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MATH 2230 Complex Variables with Applications
Homework 2
(due on Sept. 19)

Sect. 18 No. 5, 10, 11
Sect. 20 No. 8. 9
Sect. 24 No. 2

Sect. 18 Exercises

5. Show that the function
$$f(z) = \left(\frac{z}{\bar{z}}\right)^2$$
has the value 1 at all nonzero points on the real and imaginary axes, where $z=(x,0)$ and $z=(0,y)$, respectively, but that it has the value -1 at all nonzero points on the line $y=x$, where $z=(x,x)$. Thus show that the limit of $f(z)$ as z tends to 0 does not exist. [Note that it is not sufficient to simply consider nonzero points $z=(x,0)$ and $z=(0,y)$, as it was in Example 2, Sec. 15.]

10. Use the theorem in Sec. 17 to show that

(a) $\lim\limits_{z\to\infty} \dfrac{4z^2}{(z-1)^2} = 4$ (b) $\lim\limits_{z\to 1} \dfrac{1}{(z-1)^3} = \infty$ (c) $\lim\limits_{z\to\infty} \dfrac{z^2+1}{z-1} = \infty$

11. With the aid of the theorem in Sec. 17, show that when
$$T(z) = \frac{az+b}{cz+d} \quad (ad-bc \neq 0)$$
(a) $\lim\limits_{z\to\infty} T(z) = \infty$ if $c=0$;
(b) $\lim\limits_{z\to\infty} T(z) = \dfrac{a}{c}$ and $\lim\limits_{z\to -d/c} T(z) = \infty$ if $c \neq 0$

Sect. 20 Exercises

8. Use the method in Example 2, Sec. 19, to show that $f'(z)$ does not exist at any point z when

(a) $f(z) = \operatorname{Re} z$ (b) $f(z) = \operatorname{Im} z$

9. Let f denote the function whose values are

$$f(z) = \begin{cases} \overline{z}^2/z & \text{when} \quad z \neq 0, \\ 0 & \text{when} \quad z = 0. \end{cases}$$

Show that if $z = 0$, then $\Delta w/\Delta z = 1$ at each nonzero point on the real and imaginary axes in the Δz, or $\Delta x\ \Delta y$, plane. Then show that $\Delta w/\Delta z = -1$ at each nonzero point $(\Delta x, \Delta x)$ on the line $\Delta y = \Delta x$ in that plane (Fig. 29). Conclude from these observations that $f'(0)$ does not exist. Note that to obtain this result, it is not sufficient to consider only horizontal and vertical approaches to the origin in the Δz plane. (Compare with Exercise 5, Sec. 18, as well as Example 2, Sec. 19.)

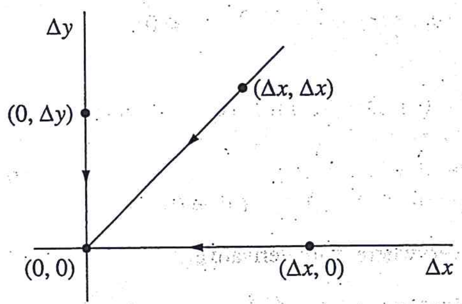

FIGURE 29

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Sect. 24 Exercises

2. Use the theorem in Sec. 23 to show that $f'(z)$ and its derivative $f''(z)$ exist everywhere and find $f''(z)$ when

(a) $f(z) = iz + 2$ (b) $f(z) = e^{-x}e^{-iy}$

(c) $f(z) = z^3$ (d) $f(z) = \cos x \cosh y - i \sin x \sinh y$

Ans. (b) $f''(z) = f(z)$; (d) $f''(z) = -f(z)$