

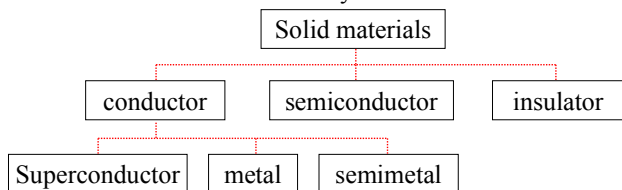
### Ch.3 Superconductivity

Read one of the following references:

1. F. J. Owens and C. P. Poole, Jr., "The new superconductors", (QC611.95 .O89 2002 **ebook**).
2. V L Ginzburg & E A Andryushin, "Superconductivity" 2/e (QC611.92 .G56 2004 **ebook**)
3. G. Vidal, Superconductivity: the next revolution? (1993) (QC611.92V53)
4. P.F. Dahl, Superconductivity (QC611.92D34 1992)

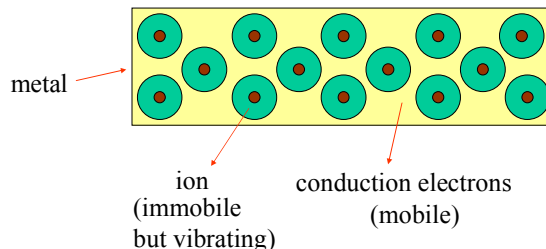
#### (I) Introduction

- Solids have different conductivity:



- Superconducting materials: elements, alloys, compounds. (classified as Type I & Type II)
- Type I superconductor has properties:
  - perfect conductor ( $\rho = 0$ )
  - Meissner's effect ( $B = 0$ )
- Applications: superconducting magnet, Maglev, SQUID, ...

#### Origin of conductivity in metal



Typically, the number of conduction electrons in a solid  $n \approx 10^{22}$  electrons/cm<sup>3</sup>.

Note: At higher pressure,  $n$  (and thus conductivity) increase. e.g. hydrogen can be forced to become metallic at ~ 1-10 megabars. (1 bar  $\approx$  1 atmospheric pressure)

(<http://www.nature.com/news/1998/000622/full/news000622-6.html>)

Motivation:

As  $T \rightarrow 0$ , gas becomes liquid, then solid in which atoms have only minimum thermal vibrations.

How about conduction electrons in metals at low temperature?

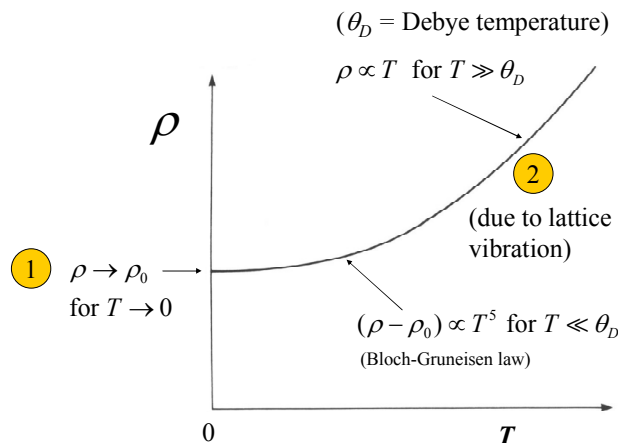
To answer this question, need QM.

Electron sees periodic potential and has wave behavior (wave-particle duality).

By solving Schrödinger's equation (a wave equation), we can find allowed energy states for electrons and electron in these states can travel through the solid more freely!

Typical result (covered in PHY 4450) is shown in a graph:

#### Resistivity of a metal as a function of temperature:



#### Origin of resistance:

- 1  $\rho_0$  (residual resistivity) due to defects, impurities, etc.

- 2 temperature dependence of  $\rho$  depends on electron-ion scattering & lattice vibrations.

$$\rho \propto T \text{ for } T \gg \theta_D$$

$$(\rho - \rho_0) \propto T^5 \text{ for } T \ll \theta_D$$

$$\theta_D = 315\text{K for Cu}$$

$$\theta_D = 230\text{K for Pt}$$

#### What happens when $T \rightarrow 0$ ?

If there are no defects and impurities then  $\rho = \rho_0 = 0$  at  $T = 0$ .

**Ideal periodicity  $\Rightarrow$  No resistance!**  
(impossible!)

Classical picture: At  $T = 0$ , all ions are standing still.

Quantum picture: Even at  $T = 0$ , electrons are still moving inside atoms while the atoms can almost stand still.

#### Conjectures near 1900:

James Dewar:  $\lim_{T \rightarrow 0} \rho(T) = 0$

(no lattice vibration  $\Rightarrow$  very pure metal is a perfect conductor)

Lord Kelvin, Kamerlingh Onnes:  $\lim_{T \rightarrow 0} \rho(T) = +\infty$   
(freezing of electron gas)

No theory was known at that time!

Need experimental observation.

#### (II) The big chill: low temperature experiments.

1883 Olszewski & Wroblewski liquefied oxygen (90.2 K) (boiling point at 1 bar) and nitrogen (77.3 K).

1898 James Dewar liquefied hydrogen (20.5 K).

1908 K. Onnes liquefied helium (4.2 K), obtained 1.7 K at reduced pressure.

Onnes obtained Nobel prize in 1913.

Lowest temperature recorded on Earth: in Vostok (Russian)

-89.2 °C (confirmed): July 21, 1983

-91 °C (unconfirmed): 1997 (colder than dry ice!)

The coldest temperature achieved in physics labs:

100 pK (in 2000)

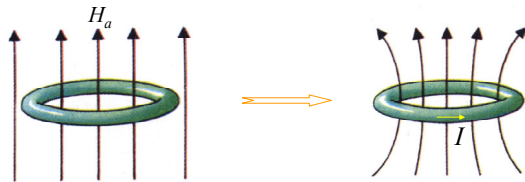
([http://lth.tkk.fi/wiki/LTL/World\\_record\\_in\\_low\\_temperatures](http://lth.tkk.fi/wiki/LTL/World_record_in_low_temperatures))

Temperature in deep space  $\sim$  2.735 K.

PHY 3202 (3-2)

A simple approach to test  $R=0$  is to use a **superconducting ring** (first by Tuyn and Onnes).

The current in the ring can be induced by Faraday's law:



- ① First cool the ring to  $T < T_C$
- ② Then remove the applied field  $H_a$ .
- ③ The induced current in the superconducting (SC) ring flows **persistently**. (If the ring is made of ordinary metal, the induced current decays rapidly.) Tuyn and Onnes did not observe any decay of current in **24 h**.

This can be understood by Faraday's law.

The ring is equivalent to an  $RL$  circuit.

$R$  = the total resistance of the ring (assuming  $R \neq 0$  first).

$L$  = the inductance of the ring

In an external magnetic field  $\vec{B}_a = \mu_0 \vec{H}_a$ ,

the induced voltage in the ring is  $V = -\frac{d\Phi_a}{dt} = L\frac{dI}{dt} + IR$

where  $\Phi_a = \int_{\text{ring}} \vec{B}_a \cdot d\vec{a}$

If  $\frac{dB_a}{dt} = 0$ , then  $-\frac{d\Phi_a}{dt} = 0$  or  $I(t) = I_0 e^{-(R/L)t} = I_0 e^{-t/\tau}$ .

The decay time constant is  $\tau = \frac{L}{R}$

If  $R = 0$ , then  $I(t) = I_0$  (persistent current).

Can  $R = 0$  be verified by the experiment on SC ring?

For a ring of radius  $r$  and cross-sectional radius  $a$ ,

$$L = \mu_0 r [\log_e(8r/a) - 2]$$

and  $R = 2r\rho/a^2$

where  $\rho$  is the resistivity of the ring material.

Thus  $\rho\tau = \frac{1}{2}\mu_0 a^2 [\log_e(8r/a) - 2]$  [1]

For a Cu ring at room temperature

( $\rho \approx 1.56 \mu\Omega\cdot\text{cm}$ ,  $a = 1.0 \text{ mm}$  and  $r = 10 \text{ cm}$ ),  $\tau = 0.19 \text{ ms}$ .

For a superconducting ring with the same  $a$  and  $r$ , if, for over a period  $t$ , the supercurrent persists without appreciable change (assuming  $<1\%$  of drop in  $I$ , which can be easily detected by a Hall effect sensor), then  $\tau$  is determined by  $I(t) = I_0 e^{-t/\tau}$ .

Assume  $I(t) > 0.99I_0$  in  $t = 1 \text{ year}$ . Then  $\tau > 3 \times 10^9 \text{ s}$  and thus by Eq. [1],  $\rho < 1 \times 10^{-19} \Omega\cdot\text{cm}$ .

This is how we estimate an

**upper limit of resistivity of superconductor.**

One can never show  $R = 0$  by experiment.

The measurement of  $I(t)$  in a ring sample is thus the most sensitive method to set an upper limit of resistivity.

In 1962, the measured upper limits are

$< 3 \times 10^{-23} \Omega\cdot\text{cm}$  for conventional Type I superconductor (J. Appl. Phys, **33**, 748 (1962)),

$< 2 \times 10^{-19} \Omega\cdot\text{cm}$  for conventional Type II superconductor (Phys. Rev. Lett. **9**, 306 (1962)).

(The classification of superconductors will be given later.)

## (IV) Magnetic property of superconductor

### Onnes' Dream

Novel property  $\Rightarrow$  Novel application:

$R = 0$  in superconductor  $\Rightarrow$  we can pass very high current through a superconducting wire.

This suggests that we can use superconducting wire to build a magnet (**superconducting magnet, SM**) to generate strong and stable magnetic field. SM is just a solenoid of superconducting wire with high current.

Onnes received a grant to build a 10 T SM.

But it is not that simple! He never attained the goal because he worked only on elements (now classified as Type I superconductors). It took half a century for other scientists to achieve this goal.

Instead he found **critical current**  $I_C$  and **critical field**  $H_C$ .

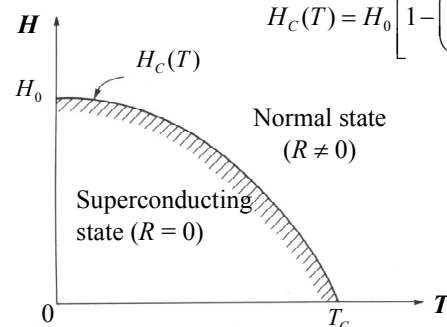
The superconducting state can be maintained only when  $I < I_C$  and  $H < H_C$ .

For all pure metals Onnes studied,

$$H_C \sim \text{few } 10^2 \text{ Oe } (B_C \sim 10^{-2} \text{ T}).$$

$H_C$  depends on temperature:

$$H_C(T) = H_0 \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right] \quad [2]$$



### Current-carrying wire generates magnetic field:



The magnetic field on the wire surface is  $B = \frac{\mu_0 I}{2\pi r}$ .

Assume there is no external magnetic field ( $H_a = 0$ ).

In order to maintain superconducting state, the current must not exceed a critical value  $I_C$  so that the generated field on wire surface  $B \leq \mu_0 H_C$ .

**Critical current:**  $\frac{\mu_0 I_C}{2\pi r} \approx \mu_0 H_C \Rightarrow I_C \approx 2\pi r H_C$

We will show later that in a (Type I) superconductor, the supercurrent is confined to the surface.

due to  $I$

If  $H_a \neq 0$  then we need  $|\vec{H}_a + \vec{H}_I| < H_C$

For applied magnetic field  $H_a = 0$ ,  $I_C \propto H_C$ .

$$\therefore I_C = I_0 \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right]$$

For different applied

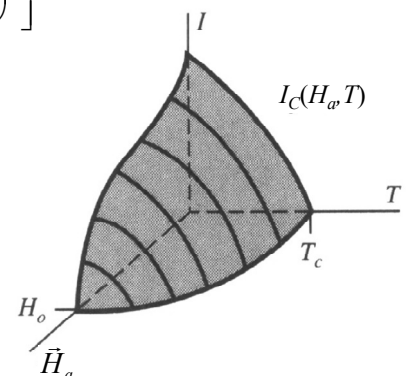
field  $H_a$ , the

superconductor has

different  $I_C(T)$  curve.

In general, the  $I_C(H_a, T)$

surface depends on  $\vec{H}_a$  direction.



## (V) The Meissner effect

- Is superconductor just a perfect conductor only?  $\sigma = \infty$   
No! The evidence: Some alloys and even some amorphous metals (with low conductivity) are superconductors. Superconductor has peculiar magnetic property. It is now known as the Meissner effect.
- Meissner & Ochsenfeld (1933) used a **solid** Pb cylinder in order to
  - look for any change in magnetic properties in association with the superconducting transition.
  - find out whether the supercurrent shielded the magnetic field completely.

### What is $\vec{B}$ inside SC?

For perfect conductor,  $\sigma = \infty$ .  
Since  $\vec{J} = \sigma \vec{E} < \infty$ ,  $\vec{E} = 0$ .  
(Note: in electrostatics,  $\vec{E} = 0$ )

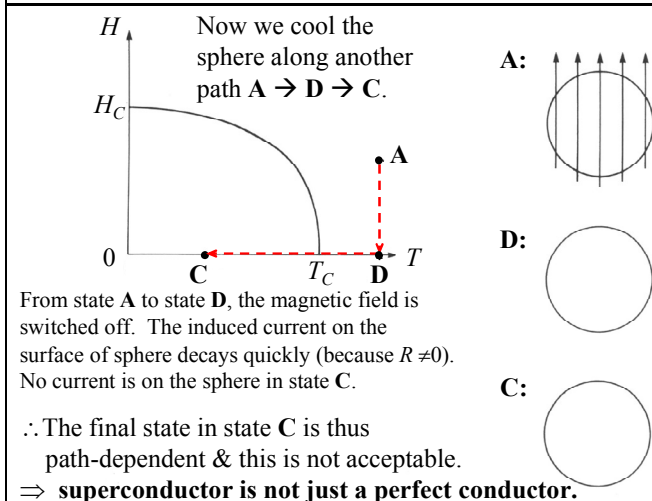
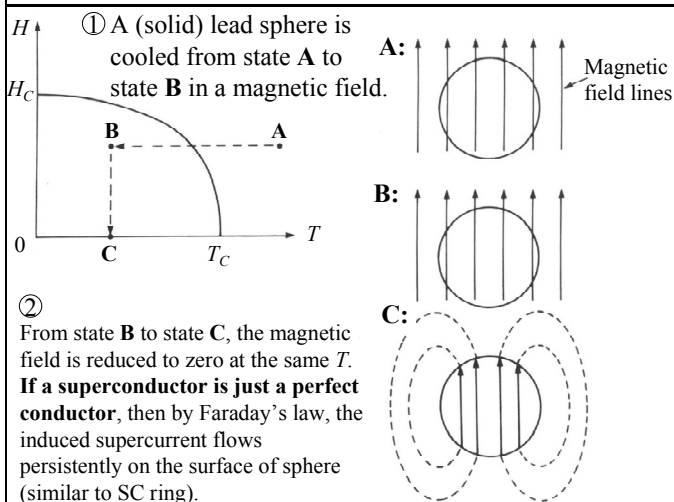
Faraday's law:  $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$   
 $\therefore \frac{\partial \vec{B}}{\partial t} = 0$ .

$\Rightarrow \vec{B}$  inside a perfect conductor is time-independent.

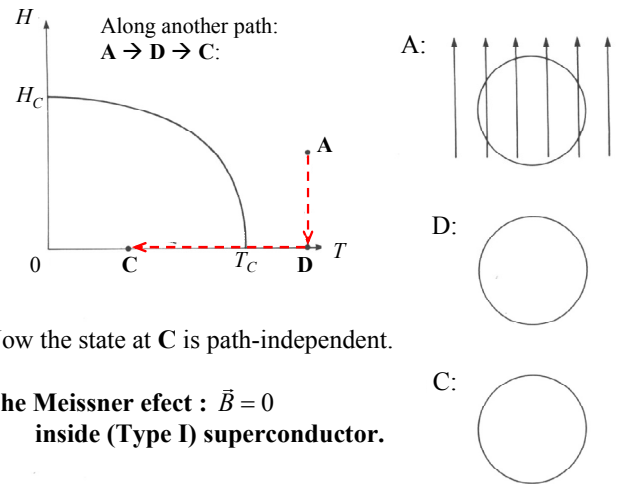
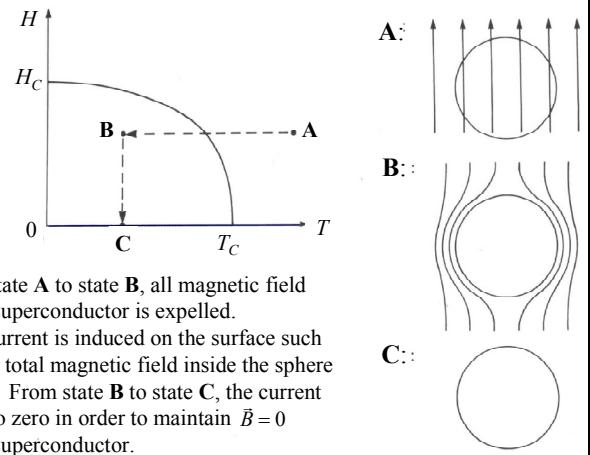
But  $\vec{B} = ?$

Are superconductors simply perfect conductors?

Now consider the cooling process:



## Meissner & Ochsenfeld's discovery: $\vec{B} = 0$ inside SC



## Supercurrent in superconductors:

The Meissner effect:  $\vec{B} = 0$  inside (Type I) SC.

By Maxwell-Ampere's law,  $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

or  $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enclosed}} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{a}$   
 (for a closed loop inside SC.)

Remember that we have shown that  $\vec{E} = 0$  inside SC.

$\therefore \vec{J} = 0$  At any point inside SC

Current in a (Type I) SC is confined to its surface.

Therefore if we put a SC in a magnetic field  $H < H_c$ , then a supercurrent is induced on the SC surface to ensure that the total magnetic field  $\vec{B}$  inside the SC is zero.

This current is called the screening current.

**Screening current distribution** is unique & can be calculated:

Consider a (Type I) superconducting **sphere** (at  $T < T_c$ )

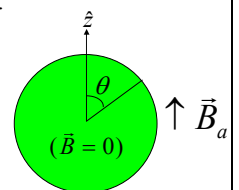
placed in a uniform magnetic field  $\vec{B}_a = \mu_0 \vec{H} = \mu_0 H \hat{z}$ .

Because of the Meissner effect, a surface current is induced so that inside the superconductor  $\vec{B} = 0$ .

$\vec{B} = \vec{B}_a + \vec{B}_i = 0$

$\vec{B}_i$  is due to the surface current density

$\vec{K} = -\frac{3}{2\mu_0} B_a \sin \theta \hat{\phi}$



(Here we use spherical coordinates  $(\hat{r}, \hat{\theta}, \hat{\phi})$ .)

For details, see [Appendix 10.](#)



### Penetration depth:

**Meissner effect:**  $B = 0$  inside (Type I) superconductor.

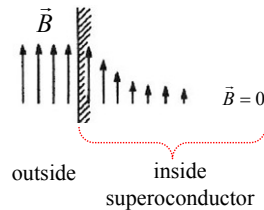
Actually the magnetic field can penetrate an **ultra-thin** depth underneath the surface of superconductor. (The thickness of this layer is characterized by a penetration depth  $\lambda$ ). The supercurrents also flow within this layer.

Penetration depth depends on  $T$ :

$$\lambda(T) = \lambda_0 \left[ 1 - \left( \frac{T}{T_c} \right)^4 \right]^{-1/2}$$

constant

(Pb:  $\lambda_0 = 370 \text{ \AA}$ )



### (VI) Superconductors are diamagnetic.

By definition,

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

Inside SC,  $\vec{B} = 0$

$$\Rightarrow \vec{M} = -\vec{H} \quad (\text{diamagnetic})$$

or  $\vec{B} = \vec{B}_a + \vec{B}_i$

applied field      magnetization due to induced current

$\vec{B} = 0$  inside SC

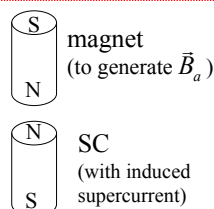
$$\Rightarrow \vec{B}_i = -\vec{B}_a$$

$\therefore$  SC can be levitated by a magnetic field! (a strong force)

Magnetization ( $\vec{M}$ ) is the magnetic moment per unit volume:

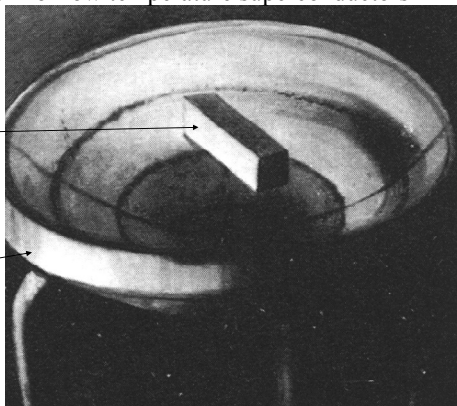
$$\vec{M} = \lim_{V \rightarrow 0} \frac{\sum_i \vec{m}_i}{V}$$

( $H$  is the applied magnetic field.)



### Magnetic levitation for low temperature superconductors

magnet  
low temperature superconductor at  $T < T_c$ .



(Note: We will see that magnetic levitation for high temperature superconductors is different.)

### (VII) Summary

Superconductor possesses

1. infinite conductivity ( $\rho = 0$ )
2. perfect diamagnetism (the Meissner effect  $\vec{B} = 0$ )

provided that

$T < T_c$  (critical temperature or transition temperature)

$H < H_c$  (critical field)

$I < I_c$  (critical current)

The transition is an example of **critical phenomenon**.

Many properties change at  $T_c$ : specific heat, thermo-emf, Hall effect, absorption of ultrasound, absorption of infrared, ...

### (VIII) Classification of superconductors

**Type I superconductors:** The superconductors that are metal elements except Nb & V. They have the properties listed in (VII).

**Type II superconductors:** Nb, V, alloys & compounds.

1930s: Schubnikow (Russian) discovered that some **Pb alloys** had  $H_c$  much higher than  $H_c(\text{Pb})$ . His paper was not noticed by the West due to the World War II.

1961: Scientists in Bell Lab discovered  $\text{Nb}_3\text{Sn}$  (an intermetallic compound) with high  $I_c$  and  $H_c$ .

These superconducting alloys and compounds have interesting properties: magnetic field can penetrate the material in a novel way.

(History: J.K. Hulm, J.E. Kunzler and B.T. Matthias, "The road to superconducting materials", Physics Today, Jan. 1981, pp.34-43.)

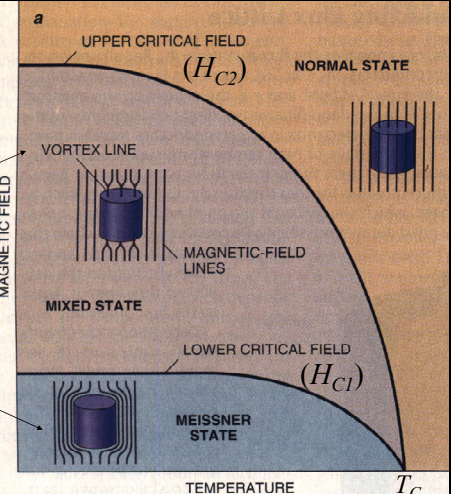
For Type II superconductor, there are two critical fields:  $H_{c1}(T)$  &  $H_{c2}(T)$

#### mixed state

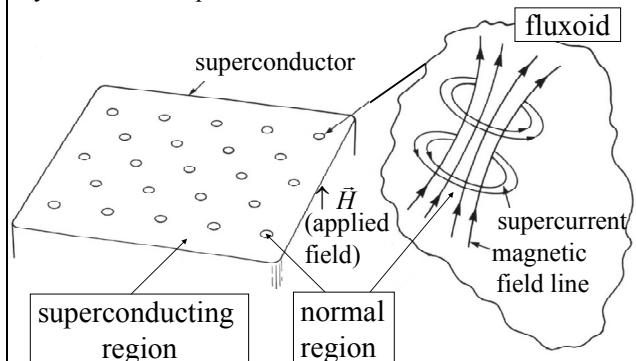
( $H_{c1} < H < H_{c2}$ ) is neither fully superconducting nor fully normal.

#### Meissner state

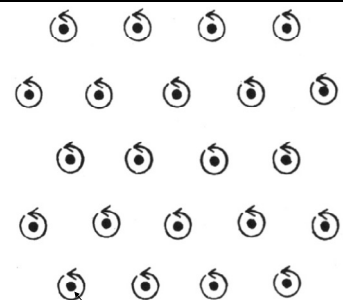
( $0 < H < H_{c1}$ ) complete Meissner state



In mixed state, magnetic field penetrates the superconductor in a form of regular triangular array of fluxoids (magnetic field line bundles). Each fluxoid contains a normal core, surrounded by a vortex of supercurrent.



**Vortices in mixed state** when viewed along  $\vec{H}$ :



The magnetic flux of each fluxoid:  $\phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15} \text{ T}\cdot\text{m}^2$

For an average magnetic field  $B$  in the sample, the number of fluxoids per unit area is  $B/\phi_0$ .

$\phi_0$  depends on  $h$ ; indicating that we need QM to understand its origin.

The mixed state allows very high  $H_{C2}$ :

	$B_{C2} = \mu_0 H_{C2}$ at $T = 0$	$T_C$
<b>Nb<sub>3</sub>Ge</b>	38 T	23 K
<b>Nb<sub>3</sub>Sn</b>	24.5 T	18 K
<b>V<sub>3</sub>Si</b>	23.5 T	16.9 K

Tesla

Like Type I superconductor, Type II superconductors have small  $H_{C1}$ .

Why is  $H_C$  of Type I superconductors or  $H_{C1}$  of Type II superconductors so small?

Recall: Magnetic field stores energy with energy density

$$B^2 / 2\mu_0$$

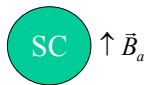
Consider a superconductor (of volume  $V$ ) in a uniform magnetic field  $\vec{B}_a$ .

In normal state, the energy of magnetic field within the superconductor is  $(B_a^2 / 2\mu_0)V$

In superconducting state (at  $T < T_C$ ),  $\vec{B} = 0$  inside SC.

$\Rightarrow$  Need to exclude energy  $((\mu_0 H_C)^2 / 2\mu_0)V$  out of superconductor.

That explains why  $H_C$  &  $H_{C1}$  are so small ( $\sim 10^2$ – $10^3$  Oe).



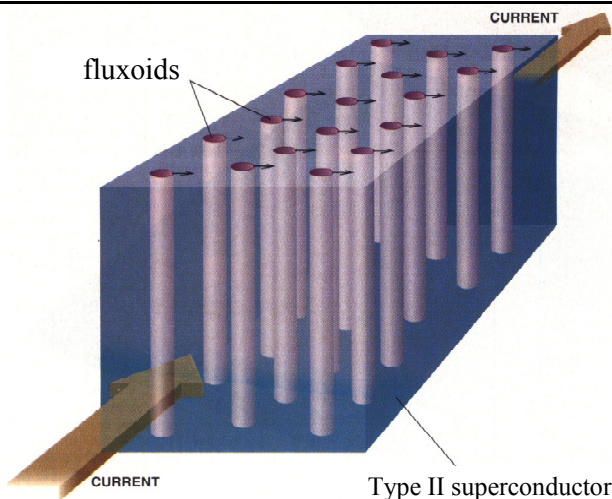
#### • Fluxoids can move around:

When a current is passing through a superconductor, it may hit the fluxoids and interact with the magnetic field in the fluxoids. Or the magnetic field of this current interacts with the supercurrents surrounding the fluxoids. As a result, the fluxoids can be moved by Lorentz force, resulting in energy loss & very small  $I_C$ .

#### • Flux pinning:

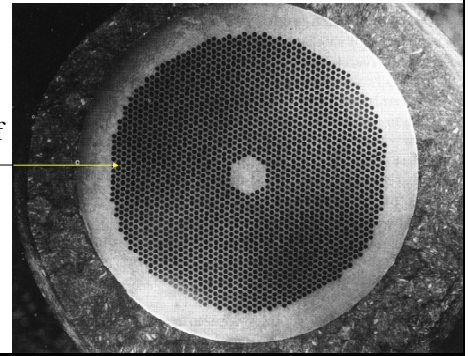
The fluxoids can be “pinned” (or fixed) by crystal imperfections (grain boundaries, defects, impurities, etc.), resulting in very high  $I_C$ .

• **Stable magnetic levitation demo** with high temperature superconductors (HTS) is due to strong pinning force.

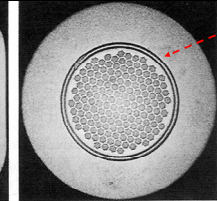
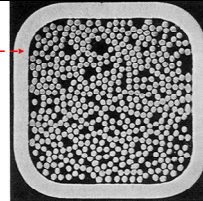


Immediately after superconductor Nb<sub>3</sub>Sn was found in 1961, high field superconducting magnet (SM) (Onnes’s dream) was built at General Electric using Nb<sub>3</sub>Sn, and later in other labs using superconductor Nb-Ti.

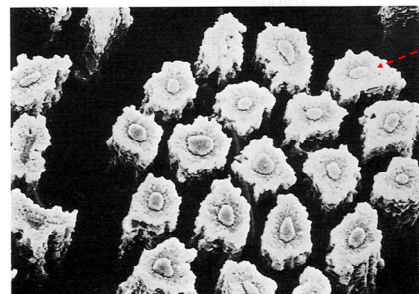
Modern SMs are built using superconducting cable composed of Ni-Ti filaments embedded in pure Cu.



Cable (composed of many wires)



Wire (composed of many filaments)



filament

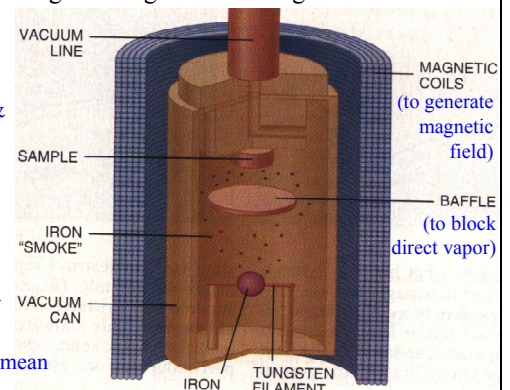
(The New Quantum Universe, Fig. 7.23)

**(IX) Decoration experiment** (used to visualize the fluxoids) (first done in 1966): like mapping the magnetic field distribution of a magnet using Fe dust in high school.

The sample (superconductor) must be in mixed state first ( $T < T_C$  &  $H_{C1} < H < H_{C2}$ ).

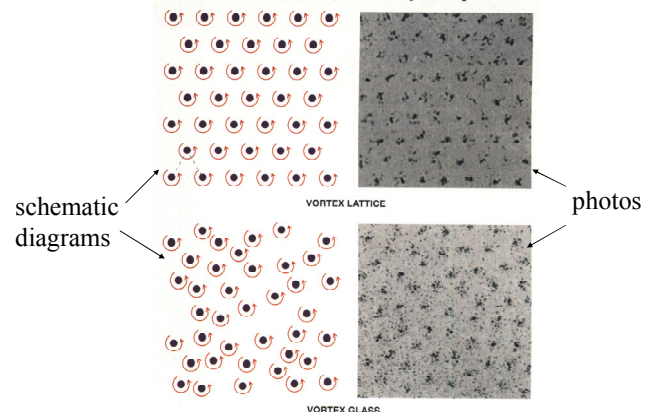
Iron vapor (“smoke”) is generated when the tungsten filament is heated.

Vacuum is required for large mean free path of vapor.



#### States of a Vortex Solid

A superconductor in a magnetic field “freezes” solid in two ways. If the material is clean, the vortex lines will fall into a regular triangular array, forming a vortex lattice. If the substance has many defects or impurities, the lines will develop a disordered pattern, forming a vortex glass.





### The flux quantum ( $\phi_0$ )

The flux quantum  $\phi_0 = h/2e$  is not just for Type II SC. It was measured by two groups using **Type I SC**:

1. Doll & Nābauer (1961) used a Pb tube (actually a Pb coating on a quartz rod) ( $T_C(\text{Pb}) = 7.19 \text{ K}$ ).
2. Deaver & Fairbank (1961) used a Sn tube (actually a Sn coating on a Cu wire) ( $T_C(\text{Sn}) = 3.72 \text{ K}$ ).

Both samples had diameter  $\approx 10 \mu\text{m}$ .

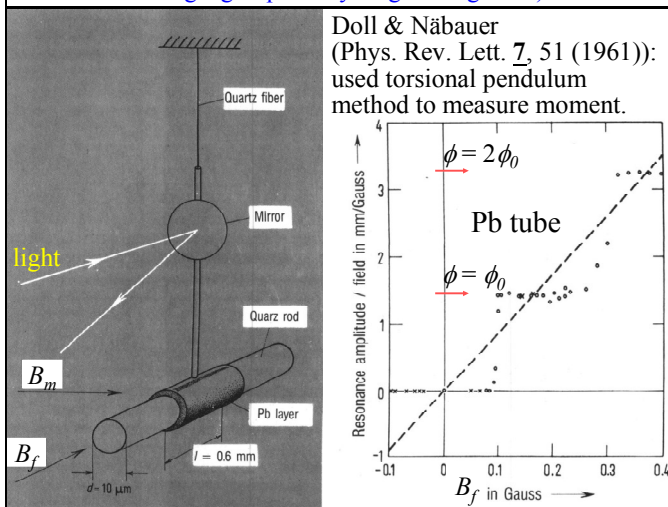
Their experiments are quite similar.

1. At  $T > T_C$ , a magnetic field  $B_f$  (called the freezing field) is applied along the axis of the tube (in normal state). Therefore the magnetic flux in the tube  $\neq 0$ .
2. Then cooled the tubes in  $B_f$  to  $T = T_0 < T_C$ .
3. Finally removed  $B_f$  at  $T_0$ . As a result, a persistent current is induced on the tube's **inner** surface. (Why?)

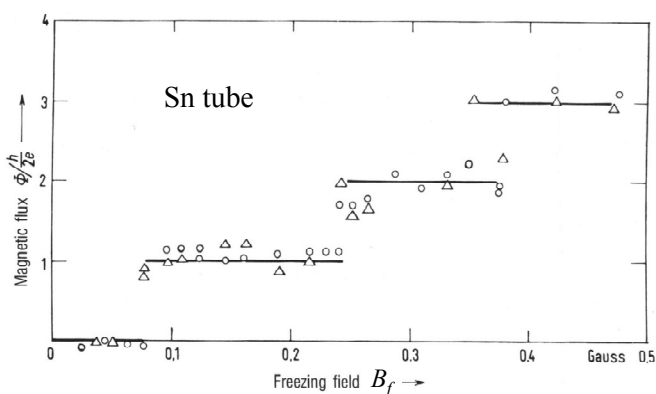
The magnetic flux trapped in the tube is adjusted automatically to become quantized:  $\phi = n \phi_0$ , where  $n$  is an integer.

(Note:  $\phi < B_f A$ , where  $A$  is the cross-section area of the tube.) In other words, the tube (in superconducting state) becomes a tiny magnet. The next step is to measure its magnetic moment, from which  $\phi$  is determined. Note:

- (1) The field is so small, even a Type II SC is used for this experiment, it is in Meissner state.
- (2) We can estimate the required  $B_f$  for these experiments:  
To get a flux  $= \phi_0 = 2 \times 10^{-15} \text{ T.m}^2$ ,  
the applied field is  $B_f = 2.5 \times 10^{-5} \text{ T}$ .  
Recall: Earth field  $\approx 5 \times 10^{-5} \text{ T}$ . (1 T =  $10^4$  Gauss)
- (3) In mixed state of type II superconductor, the magnetic flux of each fluxoid is  $\phi_0$  in order to minimize the total energy. (Why? Hint: treat each fluxoid as a tiny magnet and think about holding a group of tiny magnets together.)



Deaver & Fairbank (Phys. Rev. Lett. **7**, 43 (1961)):



### (X) Theories for superconductivity (Type I & Type II)

Questions:

- Why  $T_C$  so low?
- Is superconductivity simply the state of a conductor with very high purity?
- Why are good metals not superconducting?
- Why is the transition so abrupt?
- Do we need quantum mechanics (QM) developed in 20s-30s to explain superconductivity?  
Yes! Superconductivity is due to conduction electrons. Bloch (1928) used QM to show that conduction electrons in crystal move freely through the crystal like a wave (a quantum phenomenon).
- Why  $R = 0$ ?

#### (a) Cooper pair (1956)

Cooper assumed a weak attractive force between two conduction electrons at 0 K, & then showed theoretically that the two electrons:

one with  $\vec{p} \uparrow$  (momentum  $\vec{p}$  and spin up),

one with  $-\vec{p} \downarrow$  (momentum  $-\vec{p}$  and spin down)

can form “bound” state (just like one proton and one electron form a bound state, the hydrogen atom).

The weakly bound electron pairs (now called Cooper pairs) in the superconductor are boson & can move cooperatively without resistance.

**Note:** Since the momentum of the pair is well defined, the Cooper pair must spread out in space. (Uncertainty principle)

#### Properties of Cooper pair:

total charge =  $2e$  (Note:  $\phi_0 = h/2e$ )

total momentum = 0

and total magnetic moment = 0.

Similar to two-fluid model, Cooper pairs appear at  $T < T_C$  and increase in number as  $T$  is lowered. Only those electrons near  $E_F$  can form Cooper pairs.

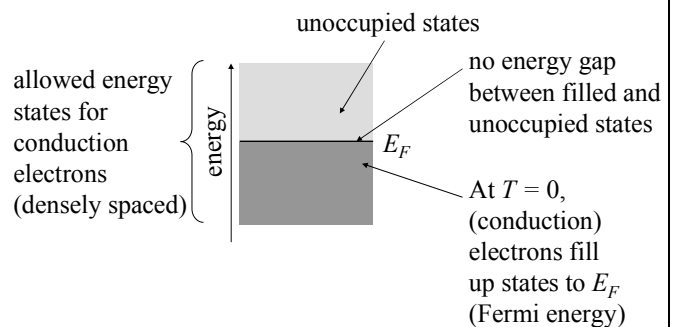
The Cooper pair has a binding energy per electron ( $\Delta(T)$ , called the pairing energy)  $\sim 10^{-4} - 10^{-3} \text{ eV}$ .

The energy required to break up a Cooper pair (to become two normal electrons) is  $2\Delta(T)$ .

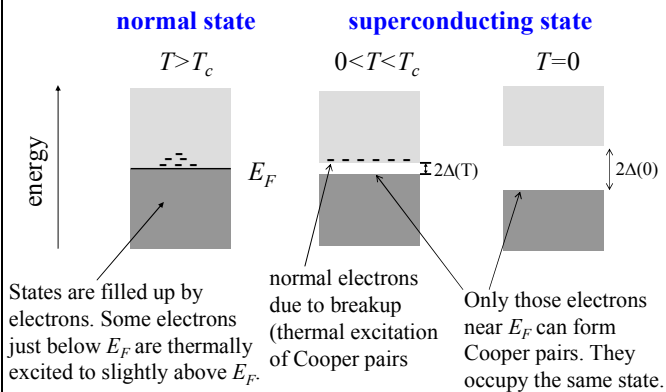
The two electrons in a Cooper pair are separated by few hundred Å. This size is much larger than the separation of two pairs. So all Cooper pairs overlap appreciably.

Cooper pair is the basis of the BCS theory.

#### Recall: Energy diagram of electrons in normal metals

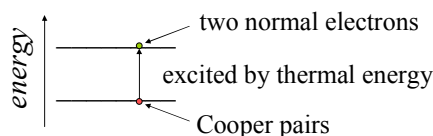


The binding energy for superconductor introduces an energy gap  $2\Delta(T)$  near the Fermi energy  $E_F \sim eV$   
 $2\Delta(0) \sim 10^{-4} E_F$



### Low $T_C$ is related to small energy gap $2\Delta(T)$ :

In superconducting state, the thermal energy of lattice  $k_B T < 2\Delta(T)$ , otherwise most Cooper pairs are destroyed by thermal energy.



Now  $2\Delta(0) \sim 1 \text{ meV}$  (see data shown later)  
 and for estimation, we take  $k_B T_C \sim 2\Delta(0)$   
 (More exact calculation gives  $3.5 k_B T_C = 2\Delta(0)$ )  
 $\Rightarrow T_C \sim 12 \text{ K}$ .

What is the state of superconducting electrons in a superconductor?  
 Late 40s – early 50s, it was believed that electrons should have a new type of interaction in addition to the well-known repulsive Coulomb force.



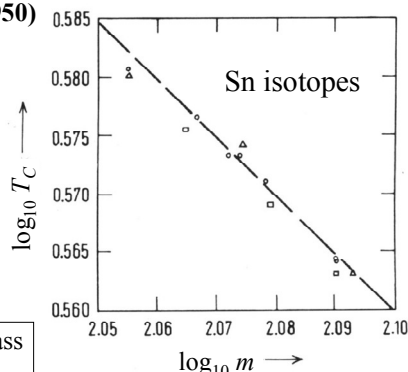
Fröhlich (1950) & Bardeen (1950) believed that the **lattice vibration** should play a major role. The idea was based on some experimental results.

### E.g., Isotope effect (1950)

For superconducting element with more than one isotope

$$T_C \propto m^{-1/2}$$

where  $m$  is the atomic mass of the isotope.



Recall: For a spring-mass system,

$$\omega = \sqrt{k/m}$$

This confirmed the importance of lattice vibration and the electron-lattice interaction should be considered.

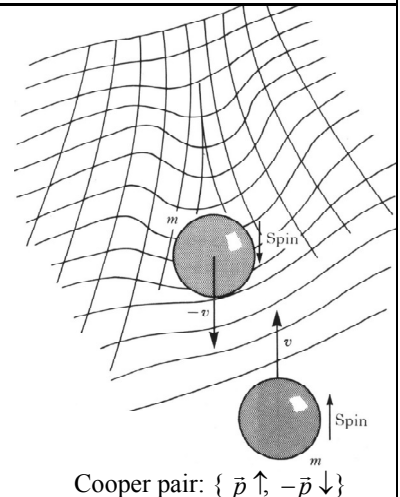
Why the electron-lattice interaction supplies the attractive force?

**“The mattress effect”**: When a bowling ball falls onto a mattress, it distorts the mattress (床褥). Another bowling ball nearby may roll toward the first ball because of this distortion. It appears that the two bowling balls attract each other.

Now the analogy:

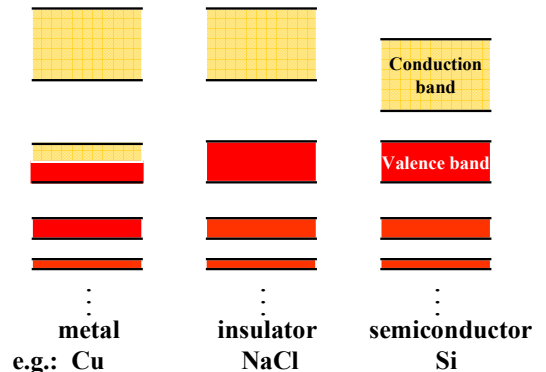
conduction electrons in superconductor  $\leftrightarrow$  bowling balls  
 crystal lattice of superconductor  $\leftrightarrow$  mattress

The crystal lattice is polarized by an electron, resulting in an effective positive charge to attract another electron. These two electrons form a bound state: the Cooper pair.

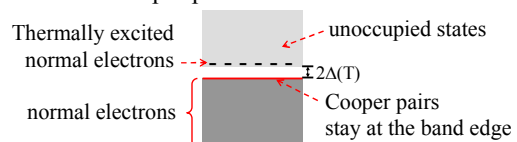


### Why $R = 0$ ?

Recall:



**Why  $R = 0$ ?** The following argument is quite tricky. We use the following energy diagram for both normal electrons and Cooper pairs:



First, because all Cooper pairs (bosons) have the same binding energy, they are all gathering at the band edge. If  $R \neq 0$  (with power dissipation), then when we apply an electric field, the Cooper pairs in the current can scatter with the lattice & change their states. But there is an energy gap above the original state.  
 $\Rightarrow$  Their states cannot be changed.  
 $\Rightarrow$  no scattering  $\Rightarrow R = 0$ .



### (b) BCS theory of superconductivity

Many best minds in physics (including Einstein) tried to understand superconductivity, but only 40 years after Onnes's discovery, the theory was established.

BCS theory (1957) was derived using second quantization technique (taught in QMII):

$$H_{BCS} = \sum_k \epsilon_k (c_k^\dagger c_k + c_{-k}^\dagger c_{-k}) - \sum_{kk'} V_{kk'} c_k^\dagger c_{-k}^\dagger c_{-k'} c_{k'}$$

They extended the Cooper model to all conduction electrons near  $E_F$  in the solid. The BCS theory explained successfully almost everything that was known about low-temperature superconductivity and provided the microscopic origin (i.e., the mechanism).

### Nobel Prize (1972)

John Bardeen  
(1908-1991)

Leon Neil  
Cooper  
(1930 - )

John Robert  
Schrieffer  
(1931- )



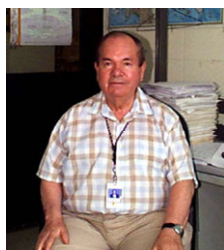
"for their development of a theory of superconductivity"

### Nobel Prize (2003)

Alexei A.  
Abrikosov

Vitaly L.  
Ginzburg

Anthony J.  
Leggett



"for pioneering contributions to the theory of superconductors and superfluids"

### (XI) Tunneling effect:

The energy gap  $2\Delta(T)$  can be determined by tunneling experiments.

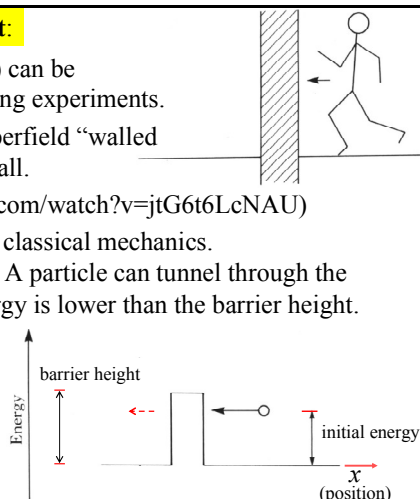
Magician David Copperfield "walled through" the Great Wall.

(<http://www.youtube.com/watch?v=jtG6t6LcNAU>)

This is not allowed in classical mechanics.

Quantum mechanics: A particle can tunnel through the barrier even if its energy is lower than the barrier height.

It is due to uncertainty principle.



The tunneling effect can be observed using thin film samples:

NIN, SIN or SIS junctions, where

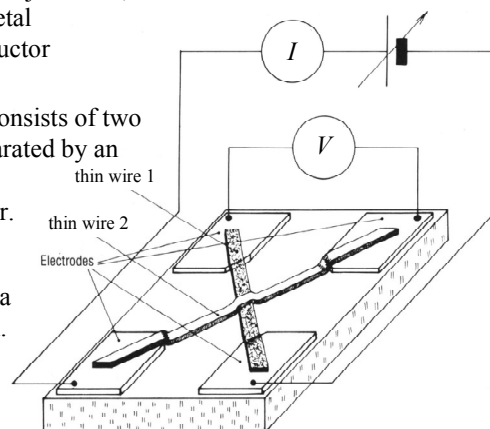
N = normal metal

S = superconductor

I = insulator.

Each sample consists of two thin wires separated by an ultrathin insulating layer.

Each wire has a width  $< 1$  mm.



Consider **NIN junction** with layer thicknesses:

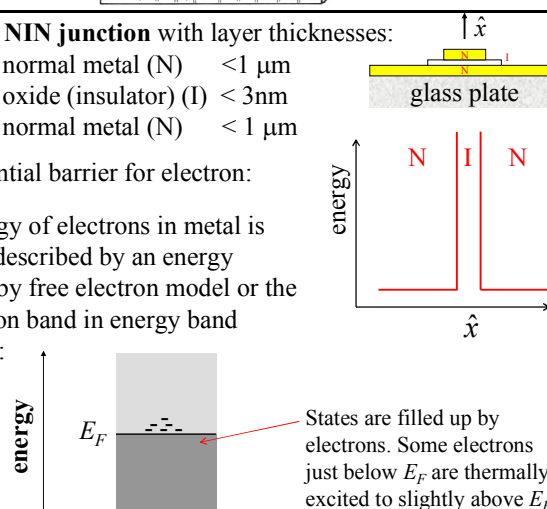
layer 1: normal metal (N)  $< 1 \mu\text{m}$

layer 2: oxide (insulator) (I)  $< 3\text{nm}$

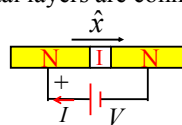
layer 3: normal metal (N)  $< 1 \mu\text{m}$

The potential barrier for electron:

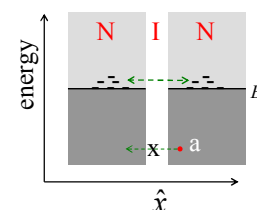
The energy of electrons in metal is limited (described by an energy diagram by free electron model or the conduction band in energy band diagram):



The metal layers are connected to a voltage source.



The energy diagram of the sample as a function of position  $x$  for  $V = 0$ :



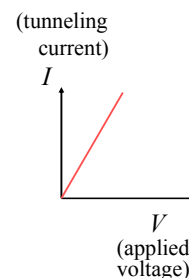
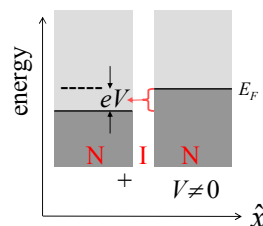
Not all electrons can tunnel through the barrier, e.g., electron **a** cannot tunnel because there is no available state at the same energy in the left layer.

(We will ignore the lower parts of the energy diagrams because no tunneling can occur in this region.)

Tunneling can occur for electrons above  $E_F$  but  $I = 0$ . Why?

When a voltage  $V$  is applied, the energy levels are shifted. (Why?)

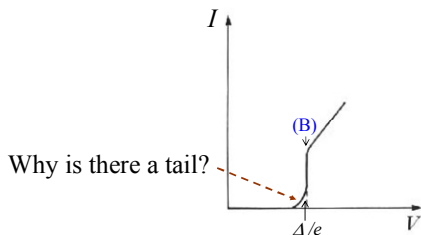
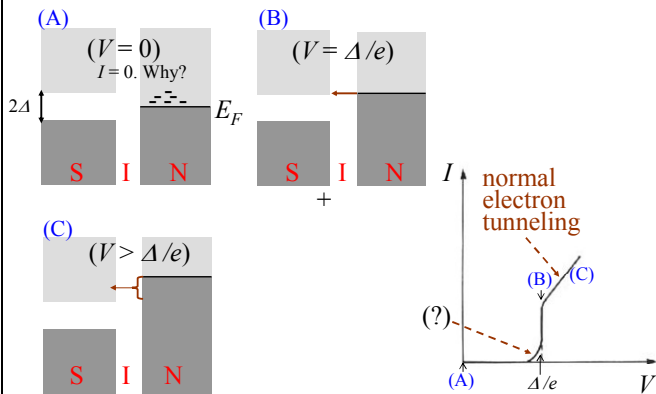
(Here  $e > 0$ ,  $V > 0$ )



Electrons in the right layer with energy  $E_F - eV \leq E \leq E_F$  can tunnel through the insulating layer into the left layer.

Result: Tunneling current increases linearly with increasing  $V$ . Tunneling is enhanced by electric field.

**SIN junction:** layer 1: superconductor (S)  
layer 2: oxide (insulator) (I)  
layer 3: normal metal (N)

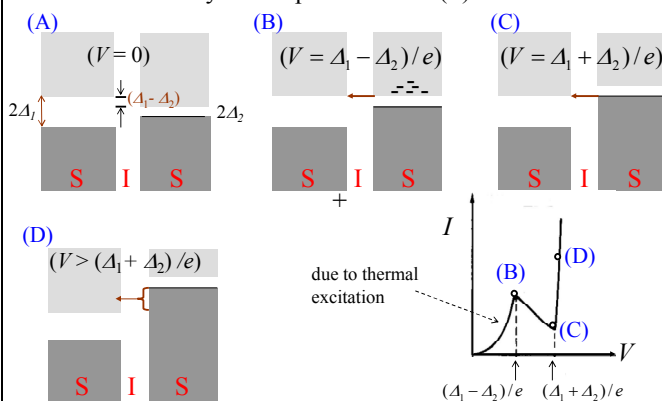


Recall: At  $T > 0$ , some electrons can be found with  $E > E_F$  due to thermal excitation. The tunneling current in the tail is due to these electrons.

The tail is weaker at lower temperature.

See curves 2 & 3 of data for Al-Al<sub>2</sub>O<sub>3</sub>-Pb junction below.

**SIS junction:** layer 1: superconductor (S)  
layer 2: oxide (insulator) (I)  
layer 3: superconductor (S)



e.g. Al-Al<sub>2</sub>O<sub>3</sub>-Pb junction  
is an SIS junction.  
 $T_C$  (Pb) = 7.2 K  
 $T_C$  (Al) = 1.2 K

$I$ - $V$  curves were measured at  
different temperature  $T$ :

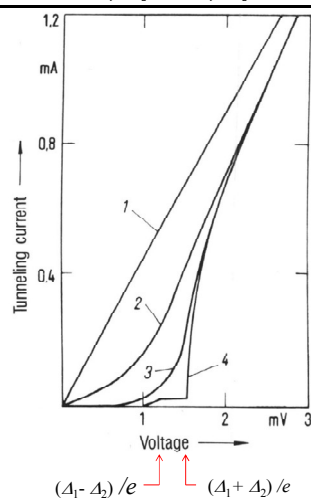
curve 1:  $T = 10$  K  
(NIN junction)

curve 2:  $T = 4.2$  K  
(SIN junction)

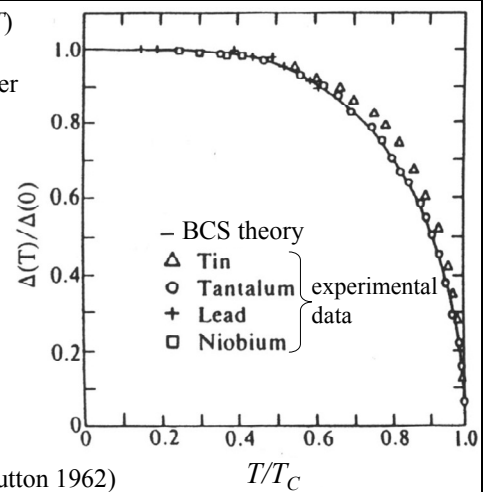
curve 3:  $T = 1.64$  K  
(SIN junction)

curve 4:  $T = 1.05$  K  
(SIS junction)

(Giaever & Megerle 1961)



Energy gap  $2\Delta(T)$   
can also be  
measured by other  
methods.  
 $\Delta(T)$  depends on  
temperature:

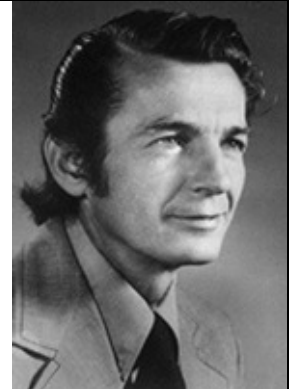


(Townsend & Sutton 1962)

**Nobel Prize (1973)**

Ivar Giaever (1929 - )

“for his experimental  
discoveries regarding  
tunneling phenomena in  
superconductors”



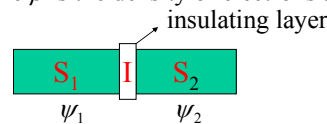
You will enjoy reading his Nobel lecture:

[www.nobel.se/physics/laureates/1973/giaever-lecture.html](http://www.nobel.se/physics/laureates/1973/giaever-lecture.html)

## (XII) The Josephson effect (1962)

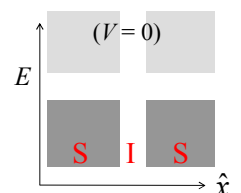
Josephson further explored the theory for an SIS junction.

The two superconductors are the same, i.e.,  $S_1 = S_2$  &  $\Delta_1 = \Delta_2$ .  
At that time, people already knew that all Cooper pairs are in  
the same QM state, described by a single wavefunction (a  
complex number):  $\psi(\vec{r}) = \sqrt{\rho}e^{i\theta}$   
where  $\rho$  is the density of electrons and  $\theta$  is the phase.



In superconductor  $S_1$ :  $\psi_1(\vec{r}) = \sqrt{\rho_1}e^{i\theta_1}$

In superconductor  $S_2$ :  $\psi_2(\vec{r}) = \sqrt{\rho_2}e^{i\theta_2}$



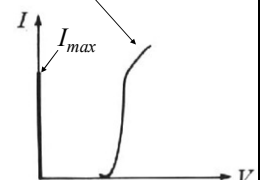
When a small  $U_0$  is applied, a dc  
supercurrent  $I$  tunnels through the  
barrier with  $V = 0$ , and its direction  
depends on the polarity of  $U_0$ .

$$I = I_{\max} \sin(\theta_2 - \theta_1)$$

$I$  increases for higher  $U_0$  until  $I = I_{\max}$ .  
For even higher  $U_0$ ,  $V \neq 0$   
and the  $V$ - $I$  curve is

This is the dc Josephson effect.

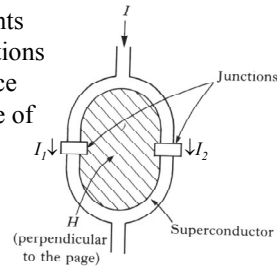
(For a simple derivation, see The  
Feynman lectures on physics, vol. 3,  
p.21-14, 1963.)



Josephson also predicted the generation of a high frequency alternating current at the junction: the frequency  $\nu_J = V/\phi_0$ . This ac Josephson effect was confirmed by Giaever (1965). The Josephson effect is a quantum behavior seen in macroscopic object (the superconductor).

SQUID (superconducting quantum interference device) is composed of two Josephson junctions. The two currents ( $I_1, I_2$ ) passing through the two junctions can have very interesting interference behavior in magnetic field. Because of this exotic property, SQUID is an extremely sensitive magnetic field sensor.

(see applications)



### Nobel Prize (1973)

Brian David Josephson  
(1940 - )

“for his theoretical predictions of the properties of a supercurrent through a tunnel barrier, in particular those phenomena which are generally known as the Josephson effects”



### (XIII) Limit of $T_C$ and new surprises

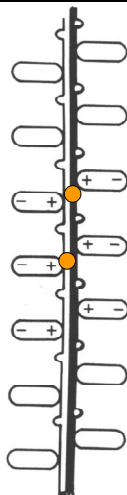
- BCS theory was further perfected so that given the energy spectrum of lattice vibration and the electronic properties of a material, one can predict the value of its  $T_C$ .
- An example is McMillan equation (1968) from which the maximum  $T_C$  was estimated to be about 30-40 K. Before 1986, the record  $T_C$  was found to be 23 K in  $Nb_3Ge$ .
- By 80s, people believed that superconductivity is a “mature” physics!
- Is high  $T_C$  superconductivity possible?  
Some people are optimistic.  
Theorists looked for new superconducting mechanisms.  
Experimentalists searched for new materials.

### (a) New mechanisms

Little (1964) considered a hypothetical 1D organic molecule with side branches. A conduction electron polarizes the side-branches and induces positive charges at the ends near the spine. A second electron is attracted to this region of positive charge and is therefore indirectly attracted to the first electron. The polarization is due to electron-electron interaction and thus

$$T_C \sim m_e^{-1/2} \sim \text{room temperature.}$$

mass of electron  $\ll$  mass of ion  
(Ref.: Scientific American, Feb 1965, p.21)



### (b) New materials

#### (i) Metallic hydrogen

$H_2$  is the most common element in the universe.

- 1898 liquefied hydrogen (20 K)
- 1899 solidified hydrogen (14 K) } insulating
- 1935 Wigner & Huntington predicted the existence of metallic hydrogen at 250 kbars pressure.  
(Scientific American, May 2000, pp.60-66.)
- 1960s Ashcroft predicted that metallic hydrogen would be a room temperature superconductor.  
(Physics World **8**(7), p.43 (1995).)

#### (ii) USO (unidentified superconducting objects)

- 1973 TTF-TCNQ (an organic crystal):  
“superconducting fluctuations at 60 K”
- 1978 CuCl (a semiconductor): “diamagnetic anomaly at high pressure at 150 K” (Rusakov & Chu)
- 1980 CdS: “flux exclusion at 77 K”
- 1980 TiB: “zero resistance at room temperature”
- 2000



Physics C 341-348 (2000) 723-725



www.elsevier.nl/locate/physc

Possible RT superconductivity from  $Pb_3Co_3Ag_3O$  (PACO) system

D. Djurek<sup>a</sup>, Z. Medunski<sup>a</sup>, A. Tosić<sup>b</sup> and M. Paljević<sup>c</sup>

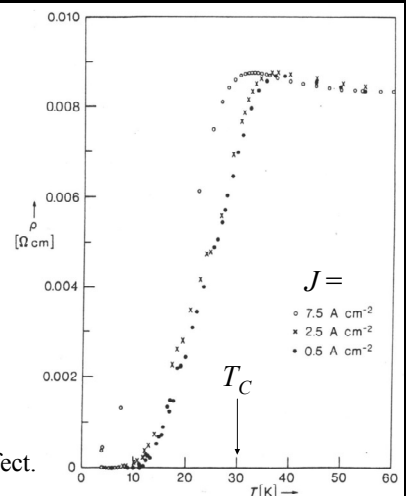
<sup>a</sup>A. Volta Applied Ceramics (AVAC), Kesten brijeg 5, HR-10080 Zagreb, Croatia

#### (iii) HTS

##### (High temperature superconductors)

J.G. Bednorz & K.A. Müller, “Possible high  $T_C$  superconductivity in the Ba-La-Cu-O system”, Z. Phys. B**64**, p.289 (1986).

No data on Meissner effect.



A few groups repeated their findings and achieved higher  $T_C$  at high pressure. This suggested new superconductors could be found by chemical substitution. Chu then became famous:

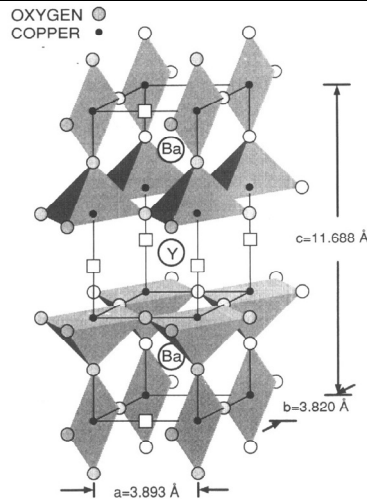
VOLUME 58, NUMBER 9 PHYSICAL REVIEW LETTERS 2 MARCH 1987

#### Superconductivity at 93 K in a New Mixed-Phase Y-Ba-Cu-O Compound System at Ambient Pressure

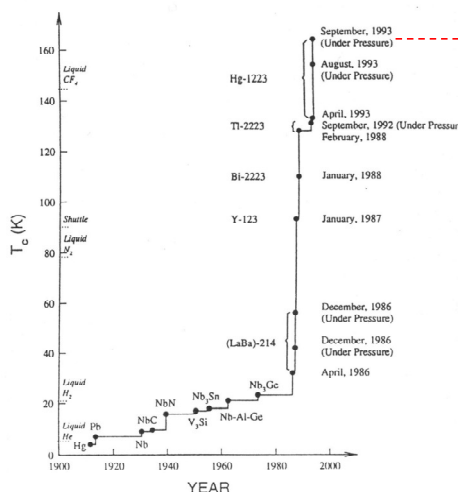
- 3 → M. K. Wu, J. R. Ashburn, and C. J. Torng  
Department of Physics, University of Alabama, Huntsville, Alabama 35899
- 4 → P. H. Hae, R. L. Meng, L. Gao, Z. J. Huang, Y. Q. Wang, and C. W. Chu<sup>(a)</sup>  
Department of Physics and Space Vacuum Epitaxy Center, University of Houston, Houston, Texas 77004  
(Received 6 February 1987; revised manuscript received 18 February 1987)
- 5 → A stable and reproducible superconductivity transition between 80 and 93 K has been unambiguously observed both resistively and magnetically in a new Y-Ba-Cu-O compound system at ambient pressure. An estimated upper critical field  $H_{c2}(0)$  between 80 and 180 T was obtained.
- 6 → I: exceptionally rapid publishing.
- 2: Y or Yb?
- 3: Who (Wu or Chu) discovered YBCO?  
(C.W. Chu, IEEE Tran. Appl. Sup. Vol. 7, p.80 (1997).)



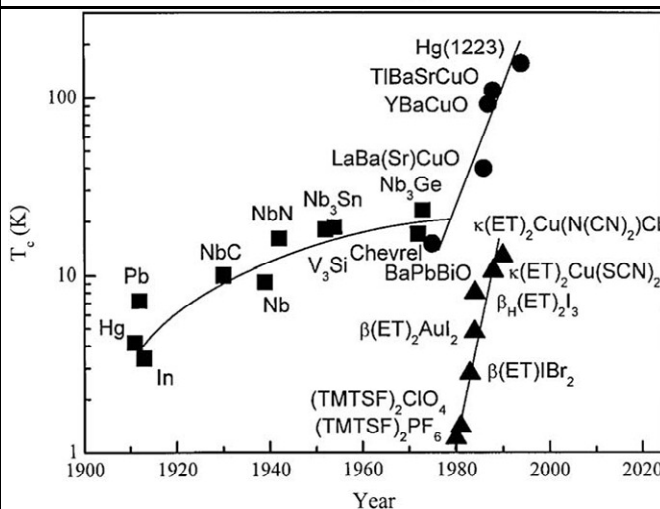
The 90 K superconductor was identified as  $\text{YBa}_2\text{Cu}_3\text{O}_7$  with a perovskite crystal structure



Many more HTS were found.



**Record  $T_c$  = 164 K**  
( $\text{HgBa}_2\text{Ca}_2\text{Cu}_3\text{O}_8$  at 30 kbars hydrostatic pressure.)

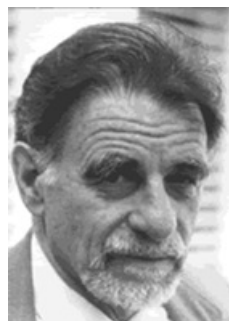


#### Nobel Prize (1986)

J. Georg Bednorz  
(1950 - )



K. Alexander Müller  
(1927 - )



"for their important breakthrough in the discovery of superconductivity in ceramic materials"

#### (iv) New Surprises

(a) **More superconducting elements at high pressures:**

S (1997), O (1998), B (2001), Fe (2001), Li (2002)

(b) **Ferromagnetic superconductors:** Superconductivity is usually destroyed by high magnetic field but materials were discovered with coexistence of superconducting and ferromagnetic properties.

2001: Fe at high pressure (Nature 406, p.316)

2001: ZrZn<sub>2</sub> alloy (Nature 412, p.58)

2001: UGe<sub>2</sub>, URhGe (Nature 413, p.613)

2000:  $\kappa$ -(BEDT-TTF)<sub>2</sub>Cu[N(CN)<sub>2</sub>]Br, magnetic-field-induced superconductivity in organic material. (Phys. Rev. Lett. 85, p.5420)

(c) Many semiconductors, oxides, hydrides, organic materials were found to be superconducting.

Figure 1 Superconductors under pressure. The colour code of this periodic table (adapted from ref. 13) shows elements that superconduct under normal, atmospheric pressure conditions (purple) and those that superconduct when subjected to high pressure (orange). Shimizu *et al.*<sup>1</sup> confirm the superconductivity of lithium at high pressure, bringing the number of such elements to 23. Under normal pressure conditions, 29 elements are superconductors.

(d) **New non-copper high temperature superconductors:**

2001:  $\text{MgB}_2$  (Jun Akimitsu)

- $T_c = 39$  K

- $\text{MgB}_2$  is available commercially. (Physics World, Jan.2002, p.29)

2008: Fe-based superconductors

$\text{SmFeAsO}_{1-x}\text{F}_x$

- $T_c = 43$  K

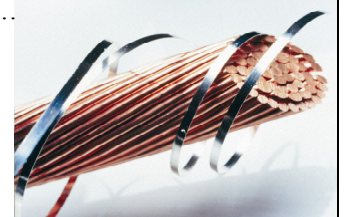
#### (XIV) Applications

(This part will not be covered in final exam.)

(1) High current applications (based on  $R = 0$ ): electricity transmission, energy storage, ...

(2) High field applications (based on superconducting magnet): MRI, Maglev, motor, generator, accelerators, research equipment, ...

(3) Josephson applications (based on Josephson effects): SQUID, supercomputers, ...



(Scientific American, Feb. 1989, p.45.)



### (a) Superconducting magnet (SM)

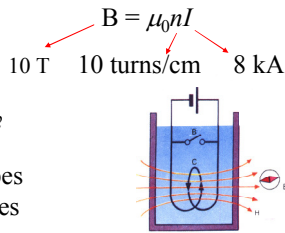
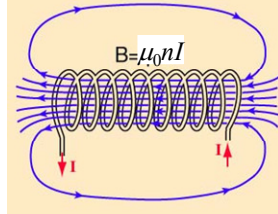
Onnes' dream: to build a 10 T SM.  
SM is a solenoid of SC wire.

Features of SM:

1. generating a stable and uniform magnetic field
2. light weight

What SC is required?

1. Type II
2. with strong pinning force
3. with high  $T_C$ , high  $I_C$ , high  $H_{C2}$
4. operated at  $T \ll T_C$  (why?)
5. can be shaped into wires or tapes
6. with good mechanical properties



Commercial SM: e.g. 15 T, 5 cm bore

1968: RCA developed a 14 T magnet using their superconducting  $Nb_3Sn$  tapes.  
(See also "Superconducting magnets above 20 Tesla", Physics Today, Aug. 2002, p.37)

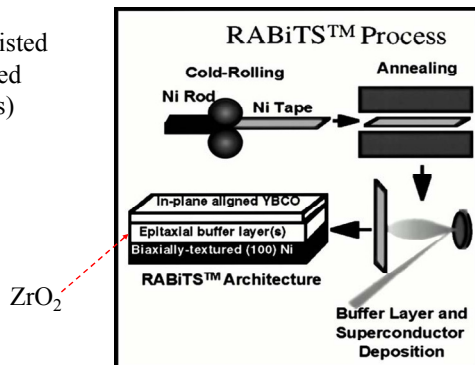


### HTS cable or tape:

The most successful process for producing long-length tapes for high current and high field applications at 77 K is called

#### RABiTS

(the Rolling-Assisted Biaxially Textured Substrate process)



### Status of HTS cable/wire/tapes:

1995: The tapes were only 4 cm long .

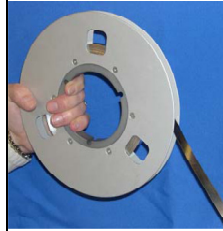
$J_C \sim 106 \text{ A/cm}^2$  (77 K)  
(Science **268**, 644 (1995))

2002: The tapes could not be made longer than 1 m.

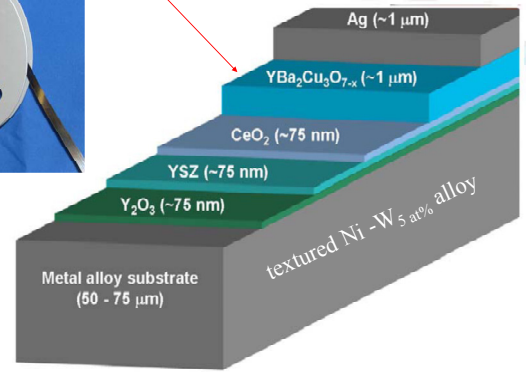
2006:  $I < 140 \text{ A}$  at 77 K for wire 100 m long and 4 mm wide  
(<http://www.amsuper.com>)

2009: National High Magnetic Field Laboratory at Florida State University was awarded nearly US\$ 3 million to build a high-temperature superconducting magnet (32 T with  $\sim 8 \text{ km}$  of HTS cable).

American Superconductor's RABiTS 2G HTS tape  
(ribbon-shaped YBCO tape)  
(2nd generation)



10 m spool



### Note: Other high field magnets:

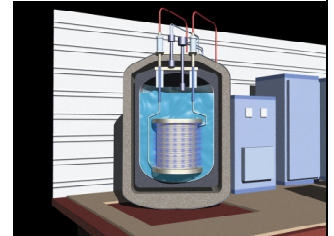
Bitter magnets (for  $B$  up to 33 T)

Hybrid magnets (Bitter + superconducting)  
(for  $B$  up to 45 T)

Pulsed magnets

(for  $B$  up to 70 T)

For an introduction, visit <http://www.magnet.fsu.edu/magnettechnology/research/magnetprojects/index.html>



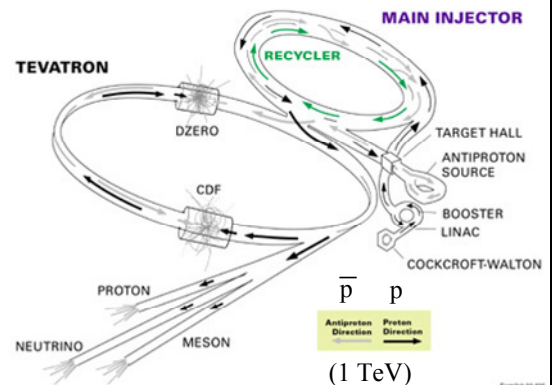
### Applications of SM:

#### (i) Particle accelerator for high energy experiments:

e.g. Tevatron  
(proton-antiproton collider) in Fermi Lab near Chicago in USA

Circumference of the circular accelerator is  $\sim 6.4 \text{ km}$ .

(<http://www.fnal.gov/>)



Need about 1000 SMs to keep the high energy particles moving in the circular accelerator.



SMs for Main Injector

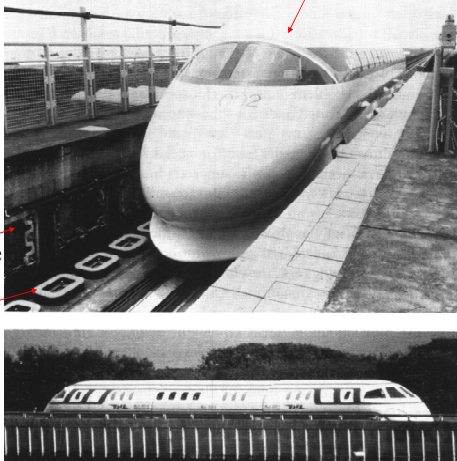
## (ii) Maglev (磁浮火車) in Japan

SMs on board

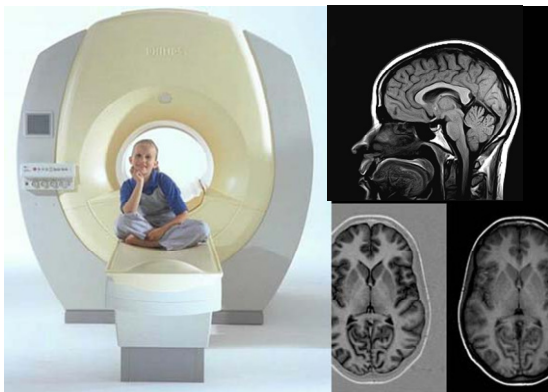
Speed record:  
518 km/h

Coils for guidance

Coils for  
levitation using  
Faraday's law.



## (iv) Magnetic Resonance Imaging (MRI) (1977)



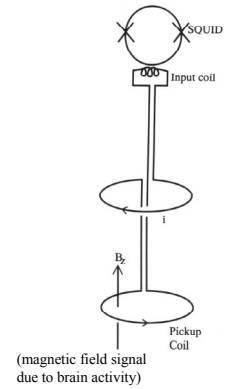
Require a strong magnetic field (0.5 – 2 Tesla), usually generated by SM.  
(Physics World Dec. 2002, pp.31-35.)

Stronger field  $\Rightarrow$  higher resolution of images  
Here is a high field MRI (4 Tesla) (for research only):

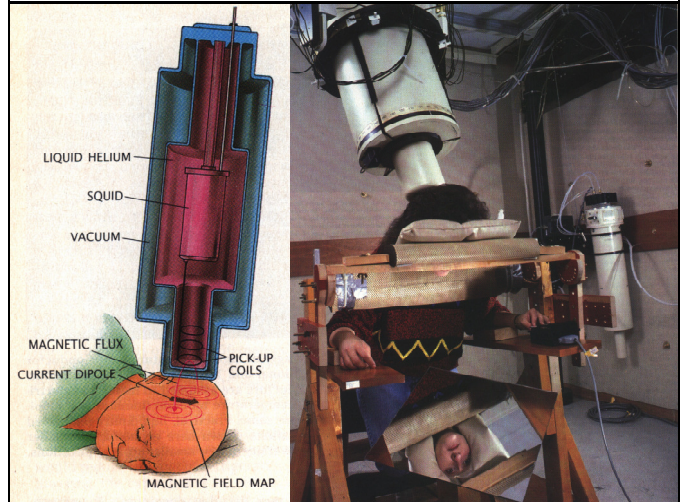


## (b) SQUID for brain research

SQUID is a sensitive magnetic field sensor.



(Scientific American, Aug. 1994, p.36.)



**Note:**  
**Magnetic field level**

