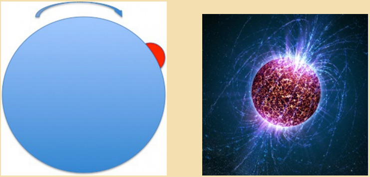

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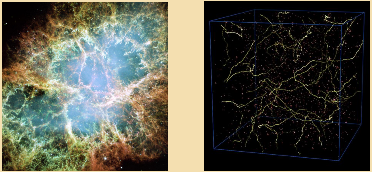

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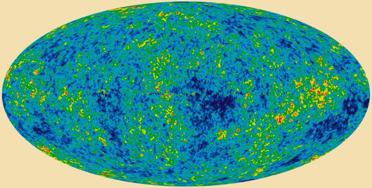


BINARY

- Two objects with masses m_1 and m_2 in a circular orbit around the common center of mass (CM),
- Angular frequency ω and separaton R.
- Gravitational waves emission

$$h_+ = \frac{4}{r}\frac{G\mu R^2\omega^2}{c^4}\,\frac{1+\cos^2(\iota)}{2}\,\cos(2\omega t_{\mathrm{ret}})n \tag{5}$$

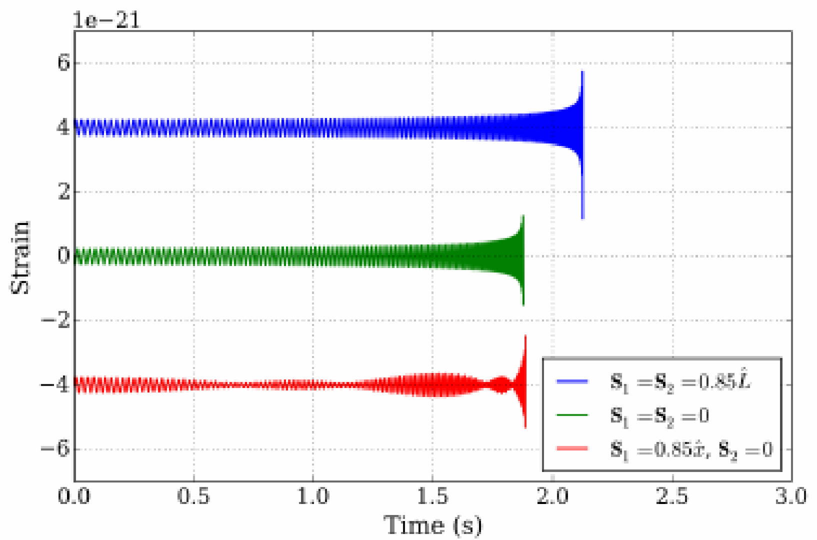
$$h_\times = \frac{4}{r}\frac{G\mu R^2\omega^2}{c^4}\,\cos(\iota)\,\sin(2\omega t_{\mathrm{ret}}). \tag{6}$$

- Decrease of rotation frequency due to GW emission

$$\frac{d(v/c)}{dt} = \frac{32\eta c^3}{5GM}\left(\frac{v}{c}\right)^9 \tag{7}$$

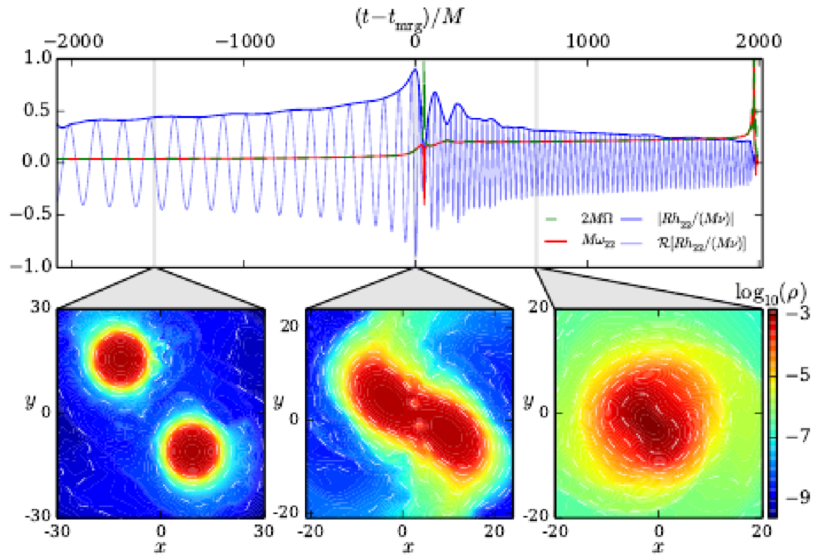
- where $v = Rw$, $M = m_1 + m_2$, $\eta = \mu/M$

Effects of Spin



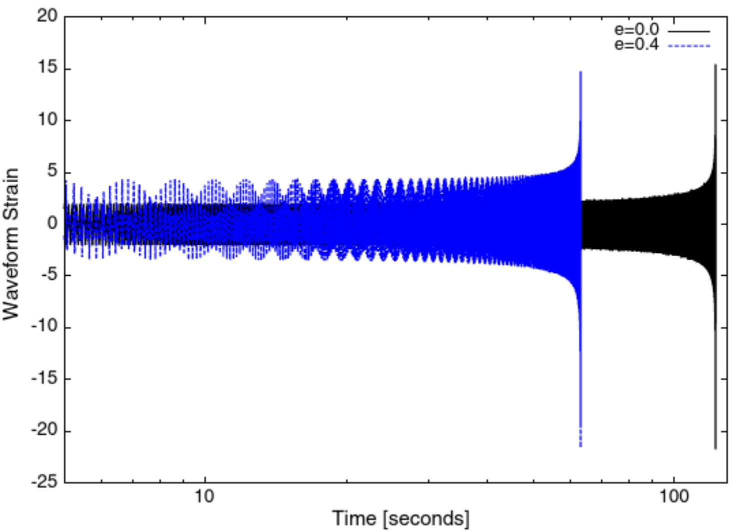
Privitera 2014

MATTER EFFECTS



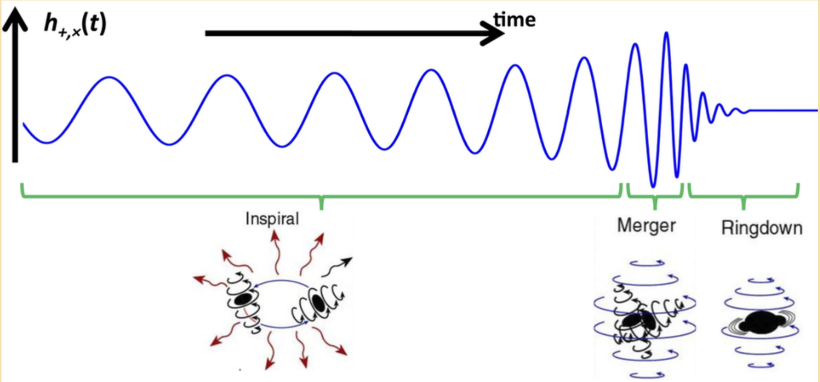
Bernuzzi et al. 2015

ECCENTRICITY EFFFECTS



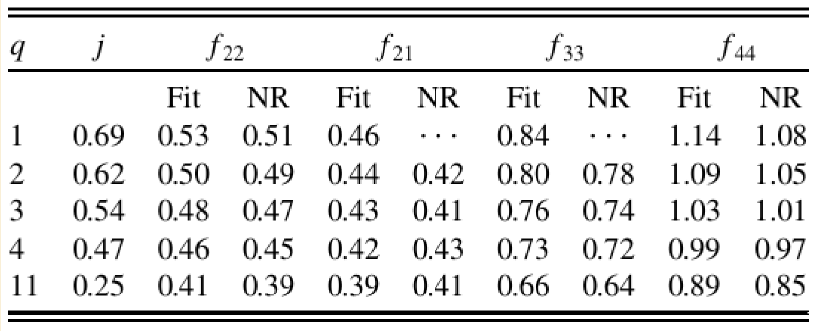
Huerta et al. 2014

FULL BINARY WAVEFORM



QUASI-NORMAL MODES

$$h(t) = \sum_{lmn} A_{lmn} e^{-t/\tau_{lmn}} \cos\left(\omega_{lmn} t + \phi_{lmn}\right) \tag{8}$$

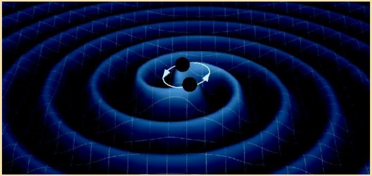


q	j	f_{22}		f_{21}		f_{33}		f_{44}	
		Fit	NR	Fit	NR	Fit	NR	Fit	NR
1	0.69	0.53	0.51	0.46	⋯	0.84	⋯	1.14	1.08
2	0.62	0.50	0.49	0.44	0.42	0.80	0.78	1.09	1.05
3	0.54	0.48	0.47	0.43	0.41	0.76	0.74	1.03	1.01
4	0.47	0.46	0.45	0.42	0.43	0.73	0.72	0.99	0.97
11	0.25	0.41	0.39	0.39	0.41	0.66	0.64	0.89	0.85

Kamaretsos et al. 2012

ASTROPHYSICAL SOURCES OF GRAVITATIONAL WAVES

Binary mergers



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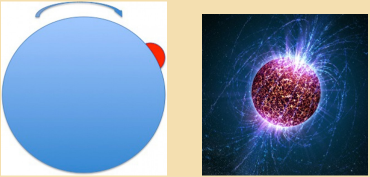

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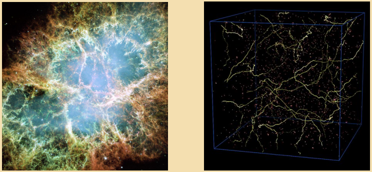

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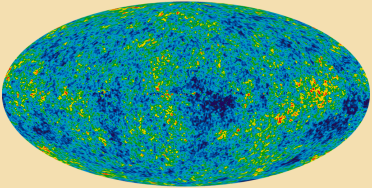
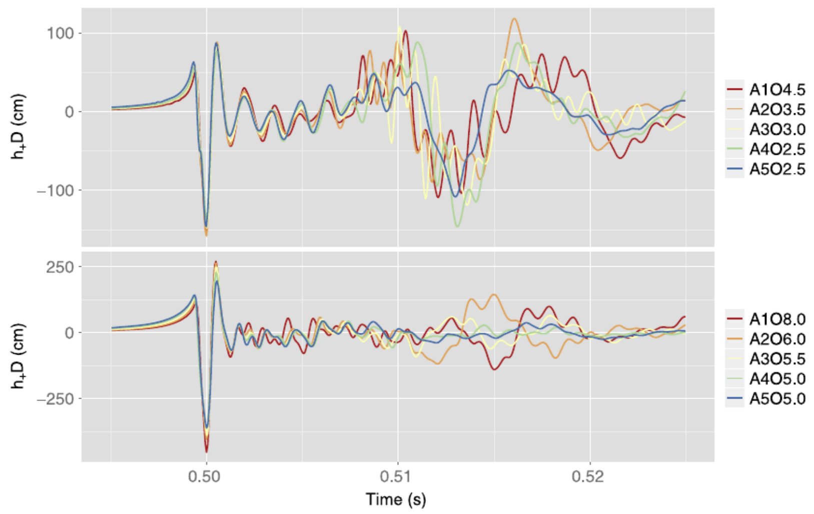


Supernova Waveforms



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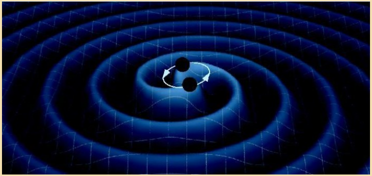

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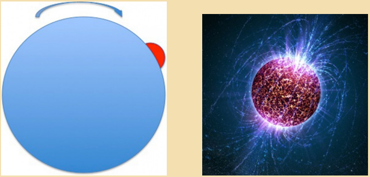

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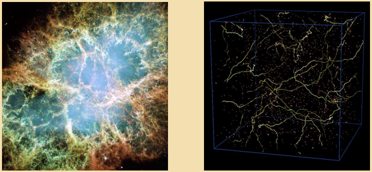

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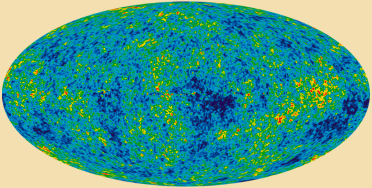


STOCHASTIC GRAVITATIONAL-WAVE BACKGROUND

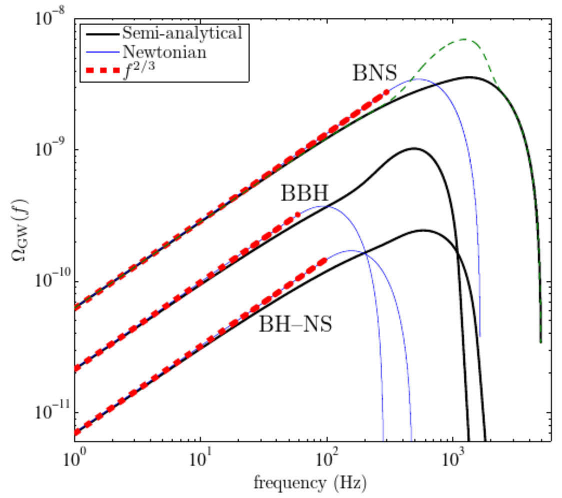
- From the Big Bang or astrophysical origin
- Assume: stationarity, Gaussian, isotropci, unpolarised

$$\left\langle h_{ij}(t)h^{ij}(t)\right\rangle = 4\int_0^\infty df\, S_h(f) \tag{9}$$

- where $S_h(f)$ is the signal power spectrum
- where $\langle \ldots \rangle$ is the essemble average
- Fractional energy density given by

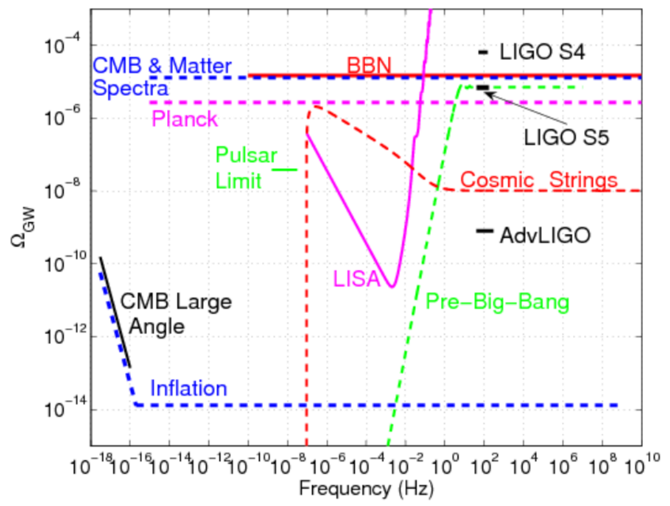
$$\Omega_{\mathrm{GW}} = \frac{4\pi^2}{3H_0^2} f^3 S_h(f) \tag{10}$$

SGWB FROM BINARIES



Zhu et al. 2012

SGWB Sources



Abbott et al. 2009