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STZ Theory of Amorphous Plasticity

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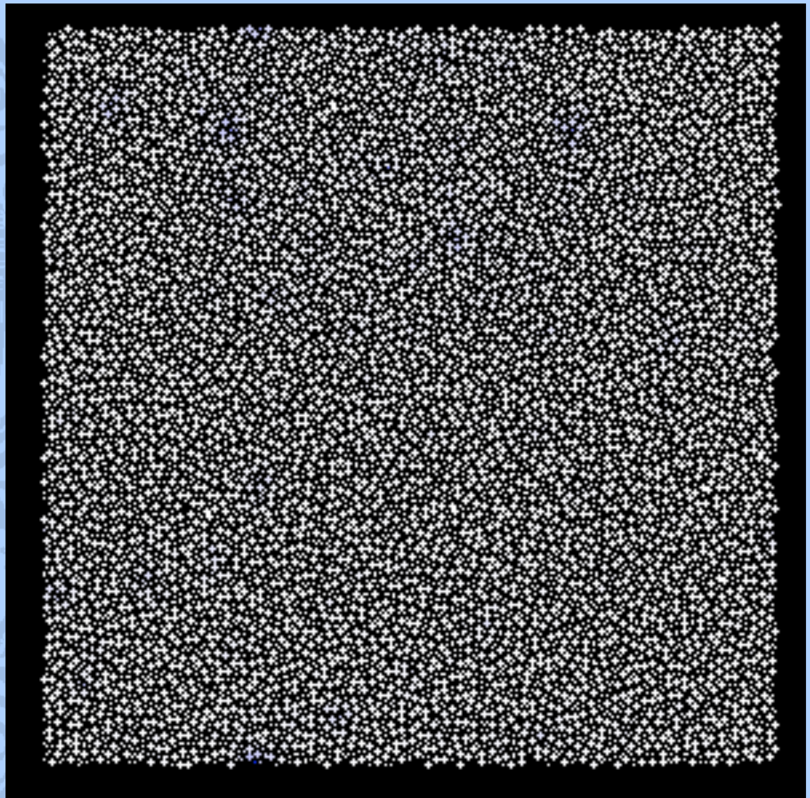
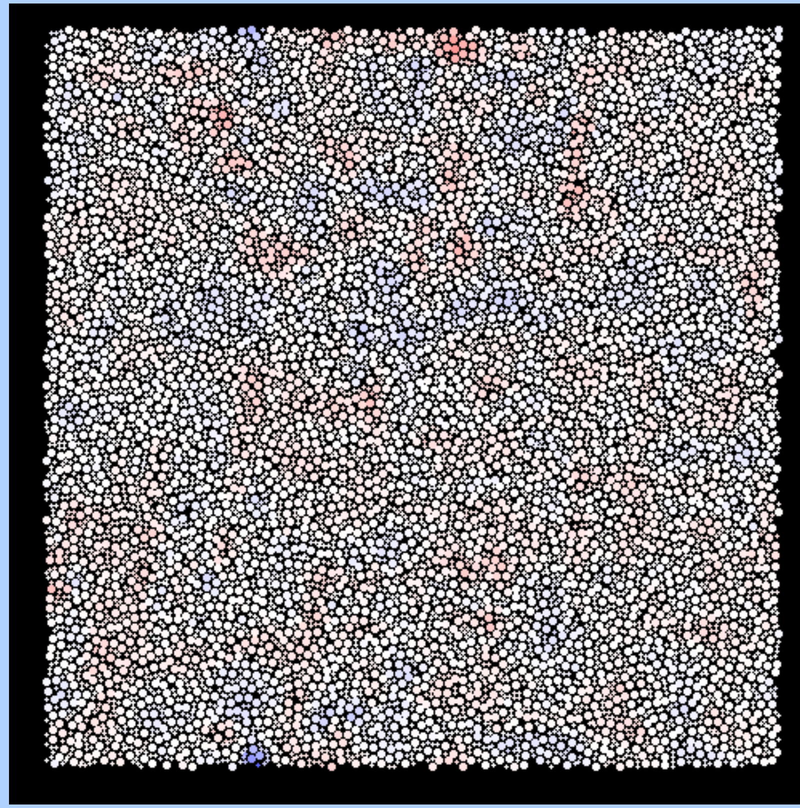
STZ Picture





STZ Picture

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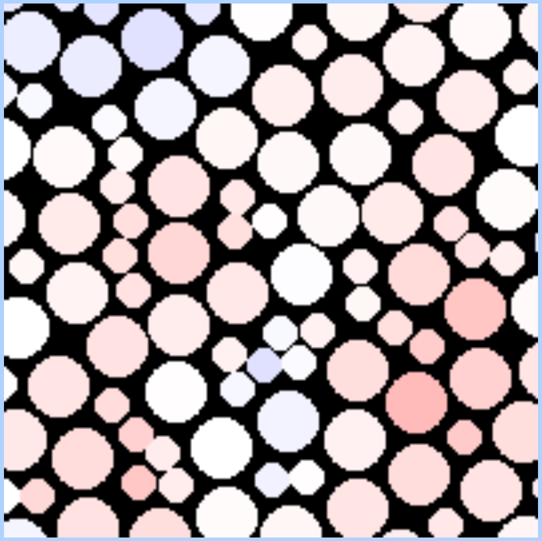
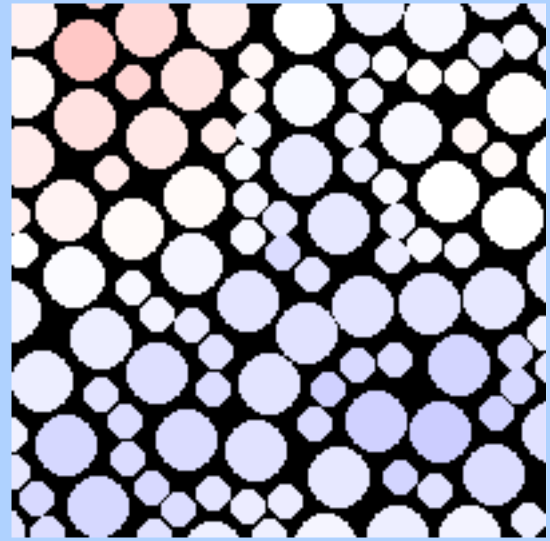
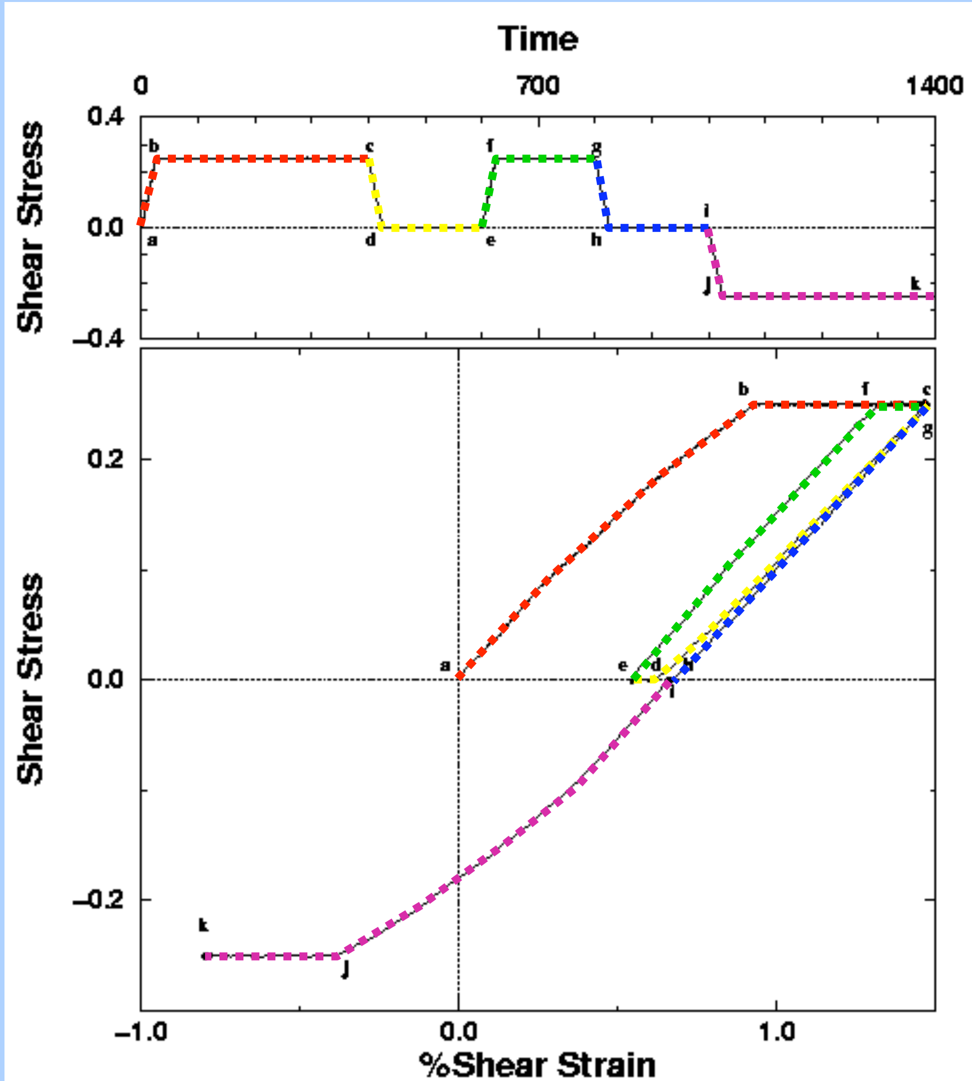




- STZs have a particular orientation. They are susceptible to shear to the extent that the shear is along this direction.
- STZs are reversible until their environment rearranges. They behave as 2-state systems.
- STZs are transient. They can be created and destroyed by neighboring plastic activity.

Homogeneous MD Simulations

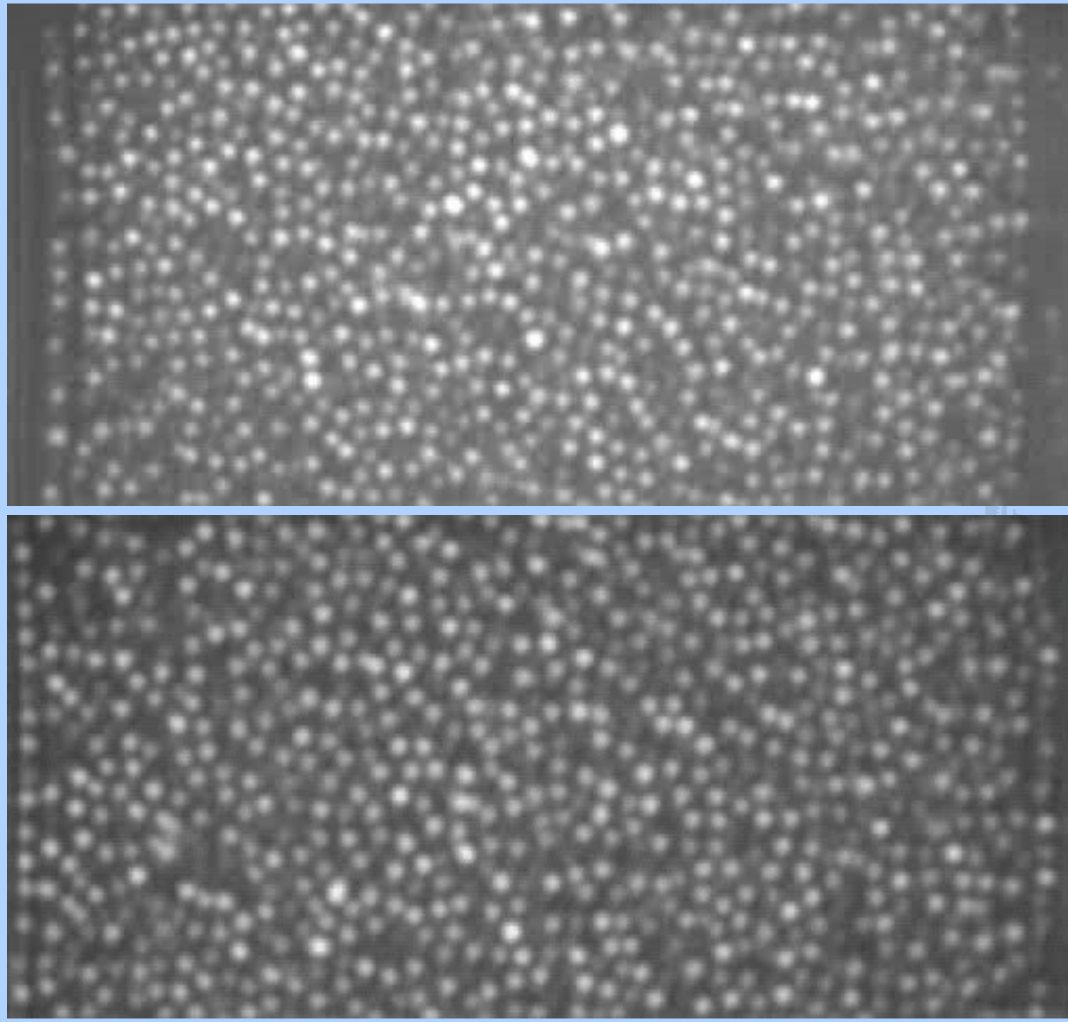


MLF and JS Langer, PRE, Vol. 57, pp. 7192-7205 (1998)

Shear Bands in Granular Media

(with W. Losert and M. Toyia)

- Taylor-Couette cell
- 102mm inner cylinder
- 44mm gap
- 1mm beads
  or 2mm beads
- Inner cylinder rotated 4-8 mm/s
- Top surface monitored with high speed camera
- Torque measured at inner cylinder



MLF, M. Toiya, W. Losert, arxiv:0802.1752 (2008)

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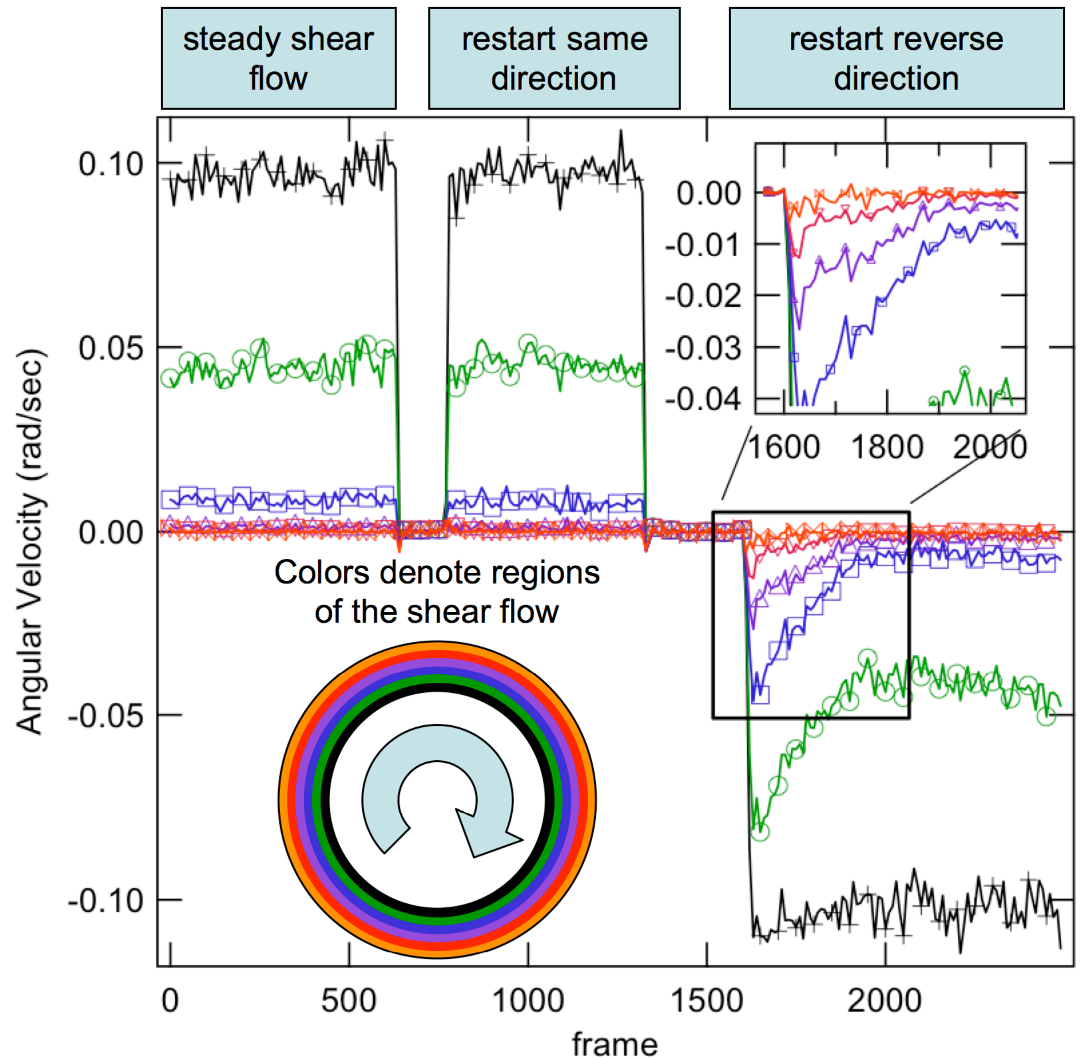

MLF, M. Toiya, W. Losert, arxiv:0802.1752 (2008)

STZ Model: Transient granular flow

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- **The physics of this system is much different:**
  - No internal time scale, requires everything to be slaved to the motion of the inner cylinder. So a‘ denotes da/dΘ, where Θ is the angle of the inner cylinder.
  - The stress is inhomogeneous $\sigma_{r\theta}(r)$.
  - We can assume that through cyclic loading the dilational degrees of freedom, described by χ, are in steady state.
  - However, during the transient following reversal the orientational degrees of freedom, described by m, are not.

$$\varepsilon'_{r\theta}\left(s_{r\theta},m_{r\theta}\right) = e^{-1/\chi_\infty} f\left(s_{r\theta},m_{r\theta}\right) = \gamma\varepsilon_0 C\left(\frac{s_{r\theta}}{s_y}\right)\left[\operatorname{sign}\left(s_{r\theta}\right) - m\right]\operatorname{sign}\left(\frac{d\Theta}{dt}\right)$$

$$m'\left(\tilde{s},m\right) = \frac{\varepsilon'_{r\theta}\left(s_{r\theta},m\right)}{\varepsilon_0}\left(1 - \frac{s_{r\theta}m}{s_y}\right)$$

MLF, M. Toiya, W. Losert, arxiv:0802.1752 (2008)

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$$m'\left(\tilde{s},m\right)=\frac{\varepsilon'_{r\theta}\left(s_{r\theta},m\right)}{\varepsilon_0}\left(1-\frac{s_{r\theta}m}{s_y}\right)$$

- **We must also define the function C**
  Bouchbinder, Procaccia and Langer, PRE 75, 036107 (2007)

$$C\left(\tilde{s};\zeta\right)=\frac{\left|\tilde{s}\right|^{\zeta+1}}{\sqrt{2\pi\zeta}}\exp\left[-\zeta\left(\left|\tilde{s}\right|-1\right)\right]+\left(\left|\tilde{s}\right|-1-\zeta^{-1}\right)P\left(\zeta+1,\zeta\left|\tilde{s}\right|\right)$$

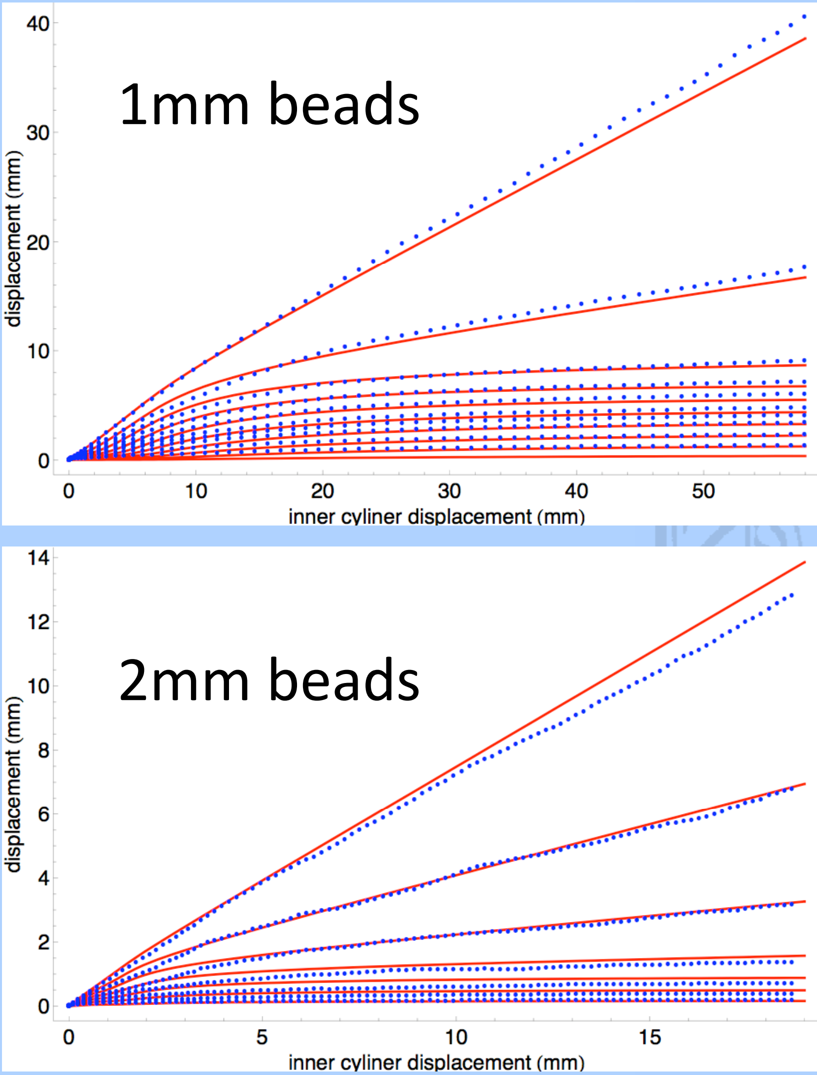
- **Now 4 parameters must be specified**
  - The radius, r_y, at which $s=s_y$.
    (116mm for 1mm beads, 127mm for 2mm beads)
  - The stress distribution of the STZs, ζ . (100)
  - The strain per STZ , ε_0. (4% for 1mm beads, 0.5% for 2mm beads)
  - γ, the attempt frequency per rate of rotation of the inner cylinder, $d\Theta/dt$. (18000)

MLF, M. Toiya, W. Losert, arxiv:0802.1752 (2008)

Comparison to Experimental Data

- The blue dots represent experimental measurements of displacement at a specified radial position, plotted as a function of the inner cylinder displacement subsequent to shear reversal.

- The red lines are the STZ predictions given the assumptions on the previous slides.



MLF, M. Toiya, W. Losert, arxiv:0802.1752 (2008)

Stress vs. Time (blue=data, red=theory)

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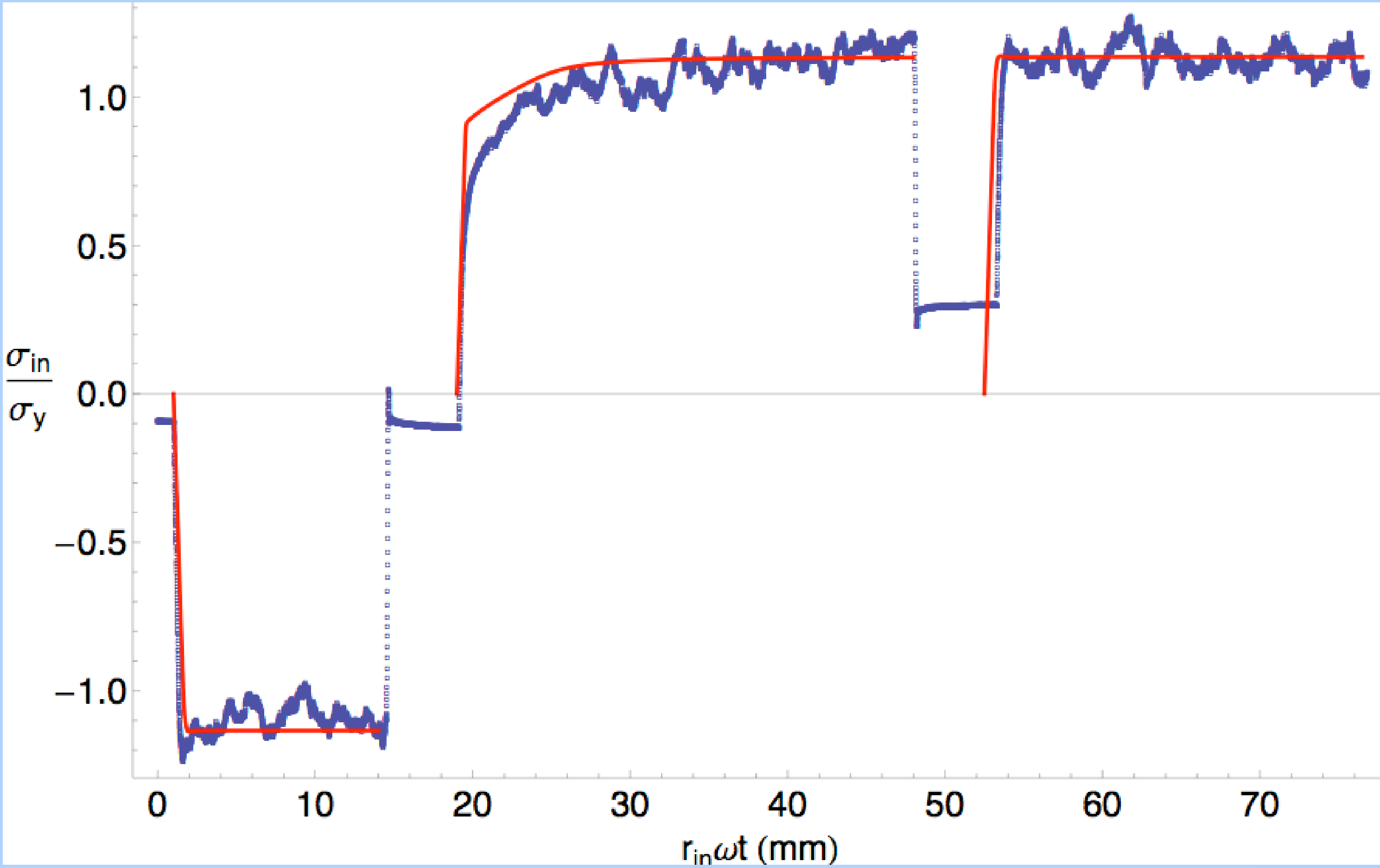
NB: time is multiplied by displacement rate of inner cylinder



MLF, M. Toiya, W. Losert, arxiv:0802.1752 (2008)

Granular Shear Bands

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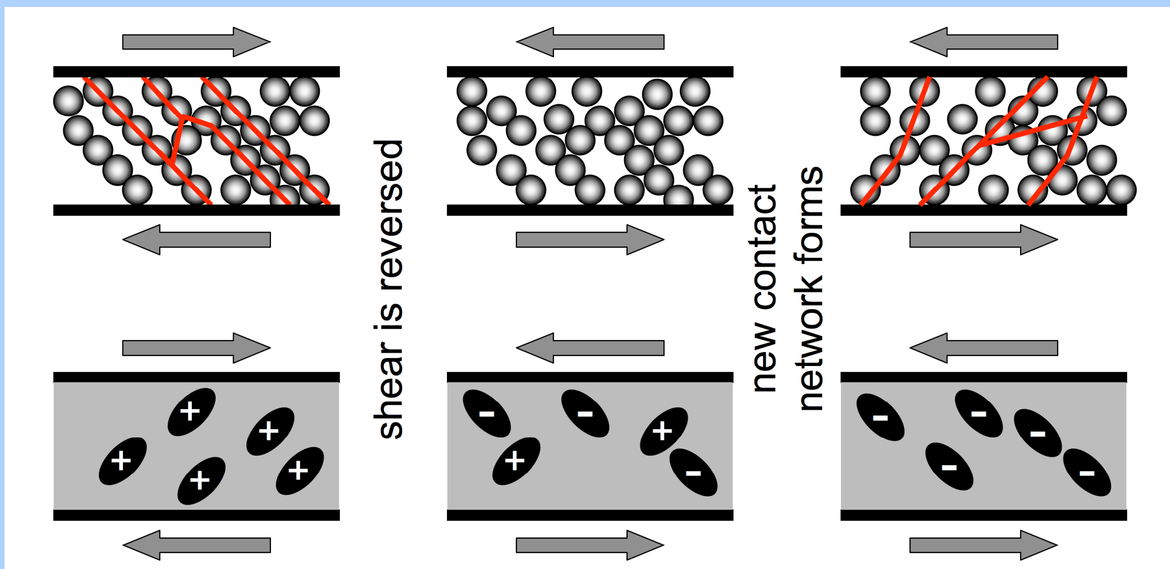
- Shear bands observed a granular medium in a Taylor-Couette cell arise due to a different mechanism than the shear bands in the metallic glass.
- These shear bands arise due to the inhomogeneous stress field, which is higher at the inner cylinder.
- The transient broadening of the shear band subsequent to shear reversal can be understood via the dynamics of the m parameter in the STZ theory, which describes STZ orientation.



MLF, M. Toiya, W. Losert, arxiv:0802.1752 (2008)

Simulations in Simple Shear (2D)

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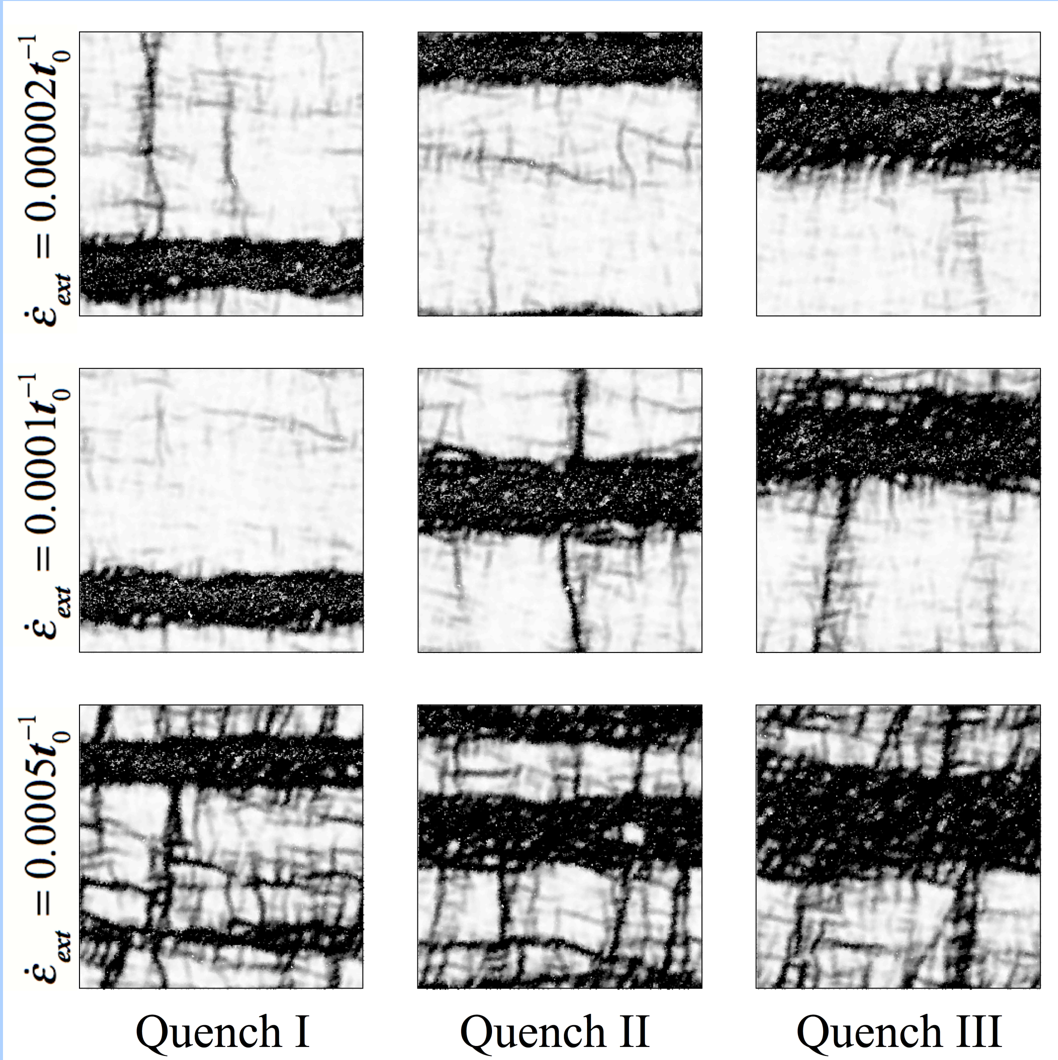
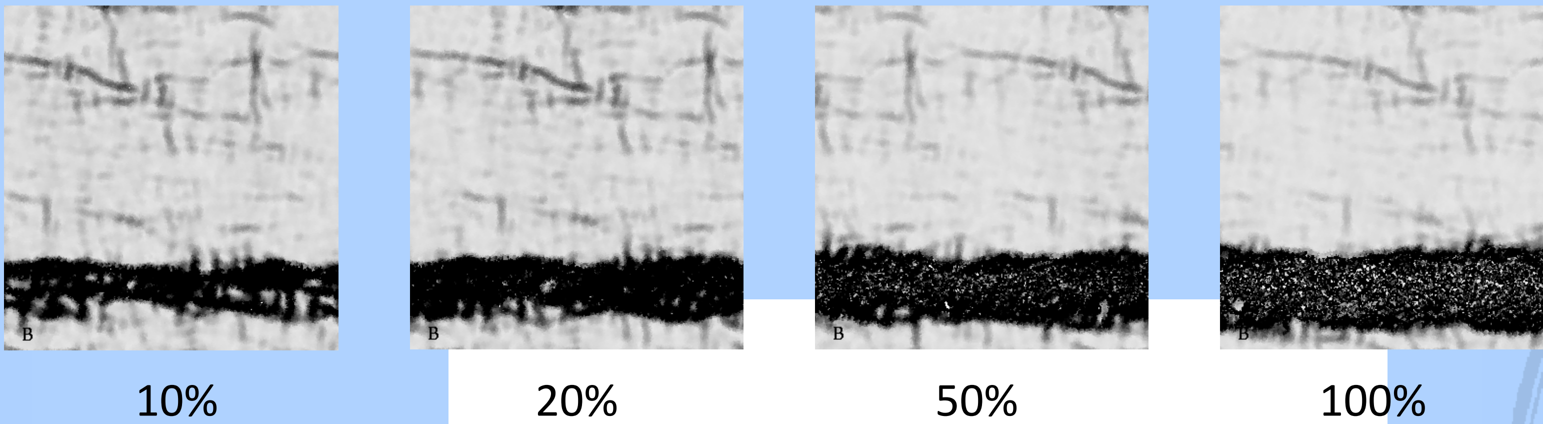
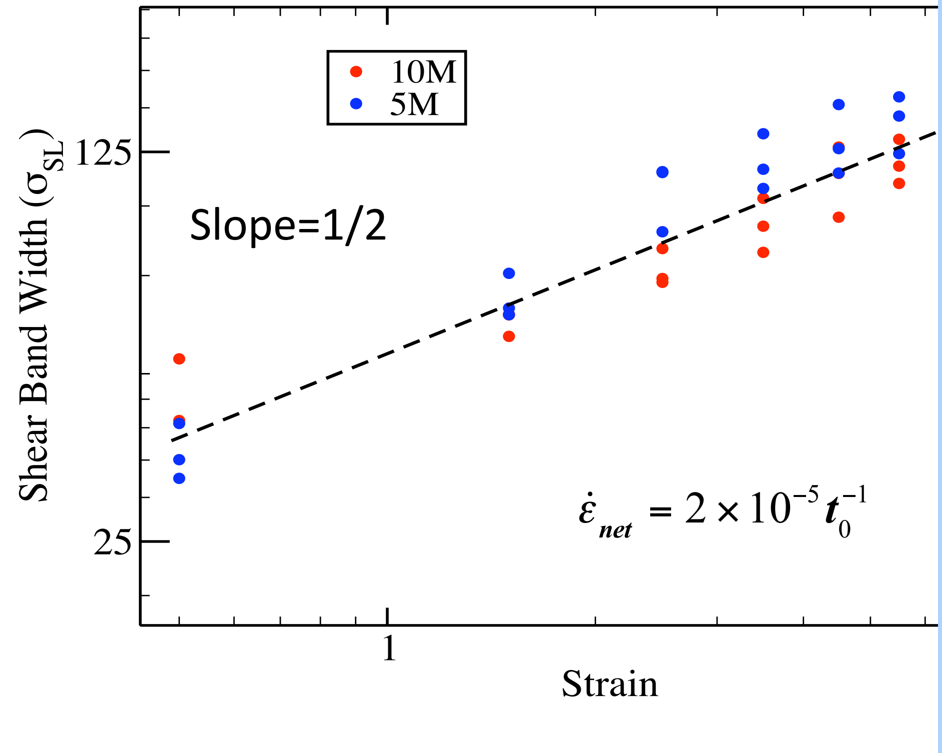


Cumulative strain up to 50% macroscopic shear

Y Shi, MB Katz, H Li, MLF, PRL, 98, 185505 (2007)

2D Simple Shear: Broadening





Shear Band MD Simulations

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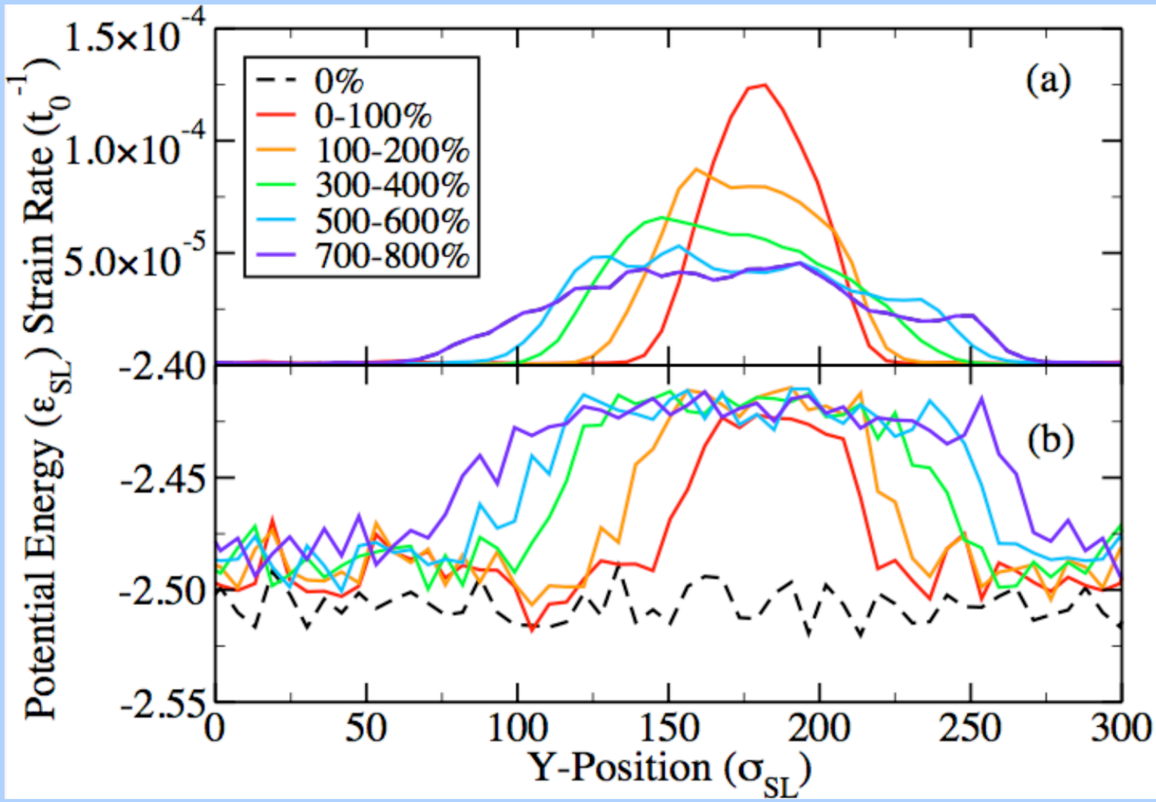
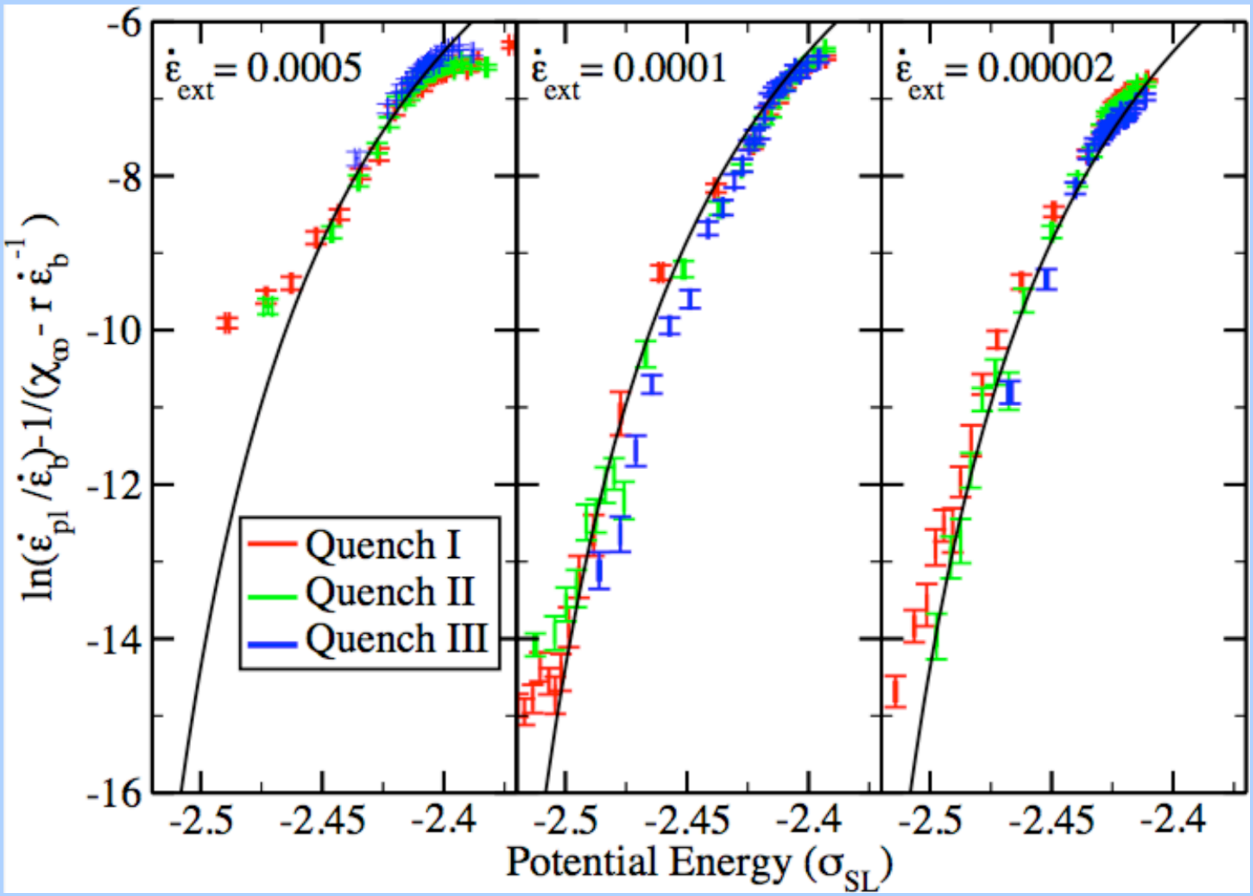


Scaling Results



Numerical Results

M Lisa Manning and JS Langer, PRE, 76, 056106(2007)

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- These equations closely reproduce the details of the strain rate and structural profiles during band formation