PHY 4910 (Radiation Lab)

(CUHK 2000)

Verification of Special Relativity and β spectroscopy

Introduction

1. Special relativity

The total energy of a particle is given by

$$E = mc^2 = T + E_o$$

and its momentum is

$$p = mv$$

where T = kinetic energy,

E_o = rest energy = $m_o c^2$

m_o = rest mass,

v = particle velocity,

c = velocity of light.

Therefore

$$m = \frac{m_o}{\sqrt{1 - v^2/c^2}}$$

and $$E^2 = (pc)^2 + E_o^2 \qquad [1]$$

The last equation can be rewritten as

$$\frac{p^2 c^2}{2T} = \left(\frac{T}{2}\right) + m_o c^2 \qquad [2]$$

We plot equation [1] in Fig. 1.

In the classical limit,

$$T = p^2/2m_o$$

In the extreme relativistic limit,

$$E = pc$$

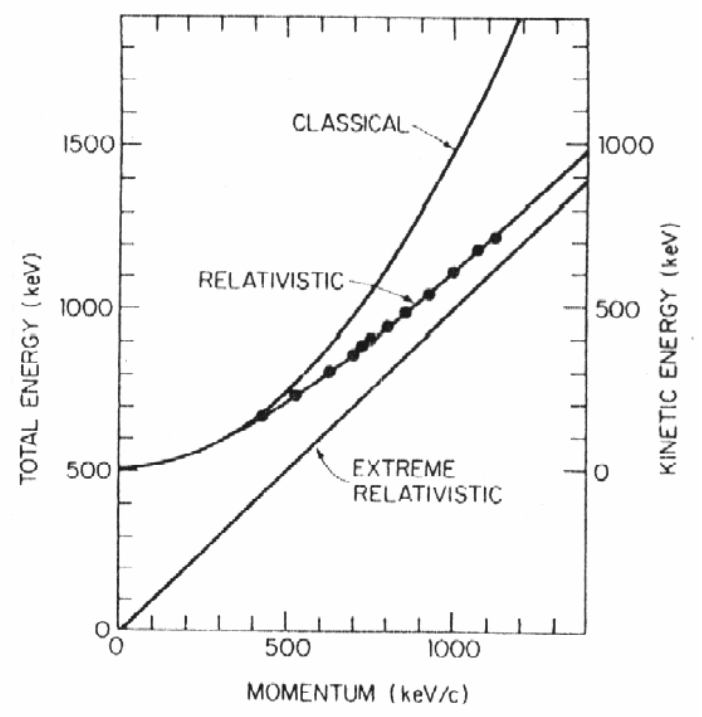

Figure 1 Energy vs. momentum (Ref.1)

2. *Source of Relativistic electrons*

To verify these equations, we need relativistic electrons which travel with speed v close to c. Such electrons are provided by many radioactive sources that undergo β decays. As described in Appendix 2, there are three types of β decay : β^+, β^- and EC. We will use a β^- source which generates a continuous spectrum of relativistic electrons. The electron spectrum is described by the Fermi's theory of β decay (Fermi 1934).

3. *Momentum spectrum of electrons (Fermi's theory)*

For β decay (${}^{A}_{Z-1}X \rightarrow {}^{A}_{Z}Y + e^- + \bar{\nu}_e$), the probability of emitting a β^- particle with momentum p_e within dp_e per unit time is given by Fermi's Golden Rule #2:

$$N(p_e)dp_e = \frac{2\pi}{\hbar}|H_{fi}|^2 \frac{dn}{dT_o} \qquad [3]$$

where H_{fi} = matrix element for transition from the initial state i (i.e. ${}^{A}_{Z-1}X$) to the final state f (i.e. ${}^{A}_{Z}Y + e^- + \bar{\nu}_e$),

dn/dT_o = density of final states available for the decay,

$T_o = T_e + T_\nu$

= end point energy = energy available for the decay,

T_e = electron energy, $(T_e)_{max} = T_o$ and

T_ν = neutrino energy, $(T_\nu)_{max} = T_o$.

It can be shown that for many β decays, equation [3] can be expressed as

$$N(p_e) \propto F(Z, p_e)\ p_e^2 (T_o - T_e)^2 \qquad [4]$$

where Z is the charge of the product nucleus after decay.

The factor $F(Z, p_e)$ takes care of the Coulomb effect between the emitted β particle and the decaying nucleus. In β^- decay, the emitted e^- is held back while in β^+ decay, the emitted e^+ is repelled from the positively charged nucleus. The effect is especially significant for low energy electrons leading to an enhancement of $N(p_e)$ for the former case and a depletion for the later case. $F(Z, p_e)$ is plotted in Fig. 2 and tabulated in some books.

Additional correction factors may be required for long-half-life radioactive nuclei. But we will not go into the detail here.

4. *Energy spectrum* $N(T_e)$:

$$N(T_e)\, dT_e = N(p_e)\, dp_e$$

$$\propto F(Z, p_e)\, p_e\, (T_o - T_e)^2\, (T_e + m_e c^2)\, dT_e \qquad [5]$$

Fig. 3 shows the momentum and energy spectra for the β decay of ^{64}Cu.

In practice, it is easier to measure $N(T_e)$ than $N(p_e)$. But you can convert $N(T_e)$ to $N(p_e)$ using the last two equations.

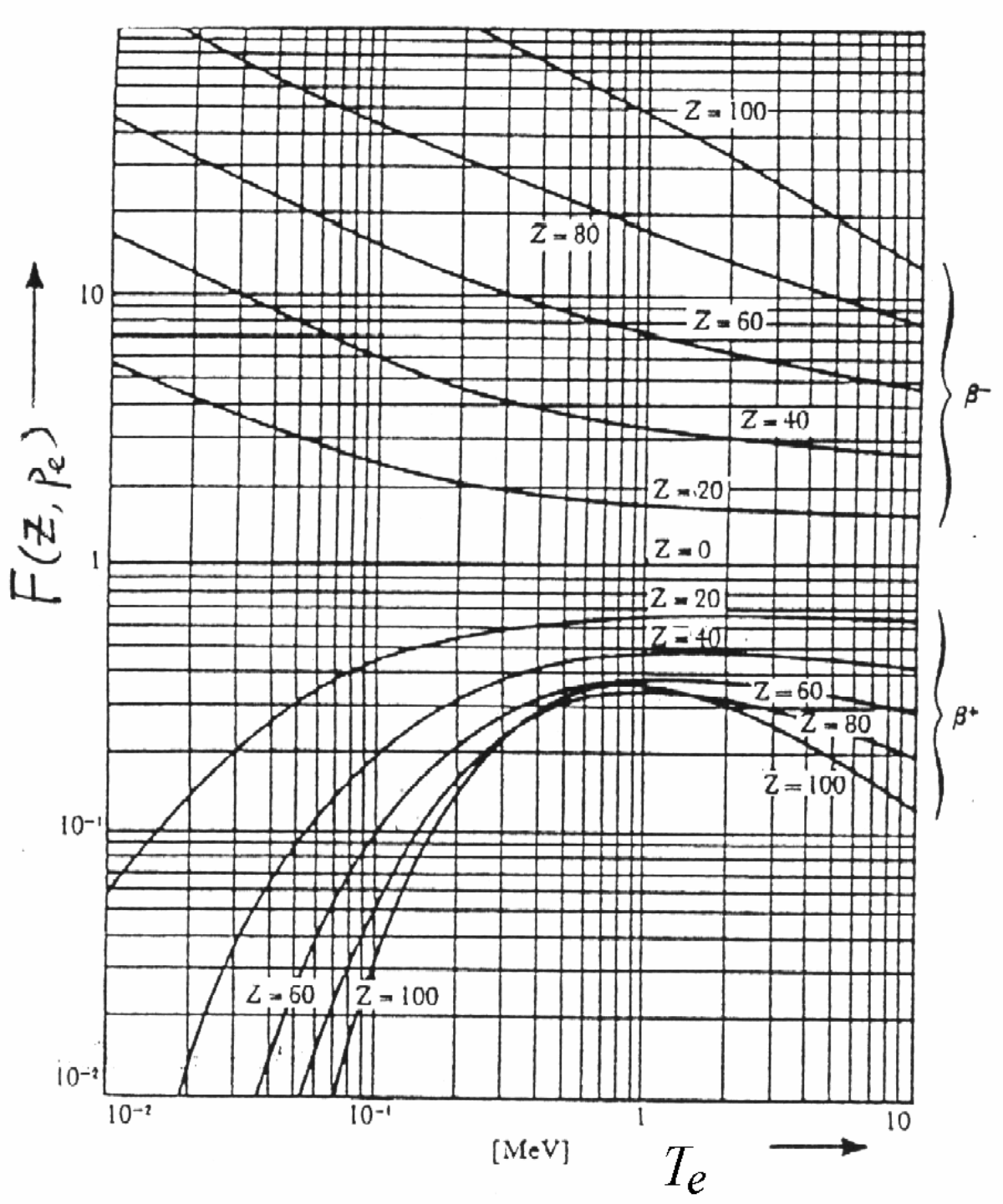

Fig. 2 Fermi function F(Z, p_e) vs T_e for β electrons and for Z = 0-100. (Ref. 2)

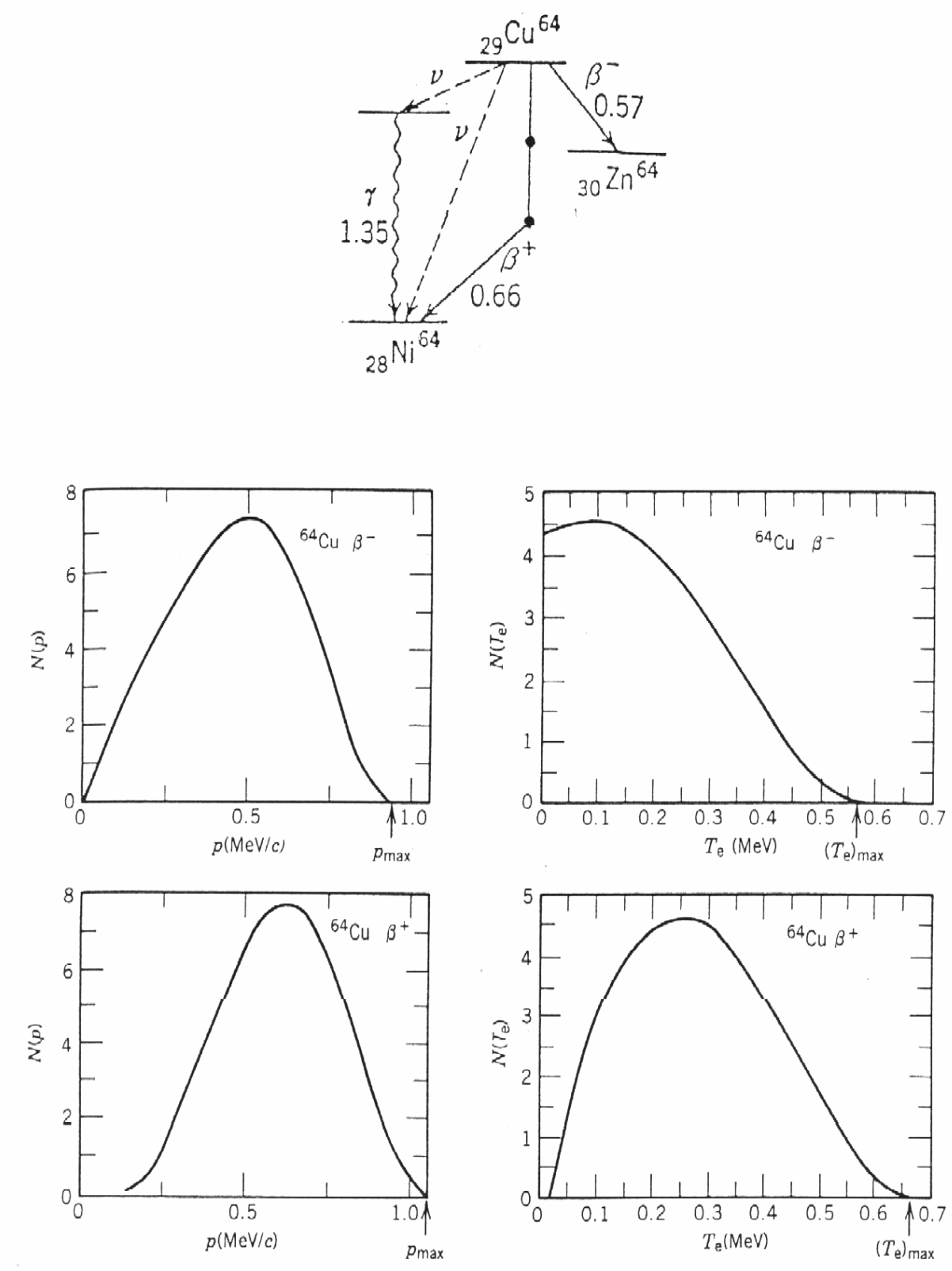

Fig. 3 Momentum and kinetic energy spectra of electrons and positrons emitted in the decay of ^{64}Cu. (Ref. 2)

5. *Kurie plot*

Equations [4] and [5] can be rewritten as

$$\left|\frac{N(p_e)}{F(Z, p_e)\ p_e^2}\right|^{1/2} \propto (T_o - T_e) \qquad [6]$$

$$\left|\frac{N(T_e)}{F(Z, p_e) p_e (T_e + m_e c^2)}\right|^{1/2} \propto (T_o - T_e) \qquad [7]$$

Therefore a plot of $\left|N(p_e) / (F\ p_e^2)\right|^{1/2}$ or $\left|N(T_e) / (F\ p_e E)\right|^{1/2}$ vs T_e gives a convenient way to verify the Fermi's theory and to accurately determine T_o as illustrated in Fig. 4.

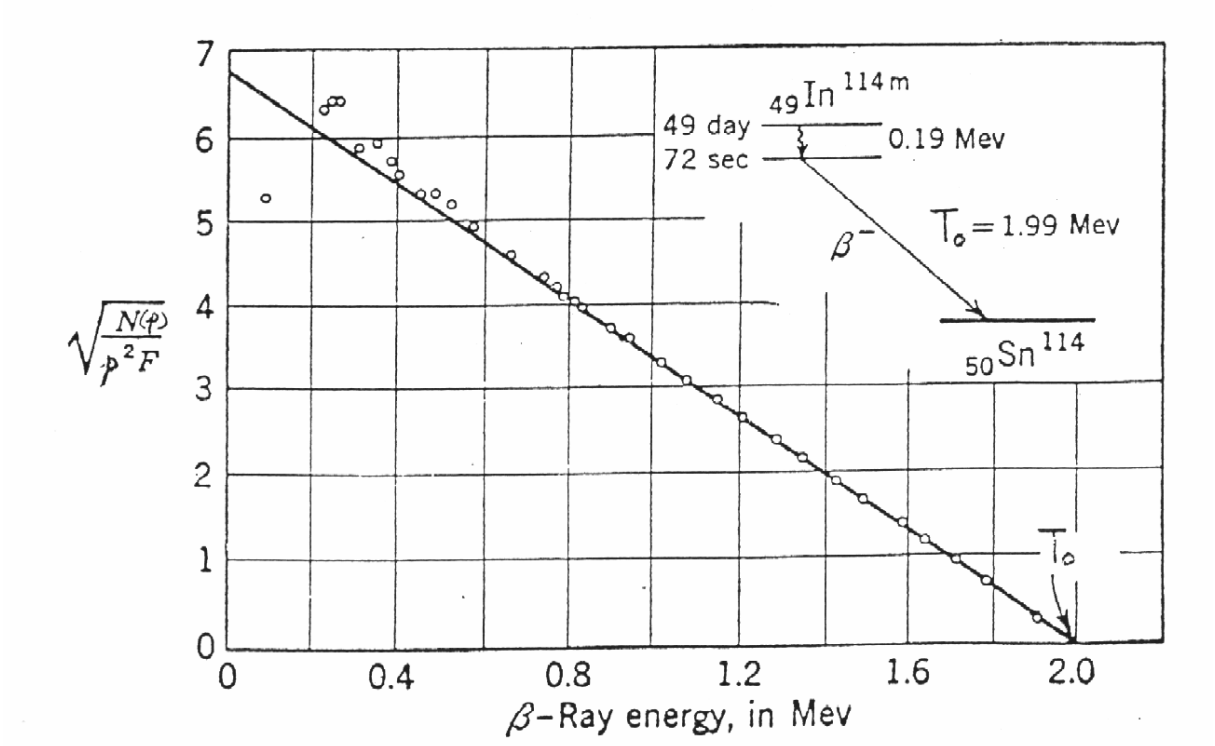

Fig. 4 Kurie plot of the β^- spectrum of ^{114}In. The inset shows the decay scheme.

Experimental method

1. Vacuum chamber

A drawing of the chamber used in this experiment is shown in Fig. 5. This arrangement is for the measurement of momentum spectrum. It contains a radioactive source of electrons (**S**) mounted at port $\mathbf{S_1}$, and a Si detector (**D**) used for energy determination. Port **V** is connected to a vacuum pump. A vacuum of ~10^{-2} Torr is good enough for this experiment. This can be achieved by means of a sorption pump or a mechanical vacuum pump. We prefer the sorption pump because it is much cleaner. Port **H** is a connector, which supports a Hall probe. The slits and baffle **B** are used to define the electron path. The chamber is supported by the poles of a standard variable-gap laboratory electromagnet. The source **S** can also be mounted at port $\mathbf{S_2}$ for direct measurement of energy spectrum.

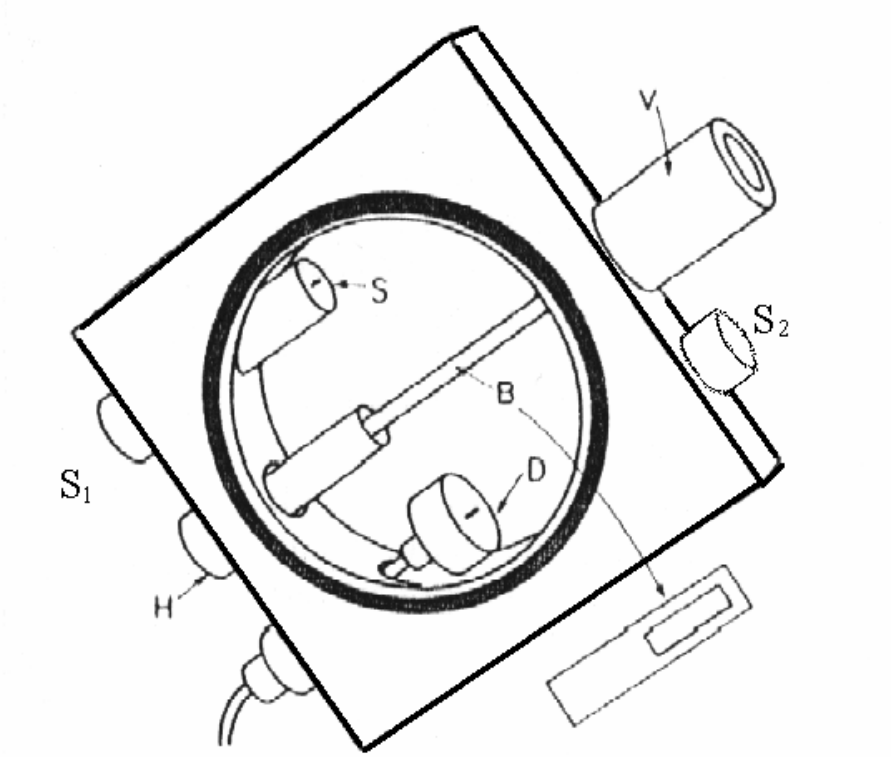

Figure 5 The vacuum chamber for the experiment.

2. β spectrometer for momentum measurement:

The vacuum chamber serves as a 180° beta-ray spectrometer. The electrons are bent by the applied magnetic field B. The geometry of 180° spectrometers offers the advantage of improved resolution which depends primarily on the dimensions of the collimators and baffle in Fig. 5. In our experiment, we place a slit (1 mm wide) in front of the source S. Electrons from the source having momentum p = eBr are selected to pass through the source slit and the baffle slit. The detector window is so small that a slit is not necessary. Because the electron charge e and path radius r are constant, momentum is varied simply by adjusting the magnetic field B of the electromagnet.

3. *Energy spectrometer for energy measurement*

The energy spectrometer consists of a Si detector, electronic components, and a multichannel analyzer (MCA). The depletion layer of the detector should be sufficiently thick to stop all β electrons. The energy spectrometer can be calibrated using radioactive sources of internal conversion electrons of known energy. Here the calibration source is mounted at port $\mathbf{S_2}$. To know more about internal conversion, refer to the manual on "Compton scattering and gamma spectroscopy". Some useful calibration sources are ^{137}Cs (624 keV), ^{133}Ba (266 and 319 keV), ^{207}Bi (480, 976, and 1047 keV) and ^{113}Sn (365 keV). Figure 6 shows the internal conversion spectrum for ^{137}Cs. The energy available for the decay is 661 keV. The 624 keV peak is due to the emission of the K shell electrons while the 656 keV peak is due to L shell electrons. Refer to the appendix of the manual on "Compton scattering and gamma spectroscopy". The full width at half maximum (FWHM) value of the calibration peaks is a measure of the energy resolution of the energy spectrometer.

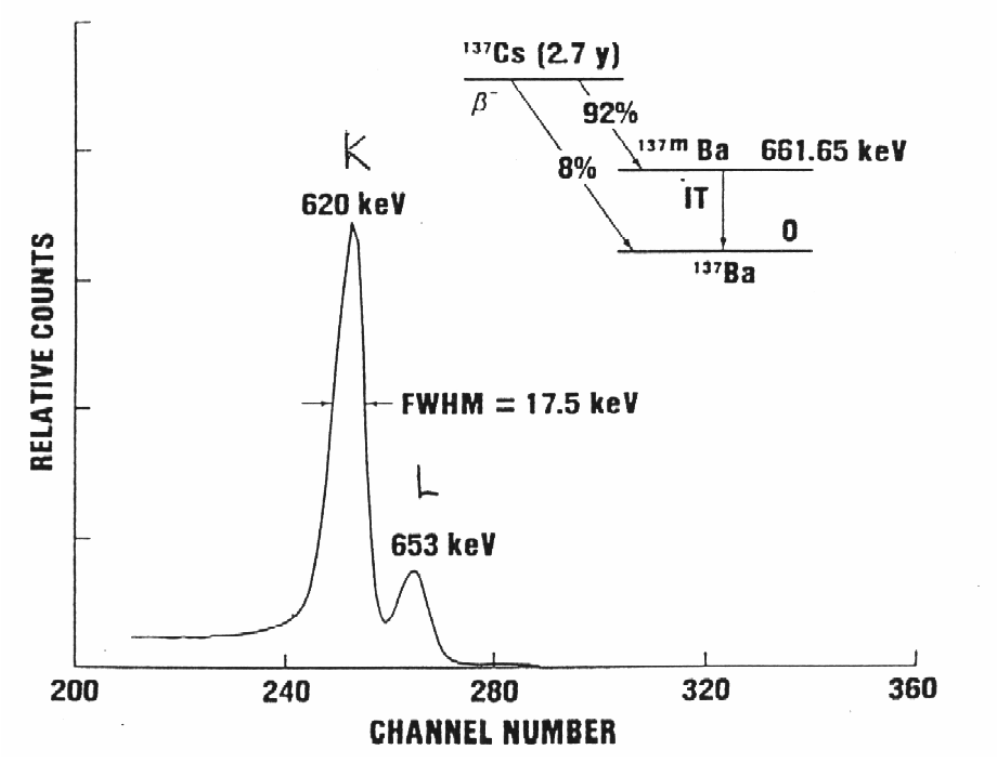

Figure 6 Internal conversion spectrum for ^{137}Cs.

4. *Source of relativistic electrons*

The source of relativistic electrons for this experiment is a ^{204}Tl beta source. We choose ^{204}Tl because it has a continuous spectrum with a high endpoint energy (766 keV) and is readily available. The continuous part of the ^{137}Cs beta spectrum is also a satisfactory source, although the endpoint energy is lower (514 keV). The ^{137}Cs gamma rays do not interfere the measurements because the detector is not sensitive to gamma rays.

Procedures

Setup: Inspect the setup. Identify the chamber, electromagnet and power supply, Hall sensor setup, Gaussmeter (Bell model 9640), source and detector plugs, pumps and fittings, power supply, preamplifier, amplifier, MCA, PC and pulse generator. Follow the cables to make sure that these modules are connected according to the schematic diagram shown in Fig. 2 of Appendix 3. We use a Si detector to measure β particles.

Note: ***Make sure that you turn off the bias voltage of the Si detector whenever you want to change samples. Apply the voltage only when the detector is in good vacuum.***

Part I. Momentum calibration

(1) We use a magnetic field to select electron momentum. The magnetic field B is measured with a calibrated Hall sensor at port **H**. The Hall sensor is already mounted on the chamber wall in such a way that its flat surface is normal to the applied magnetic field.

(2) To calibrate the Hall sensor (***in air***):
   (i) Set the Hall sensor current to 1 mA. Let it warm up for at least 10 minutes.
   (ii) Increase the electromagnet current to 1A.
   (iii) Remove the plug at port $\mathbf{S_1}$. Insert the probe of the Gaussmeter into the hole. Rotate the probe to achieve a maximum reading, from which you get the magnetic field B. Record the Hall voltage (V) of your Hall sensor.
   (iv) Repeat the steps for various electromagnet current.
   (v) Then find the calibration curve: $B = a (V - V_o)$, where a is a constant and V_o is the offset voltage.

(3) Now we can select the electron momentum simply by adjusting the magnet current. Here $p = eBr$.

(4) Measure the distance d (= 2r) between the source and the detector of the spectrometer.

Part II. (Kinetic) Energy calibration

(1) To calibrate the energy scale of the MCA, mount a ^{137}Cs source at port $\mathbf{S_2}$.

(2) Pump the chamber down to achieve a vacuum ~ 10^{-2} Torr. Read Appendix A to learn how to operate the sorption pump.

(3) Apply the required bias voltage to the Si detector and collect the energy spectrum of both β electrons and internal conversion electrons. Wait until you observe a clear conversion electron spectrum.

(4) Locate the 624 keV peak and write down its channel number (k_{624}).

(5) Identify all features of the spectrum. Save the data.

(6) ***Turn off the bias voltage. Close the sorption pump valve*** and then break the chamber vacuum.

(7) Remove the ^{137}Cs source and then connect the test input of the preamplifier to a pulse generator. Monitor the amplifier output with an oscilloscope.

(8) Set the "pulse height" of the pulse generator at 6.24. Then adjust the "attenuation factor" and the "normalize factor" until you observe a peak near k_{624} (the channel for the 624 keV peak as you observed in step (4)).

(9) Decrease the "pulse height" (H) of the pulse generator and observe the corresponding peak channel number (k). Plot H vs. k. Then get the off-set calibration curve:

$$H = b_H(k - k_o)$$

where b_H and k_o are constant.

(10) Now you can convert any MCA spectrum (counts vs. channel number) to an energy spectrum (counts vs. *kinetic* energy) using the energy calibration equation: $T = b\,(k - k_o)$ where $b = \dfrac{624}{k_{624} - k_o}$.

If you have other calibration sources, repeat steps (1)-(6) and thus find a more accurate value of b.

Part III. Energy spectrum

(1) Mount the ^{133}Ba source in front of the Si detector.
Pump the chamber down. Apply the required bias voltage to the Si detector. Then collect an energy spectrum. Identify all features of the spectrum.
Turn off the bias voltage. Close the sorption pump valve.

(2) Repeat measurement with a weak ^{204}Tl source.
Pump the chamber down. Apply the required bias voltage to the Si detector. Then collect an energy spectrum. Identify all features of the spectrum.
Turn off the bias voltage. Close the sorption pump valve.

(3) Remove the source and insert a blank plug to seal the chamber.

Part IV. Momentum spectrum

(1) Convert the energy spectrum into a momentum spectrum.

(2) The Kurie plot is optional.

Part V. Verification of special relativity

(1) A ^{204}Tl source is already mounted as shown in Fig. 5.
Pump the chamber down and then apply the required bias voltage to the Si detector.

(2) Select the electron momentum p by changing the magnet current. For each magnet current, measure the magnetic field B, calculate p and then collect an energy spectrum of these selected electrons. You will observe a peak. Determine its energy (T) using your calibration curve.

(3) Plot T vs. p.

(4) Verify equation [2] by plotting $\dfrac{p^2c^2}{2T}$ vs $\dfrac{T}{2}$,
with pc and T in units of keV.
You should get a straight line with slope = 1 and y-intercept = 511 keV.

References

1. J.G. Couch and T.K. Dorries, "Measuring relativistic electrons in the undergraduate laboratory", Am. J. Phys. 50, pp.917-921 (1982).
2. R.D. Evans, *The Atomic Nucleus* (New York: McGraw-Hill, 1955).

Useful webpages

MIT lab manual:
http://web.mit.edu/afs/athena/course/8/8.13/JLExperiments/JLExp_09.html
http://web.mit.edu/afs/athena/course/8/8.13/JLExperiments/JLExp_09.pdf

Appendix A Operation of sorption pump

Two vacuum pumps

Two pumps are available for this experiment: a mechanical pump and a sorption pump. The sorption pump operates by trapping gas at low temperature and is thus cleaner than the mechanical pump. Both pumps evacuate the chamber to 10^{-2} to 10^{-3} Torr. We will use liquid nitrogen as the cooler for the sorption pump.

Two valves

1. Mechanical-pump valve: the valve that connects to the mechanical pump.
2. Sorption-pump valve: the valve that connects to the sorption pump.

Prepare the sorption pump

1. Make sure that the bias voltage for the Si detector is off and the sorption-pump valve is closed. Open the mechanical-pump valve and close all other valves.
2. Turn on the mechanical pump and pump the system down to 10^{-1} Torr.
3. Open the sorption-pump valve and evacuate the sorption pump at room temperature.
4. Keep pumping until the system reaches 10^{-1} Torr again.
5. Close the sorption-pump valve and pour liquid nitrogen to the sorption pump container. After about 15 minutes, the sorption pump is ready for pumping.

Pump the chamber down

1. All the valves should be closed except the selected mechanical-pump valve. Turn on the mechanical pump and pump the chamber down to 10^{-1} Torr.
2. Close the mechanical-pump valve and open the sorption-pump valve. Make sure the sorption pump is still being cooled by liquid nitrogen. Now the chamber is being evacuated by the sorption pump.
3. Turn off the mechanical pump and immediately vent the mechanical pump to prevent the back-flow of mechanical pump oil.
4. Turn on the bias voltage to the Si detector when the pressure reaches 10^{-2} Torr.

Break the vacuum

1. Make sure that the bias voltage to the Si detector is turned off.
2. Close the sorption-pump valve.
3. Open the mechanical-pump valve (with the mechanical pump powered off).
3. Vent the chamber.

Appendix B Radioactive sources from Isotope Products Laboratories

BETA SOURCES (BFR SERIES)

These sources cover the range of energies from 18 to 3540 keV and provide a safe and convenient package for prototype studies, attentuation, and general laboratory use. The standard mount is in an A-2 capsule. Contained activity is supplied as a nominal value ±15%. A NIST traceable calibration of the contained value will be provided on request. Dose rate calibrations are also available on request. NIST traceability is not available for Ni-63.

STANDARD ACTIVITIES			
10 µCi	100 µCi	500 µCi	1 mCi
370 kBq	3.7 MBq	18.5 MBq	37 MBq

CATALOG NUMBER	NUCLIDE	HALF-LIFE	PRINCIPLE BETA EMISSIONS (E_{MAX} keV)	WINDOW
BFR-058	Co-58	70.9 d	2308 (β+)	1 mg/cm² polyimide (1)
BFR-063	Ni-63	100 y	66	None
BFR-147	Pm-147	2.63 y	225	1 mg/cm² polyimide
BFR-106	Ru-106 /Rh-106	1.02 y	39 3540	13.7 mg/cm² Al
BFR-022	Na-22	2.60 y	2842 (β+)	1 mg/cm² polyimide
BFR-090	Sr-90 /Y-90	28.5 y	546 2282	40 mg/cm² ss
BFR-099	Tc-99	2.13 x 10^5 y	294	None
BFR-204	Tl-204	3.78 y	763	13.7 mg/cm² Al

(1) A 9 mg/cm² polyimide cover is also available.

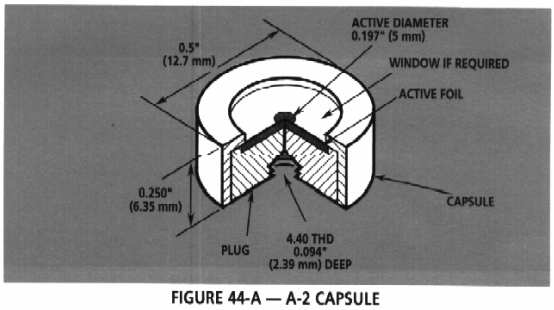

FIGURE 44-A — A-2 CAPSULE

MONO-ENERGETIC (CONVERSION) ELECTRON SOURCES (ME SERIES)

These sources are used as energy markers for the calibration of solid-state detectors and beta spectrometers. Sources are prepared by electrodeposition or evaporation of the radionuclide as a 0.2" (5.08 mm) diameter spot on very thin Mylar or nickel and protected by a 100-200 µg/cm² acrylic or gold coating. This assembly is stretched on a thin aluminum washer which in turn is mounted in an aluminum ring from which it can easily be removed (MF-1 mount).

Contained activity is supplied as a nominal value ±15%. A NIST traceable calibration of the contained value will be provided on request.

Sources will be supplied uncovered at the written request of the customer. However, the general warranty is invalid for uncovered sources.

STANDARD ACTIVITIES		
1 µCi	10 µCi	100 µCi
37 kBq	370 kBq	3.7 MBq

CATALOG NUMBER	NUCLIDE	HALF-LIFE	SUBSTRATE	METHOD OF PREPARATION	SIGNIFICANT ELECTRON ENERGIES (keV)
ME-133	Ba-133	10.5 y	Mylar (2)	Evaporation	5, 45, 75, 267, 320
ME-207	Bi-207	32.2 y	Nickel (3)	Electrodeposition	481, 975, 1047
ME-109	Cd-109	1.27 y	Nickel (3)	Electrodeposition	63, 85
ME-137	Cs-137	30 y	Mylar (2)	Evaporation	624, 656
ME-057	Co-57	272 d	Nickel (3)	Electrodeposition	7, 14, 115, 129
ME-113	Sn-113, In-113m (1)	115 d	Nickel (3)	Electrodeposition	20, 23, 364, 388
ME-119	Sn-119m	245 d	Nickel (3)	Electrodeposition	20, 23, 36, 62

(1) Sn-113 is available only as a 1 µCi (37 kBq) source.
(2) Mylar 0.9 mg/cm²
(3) Nickel 1-2 mg/cm²

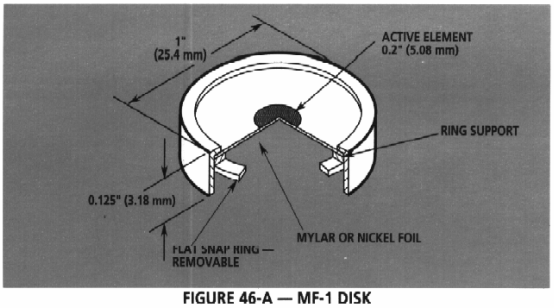

FIGURE 46-A — MF-1 DISK